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A MODIFIED ITERATIVE APPROACH FOR SOLVING LINEAR FRACTIONAL-ORDER DELAY DIFFERENTIAL EQUATIONS

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Abstract

This paper explores the application of the Modified New Iterative Method (MNIM) for solving linear fractional-order delay differential equations (FDDEs). The method is assessed through illustrative example, showcasing its effectiveness in producing accurate approximations for linear case, particularly when the fractional order approaches an integer. MNIM demonstrates strong performance in solving equations of integer and near-integer fractional order. However, the accuracy declines as the fractional order moves further from an integer, especially over larger intervals. MNIM remains a powerful and adaptable method for handling a broad spectrum of fractional differential equations involving delays.

Keywords: Modified New Iterative Method, Fractional-Order Delay Differential Equations

INTRODUCTION

This section presents an overview of existing methods for solving fractional delay differential equations (FDDEs), outlines the research problem and motivation, identifies the gap in the literature, and defines the aim, objectives, scope, and limitations of the study.

Fractional delay differential equations (FDDEs) involve both fractional derivatives and time-delay components. Unlike integer-order derivatives, fractional derivatives possess non-local properties that account for memory and hereditary effects, while time delays incorporate historical states into system dynamics. The combined use of fractional derivatives and delay terms enhances the modeling fidelity of complex real-world phenomena. FDDEs are extensively applied across various domains, including physics, chemistry, control systems, electrochemistry, bioengineering, and population dynamics (Jhinga & Daftardar-Gejji, 2019; Srivastava, 2020; Du, 2022; El-Kalla et al., 2019; Avcı, 2022). In bioengineering, for instance, fractional modeling enables improved understanding of the dynamic behavior of biological tissues and proves valuable in imaging applications such as nuclear magnetic resonance (NMR) and magnetic resonance imaging (MRI) of complex materials.

However, the non-local nature of fractional derivatives presents significant computational challenges. As such, the development of accurate, computationally efficient, and time-effective numerical methods for solving nonlinear FDDEs has become a central focus of research. Several numerical approaches have been proposed in this regard. Notably, Diethelm et al. (2002, 2004) extended the classical Adams-Bashforth scheme to formulate the fractional Adams method (FAM) for fractional differential equations (FDEs). This method was further adapted by Bhalekar and Daftardar-Gejji (2011a) to handle delay terms in FDEs. The Numerical Predictor-Corrector Method (NPCM), based on the Daftardar-Gejji and Jafari (DGJ) framework (2006), has also been developed to solve FDEs (Daftardar-Gejji et al., 2014), and later extended to FDDEs (Daftardar-Gejji et al., 2015), offering improved time efficiency.

More recently, a hybrid technique the Banach Contraction Method (BCM) was introduced by Kumar and Methi (2021), integrating the Banach contraction framework with the New Iterative Method (NIM) proposed by Daftardar-Gejji and Jafari (2006). This approach has demonstrated notable improvements in accuracy and computational speed over traditional methods such as FAM and NPCM.

Building upon this foundation, the present study aims to modify the New Iterative Method (NIM) and apply it to solve FDDEs more effectively. Despite the inherent time complexity associated with fractional operators, their ability to model memory effects remains critical for capturing the dynamics of natural systems. The proposed Modified New Iterative Method (MNIM) seeks to overcome limitations of existing techniques including NIM, VIM, FAM, and NPCM by offering rapid convergence, reduced computational cost, and enhanced solution accuracy. Ultimately, this method aspires to serve as a superior numerical tool for addressing both linear and nonlinear FDDEs with greater precision and efficiency.

METHODOLOGY

Exploring the Fundamentals of the New Iterative Method (NIM)

In order to clarify the underlying principles of the initial approach in the New Iterative Method (NIM), one can draw insights from a well-established functional equation found in the works of Daftardar-gejji & Bhalekar (2010), Ramadan & Al-luhaibi (2015), Moltot & Deresse, (2022) and Ashitha & Ranjini (2020). This perspective begins with the examination of the non-linear functional equation introduced by Daftardar-gejji & Jafari, (2006).

$$y(x) = g(x) + N[y(x)] \qquad ...(1)$$

In this context, N represents the non-linear operator, and g is a known function. Our objective is to find a solution, denoted as $y(x)$, which possesses a series representation in the following format:

$$y = \sum_{i=0}^{\infty} y_i \, . \qquad ...(2)$$

The nonlinear operator N can be decomposed as

$$N\left(\sum_{i=0}^{\infty} y_i\right) = N(y_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} y_j\right) - N\left(\sum_{j=0}^{i-1} y_j\right)\right\}. \qquad ...(3)$$

From Eqns. (2) and (3), Eqn. (1) is equivalent to

$$\sum_{i=0}^{\infty} y_i = g + N(y_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} y_j\right) - N\left(\sum_{j=0}^{i-1} y_j\right)\right\}. \qquad ...(4)$$

We define the recurrence relation:

$$\begin{cases} y_0 = g, \\ y_1 = N(y_0) \\ y_{m+1} = N(y_0 + \ldots + y_m) - N(y_0 + \ldots + y_{m-1}), m = 1,2,\ldots \end{cases} \quad \ldots(5)$$

Then

$$(y_1 + \ldots + y_{m+1}) = N(y_0 + \ldots + y_m), m = 1,2,\ldots \quad \ldots(6)$$

and

$$y = g + \sum_{i=0}^{\infty} y_i \, . \quad \ldots(7)$$

and the series $\sum_{i=0}^{\infty} y_i$ absolutely and uniformly converges to a solution of Eqn. (1).

Suitable Algorithm for Fractional Retarded Delay Differential Equation

In this section, we introduce a suitable algorithm for solving retarded Fractional Delay Differential Equations using the proposed New Iterative Method (NIM). Consider the following Fractional Delay Differential Equations:

$$\begin{cases} D^{\alpha} y(x) + Ly(x) + N[y(x-t)] = g(x), \; x > 0, \\ y^{(i)} = \delta_i, \qquad i = 0,1,2,\ldots \end{cases} \quad \ldots(8)$$

where L is a linear operator, N, represent a nonlinear operator, $g(x)$ is the source term, and D^{α} is the Caputo fractional derivative of order with $m-1<\alpha-m$. To solve Eqn. (8) by means of the proposed modification of the NIM, we apply the operator J^{α}, the inverse of the operator D^{α}, to both sides of Eqn. (8) as follows:

$$y(x) = \sum_{i=0}^{m-1} y^{(i)}(0) \frac{x^i}{i!} + J^{\alpha}[-Ly(x) - Ny(x-t) + g(x)]. \quad \ldots(9)$$

Let's consider dividing this equation into two separate parts as follows:

$$y(x) = N(y(x)) + \sum_{i=0}^{m-1} y^{(i)}(0) \frac{x^i}{i!} \quad \ldots(10)$$

where

$$N(y(x)) = J^{\alpha}[g(x) - Ly(x) - Ny(x-t)] \qquad ...(11)$$

In our quest for a solution to Eqn. (10), we seek a representation in the form of a series:

$$y(x) = \sum_{i=0}^{\infty} y_i(x). \qquad ...(12)$$

The operator N can be decomposed into the following

$$N\left(\sum_{i=0}^{\infty} y_i\right) = N(y_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} y_j\right) - N\left(\sum_{j=0}^{i-1} y_j\right)\right\}. \qquad ...(13)$$

From Eqns. (10), (12) and Eqn. (13)

$$\sum_{i=0}^{\infty} y_i = \sum_{i=0}^{m-1} y^{(i)}(0)\frac{x^i}{i!} + N(y_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} y_j\right) - N\left(\sum_{j=0}^{i-1} y_j\right)\right\}. \qquad ...(14)$$

We define the recurrence relation:

$$y_0 = \sum_{i=0}^{r} y^{(i)}(0)\frac{x^i}{i!}, \qquad ...(15)$$

$$y_1 = J^{\alpha}\left[g(x) - Ly_0(x) - Ny_0(x-t)\right]$$

...(16)

$$\begin{cases} y_2 = N(y_0 + y_1) - N(y_0) \\ = J^{\alpha}[g(x) - Ly_0(x) - Ny_0(x-t) - Ly_1(x) - Ny_1(x-t)] \\ - J^{\alpha}[g(x) - Ly_0(x) - Ny_0(x-t)] = \\ J^{\alpha}[-Ly_1(x) - Ny_1(x-t)] \end{cases} \qquad ...(17)$$

$$\begin{cases} y_3 = N(y_0 + y_1 + y_2) - N(y_0 + y_1) = \\ J^{\alpha}[g(x) - Ly_0(x) - Ny_0(x-t) - Ly_1(x) - Ny_1(x-t) - Ly_2(x) - Ny_2(x-t)] \\ - J^{\alpha}[g(x) - Ly_0(x) - Ny_0(x) - Ly_1(x) - Ny_1(x-t)] = \\ J^{\alpha}[-Ly_2(x) - Ny_2(x-t)] \end{cases} \qquad ...(18)$$

$$\begin{cases} y_4 = N(y_0 + y_1 + y_2 + y_3) - N(y_0 + y_1 + y_2) = \\ J^{\alpha}[g(x) - Ly_0(x) - Ny_0(x-t) - Ly_1(x) - Ny_1(x) - Ly_2(x) - Ny_2(x-t) - Ly_3(x) - Ny_3(x-t)] \\ - J^{\alpha}[g(x) - Ly_0(x) - Ny_0(x-t) - Ly_1(x) - Ny_1(x-t) - Ly_2(x) - Ny_2(x-t)] = \\ J^{\alpha}[-Ly_3(x) - Ny_3(x-t)] \end{cases} \quad ...(19)$$

Then k-term series solution will be in the form

$$y(x) = y_0 + y_1 + y_2 + y_3 + y_4 + y_5 + ... \quad ...(20)$$

From above Eqns., we can deduce the following:

$$\begin{cases} y_0 = \sum_{i=0}^{r} y^{(i)}(0)\frac{x^i}{i!} \\ y_1 = J^{\alpha}\left[g(x) - Ly_0(x) - Ny_0(x-t)\right] \\ y_2 = J^{\alpha}[-Ly_1(x) - Ny_1(x-t)] \\ y_3 = J^{\alpha}[-Ly_2(x) - Ny_2(x-t)] \\ \vdots \end{cases} \quad ...(21)$$

Effective Algorithm for Solving Nonlinear Fractional Delay Differential Equations

In this section, we present an appropriate algorithm for solving Nonlinear Fractional Delay Differential Equations (FDDEs) using the proposed New Iterative Method (NIM). Consider the following form of Nonlinear Fractional Delay Differential Equations:

$$\begin{cases} D^{\alpha} y(x) + Ly(x) + N\left[y\left(\frac{x}{2}\right)\right] = g(x), \quad x > 0, \\ y^{(i)} = \delta_i, \qquad i = 0,1,2,... \end{cases} \quad ...(22)$$

where L is a linear operator, N , represent a nonlinear operator, $g(x)$ is the source term, and D^{α} is the Caputo fractional derivative of order with $m-1 < \alpha - m$. To solve Eqn. (22) by means of the proposed modification of the NIM, we apply the operator J^{α} , the inverse of the operator D^{α} , to both sides of Eqn. (22) as follows:

$$y(x) = \sum_{i=0}^{m-1} y^{(i)}(0)\frac{x^i}{i!} + J^{\alpha}\left[-Ly(x) - Ny\left(\frac{x}{2}\right) + g(x)\right]. \quad ...(23)$$

Let's consider dividing this equation into two separate parts as follows:

$$y(x) = N(y(x)) + \sum_{i=0}^{m-1} y^{(i)}(0)\frac{x^i}{i!} \qquad \text{...(24)}$$

where

$$N(y(x)) = J^{\alpha}\left[g(x) - Ly(x) - Ny\left(\frac{x}{2}\right)\right] \qquad \text{...(25)}$$

In our quest for a solution to Eqn. (24), we seek a representation in the form of a series:

$$y(x) = \sum_{i=0}^{\infty} y_i(x). \qquad \text{...(26)}$$

So that, the components y_i will be determined recursively. Moreover the method defines the nonlinear terms $Ny\left(\frac{x}{2}\right)$ by the El-kalla polynomials:

$$Ny\left(\frac{x}{2}\right) = \sum_{n=0}^{\infty} \overline{A}_n \qquad \text{...(27)}$$

where $\overline{A}_n$ are the El-kalla polynomials that can be generated for all forms of nonlinearity as:

$$\overline{A}_n = f(S_n) - \sum_{i=0}^{n-1} A_i \qquad \text{...(28)}$$

where $\overline{A}_n$, are the El-kalla polynomials, $\overline{A}_0$, $\overline{A}_1$, $\overline{A}_2$,…, $f(S_n)$ is the substitution of the summation of dependent variable in the nonlinear term. For example the El-Kalla polynomials of the nonlinear term $y^2(x)$ and the nonlinear term $y^3(x)$ are shown above

El-Kalla polynomials of $y^2(x)$

$$\begin{cases} \overline{A}_0 = y_0^2(x) \\ \overline{A}_1 = 2y_0(x)y_1(x) + y_1^2(x) \\ \overline{A}_2 = 2y_0(x)y_2(x) + 2y_1(x)y_2(x) + y_2^2(x) \\ \overline{A}_3 = 2y_0(x)y_3(x) + 2y_1(x)y_3(x) + 2y_2(x)y_3(x) + y_3^2(x) \\ \overline{A}_4 = 2y_0(x)y_4(x) + 2y_1(x)y_4(x) + 2y_2(x)y_4(x) + 2y_3(x)y_4(x) + y_4^2(x) \end{cases} \qquad \text{...(29)}$$

El-Kalla polynomials of $y^3(x)$

$$\begin{cases} \overline{A}_0 = y_0^3(x) \\ \overline{A}_1 = 3y_1(x)y_0^2(x) + 3y_0(x)y_1^2(x) + y_1^3(x) \\ \overline{A}_2 = 3y_2y_0^2 + 6y_0y_1y_2 + 3y_2y_1^2 + 3y_1y_2^2 + y_2^3 \\ \overline{A}_3 = 3y_3y_0^2 + 6y_0y_1y_3 + 3y_0y_3^2 + 3y_3y_1^2 + 6y_1y_2y_3 + 3y_1y_3^2 + 3y_3y_2^2 + y_3^3 \end{cases} \quad ...(30)$$

Substituting Eqns. (26) and (27) into Eqn. (23) gives:

$$\sum_{i=0}^{\infty} y_i = \sum_{i=0}^{m-1} y^{(i)}(0)\frac{x^i}{i!} + j^{\alpha}\left(\sum_{n=0}^{\infty} \overline{A}_n\right). \quad ...(31)$$

To determined $y_i(x)$ $i \geq 0.$ first we identify the zero component $y_0(x)$ by the terms $\sum_{i=0}^{m-1} y^{(i)}(0)\frac{x^i}{i!}$ and $j^{\alpha}[g(x)]$ where $g(x)$ represents the inhomogeneous terms. Secondly, the remaining components of $y(x)$ can be determined in a way such that each component is determined by using the preceding components. In other words, the method introduces the recursive relation:

$$y_0(x) = \sum_{i=0}^{r} y^{(i)}(0)\frac{x^i}{i!} + j^{\alpha}[f(x)], \quad ...(32)$$

$$y_{n+1}(x) = J^{\alpha}\overline{A}_n \qquad , n \geq 0 \quad ...(33)$$

RESULTS

Application of the Proposed Scheme

This section evaluates the effectiveness and accuracy of the proposed approach outlined in Section 2.2 for solving fractional delay differential equations (FDDEs). To demonstrate the efficiency and validity of the Modified New Iterative Method (MNIM), we apply it to selected a linear FDDE.

Example 1 [see (Mohyud-din & Yildirim, 2010)]. Consider the LFDDE of second-order:

$$y^{\alpha}(x) = \frac{3}{4}y(x) + y\left(\frac{x}{2}\right) - x^2 + 2, \qquad 1 < \alpha \leq 2, \qquad y(0) = 0, y'(0) = 0. \quad ...(34)$$

The analytical solution is given by $y(x) = x^2$

In view of Eqn. (34) is approximately expressed as follows:

$$y(x) = J^{\alpha}\left[\frac{3}{4}y(x) + y\left(\frac{x}{2}\right) - x^2 + 2\right] + \sum_{i=0}^{r} y^{(i)}(0)\frac{x^i}{i!} \quad \text{...(35)}$$

We deduce the following recurrence relation from **section 2.2**

$$y_0(x) = \sum_{i=0}^{r} y^{(i)}(0)\frac{x^i}{i!} = 0$$

$$y_1(x) = J^{\alpha}\left[\frac{3}{4}y_0(x) + y_0\left(\frac{x}{2}\right) - x^2 + 2\right] = \frac{2}{\Gamma(\alpha+1)}x^{\alpha} - \frac{2}{\Gamma(\alpha+3)}x^{\alpha+2}$$

$$y_1\left(\frac{x}{2}\right) = \frac{x^{\alpha}}{\Gamma(\alpha+1)} - \frac{x^{\alpha+2}}{\Gamma(\alpha+3)}$$

$$y_2(x) = J^{\alpha}\left[\frac{3}{4}y_1(x) + y_1\left(\frac{x}{2}\right)\right] = \frac{3x^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{3x^{2\alpha+2}}{\Gamma(2\alpha+3)}$$

$$y_2\left(\frac{x}{2}\right) = \frac{3}{4}\frac{x^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{3}{4}\frac{x^{2\alpha+2}}{\Gamma(2\alpha+3)}$$

$$y_3 = J^{\alpha}\left[\frac{3}{4}y_2(x) + y_2\left(\frac{x}{2}\right)\right] = \frac{15}{4}\frac{x^{3\alpha}}{\Gamma(3\alpha+1)} - \frac{15}{4}\frac{x^{3\alpha+2}}{\Gamma(3\alpha+3)}$$

Now, in vision of Eqn. (20), the solution of Example 1 is

$$y(x) = y_0 + y_1 + y_2 + y_3 + \ldots =$$

$$\frac{2x^{\alpha}}{\Gamma(\alpha+1)} - \frac{2x^{\alpha+2}}{\Gamma(\alpha+3)} + \frac{3x^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{3x^{2\alpha+2}}{\Gamma(2\alpha+3)} + \frac{15}{4}\frac{x^{3\alpha}}{\Gamma(3\alpha+1)} - \frac{15}{4}\frac{x^{3\alpha+2}}{\Gamma(3\alpha+3)}$$

...(36)

TABLE 1: The approximate solution of **Example 1** using different values of α with a comparison with the exact solution when $\alpha = 2$.

x	MNIM (α=1.5)	MNIM (α=1.75)	MNIM (α=2)	Exact
0	0	0	0	0
0.1	0.048024285	0.022173159	0.010004168	0.01
0.2	0.137995110	0.075015928	0.040066733	0.04
0.3	0.258429000	0.153736931	0.090338253	0.09
0.4	0.406552134	0.256834813	0.161070872	0.16
0.5	0.581588057	0.383829931	0.252620079	0.25
0.6	0.783638301	0.534787173	0.365447038	0.36
0.7	1.013281193	0.710127092	0.500121356	0.49
0.8	1.271380694	0.910531391	0.65732413	0.64
0.9	1.558975244	1.136887783	0.837851048	0.81
1	1.877201182	1.390253278	1.042615327	1.00

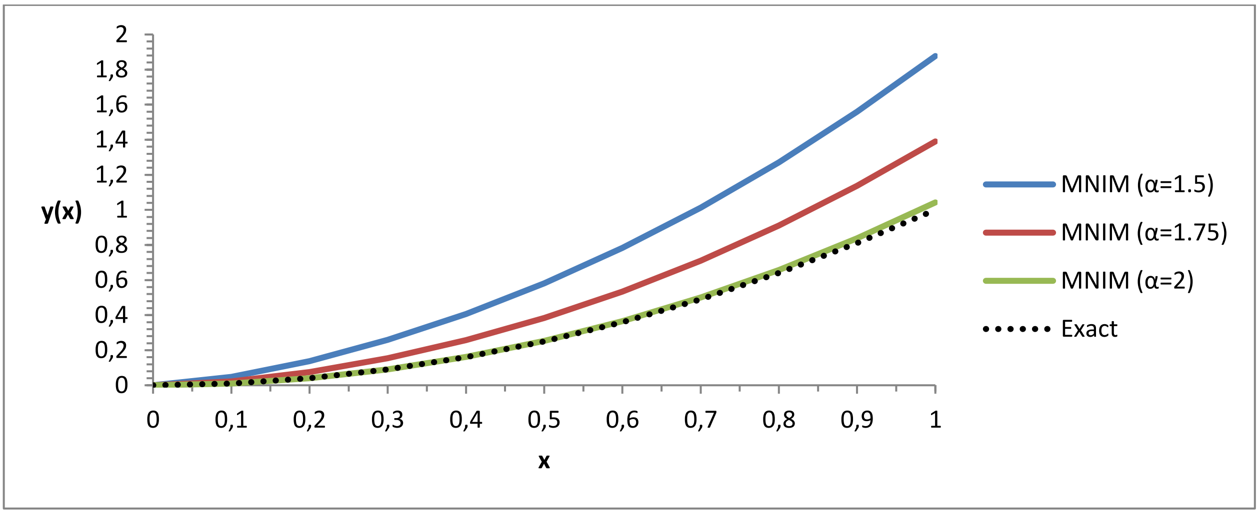

FIGURE 1: Approximate Solution obtained for different values α and the exact solution for α=2 of example 1.

DISCUSSION

The results presented in Table 1 and Figure 1 illustrate the approximate solutions obtained using the Modified New Iterative Method (MNIM) for different values of α=1.5, α=1.75, and α=2 in comparison with the exact solution when (α= 2).

From Table 1, it is evident that as x increases, the approximate solutions for different values of α progressively diverge from each other. The results for (α= 2) closely match the exact solution, with minimal numerical deviation at each step. This indicates that MNIM provides a highly accurate approximation for the given problem when (α= 2).

However, when α is decreased to 1.75 and further to 1.5, the approximate solutions increasingly deviate from the exact solution. This trend suggests that lower values of α lead to a larger numerical discrepancy, particularly for higher values of x. The error between the MNIM solution and the exact solution appears to be more pronounced as x approaches 1, where the difference between the approximate and exact values is greatest.

Figure 1 visually confirms these observations by depicting the solution curves for different values of α. The curve corresponding to (α = 2) nearly coincides with the exact solution, reinforcing the numerical accuracy of MNIM in this case. On the other hand, for (α = 1.75) and (α = 1.5), the approximate solution curves lie above the exact solution, showing an overestimation that becomes more significant for larger values of x.

Overall, the results indicate that the choice of α significantly affects the accuracy of the approximate solution, with higher values of α yielding better agreement with the exact solution. These findings highlight the importance of parameter selection in numerical methods and suggest that (α = 2) is the most suitable choice for obtaining precise results in this example.

CONCLUSION

The Modified New Iterative Method (MNIM) has been demonstrated to be a reliable and efficient technique for solving linear fractional delay differential equations. As illustrated in the example discussed, the method yields highly accurate approximations, particularly when the fractional order is close to an integer. In such cases, the MNIM-generated results exhibit strong agreement with the exact solutions across the entire domain, underscoring the method's precision and consistency.

Furthermore, when the fractional order exactly matches the integer order of the equation, MNIM continues to perform exceptionally well, reaffirming its effectiveness in solving classical delay differential equations. However, the method's accuracy begins to diminish as the fractional order decreases. This decline is notably observed in the example, where smaller fractional orders result in increasingly significant deviations from the exact solution, especially over extended domains. These discrepancies highlight the method's sensitivity to lower fractional orders and suggest challenges in accurately capturing system dynamics dominated by fractional behavior.

In summary, while the MNIM may face limitations at lower fractional orders, it remains a powerful and adaptable approach for solving a wide range of linear fractional delay differential equations with commendable accuracy.

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