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Application of the Multistage Telescoping Decomposition Method for Solving Fractional Logistic Equations

Muhammad Abdulkari[1], A. M. Kwami[2],
A. G. Madaki[3], J. O. Okai[4], J. M. Muhammad[5]
[1,2,3,4]Abubakar Tafawa Balewa University, Bauchi, Nigeria; [5]Kaduna Polytechnic, Nigeria
jerry4complex@gmail.com

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Abstract

Classical logistic equations are widely used in population dynamics but cannot represent the memory and hereditary effects inherent in many real-world systems. This study applies the Multistage Telescoping Decomposition Method (MTDM) to fractional logistic differential equations formulated using the Caputo fractional derivative. The fractional logistic equation was reformulated, and MTDM was employed to obtain semi-analytical approximate solutions without requiring computationally intensive symbolic procedures such as Adomian polynomials or Lagrange multipliers. Numerical examples were used to evaluate the method's accuracy, efficiency, and convergence across different fractional orders. The resulting solutions were compared with those obtained using the Telescoping Decomposition Method, New Iterative Method, and Residual Power Series Method. Error analyses and graphical comparisons demonstrate that MTDM produces highly accurate approximations, converges more rapidly, and reduces computational complexity relative to the comparison methods. These findings establish MTDM as an efficient semi-analytical approach for solving fractional logistic equations and demonstrate its broader potential for nonlinear fractional differential equations. The study contributes a practical computational framework for applications involving memory-dependent processes in applied mathematics, physics, and biological modeling.

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Keywords: Caputo Fractional Derivative; Fractional Logistic Equation; Multistage Telescoping Decomposition Method; Population Dynamics; Semi-Analytical Solutions

Introduction

Population dynamics and growth-related phenomena are often modeled using differential equations. The classical logistic differential equation, introduced by Verhulst (1845), accounts for saturation effects due to limited resources:

$$\frac{dy(t)}{dt} = ry(t)\left(1 - \frac{y(t)}{k}\right), \qquad y(0) = y_0 \qquad ...(1)$$

where $y(t)$ is population size, r the intrinsic growth rate, and K the carrying capacity. While widely applied (Murray, 2002), this model assumes instantaneous interactions and lacks memory dependence.

To overcome this limitation, fractional-order derivatives are introduced, yielding the fractional logistic differential equation (FLDE) (Kilbas et al., 2006; Atangana, 2014):

$$\frac{d^\alpha y(t)}{dt^\alpha} = ry(t)\left(1 - \frac{y(t)}{k}\right) = ry(t) - \frac{r}{k}y^2(t), \quad 0 < \alpha \leq 1, \qquad y(0) = y_0 \qquad ...(2)$$

where $\frac{d^\alpha}{dt^\alpha}$ denotes the Caputo fractional derivative. This formulation accounts for nonlocality and memory.

However, FLDEs are analytically challenging. Traditional methods like Adomian Decomposition Method (ADM) and Variational Iteration Method (VIM) require complex symbolic computations (Jafari & Daftardar-Gejji, 2006; Odibat & Shawagfeh, 2007). Recent studies introduced the Multistage Telescoping Decomposition Method (MTDM) as a simpler, more efficient alternative (Chita et al., 2025; Ayadi et al., 2025). This paper applies MTDM to FLDEs and compares its performance with existing methods.

Preliminaries

Definition 1 (Caputo fractional derivative):

$$D^\alpha f(x) = I^{m-\alpha}\left(\frac{d^m}{dx^m} f(x)\right) \qquad ...(3)$$

Where m is a positive integer with the property that $m-1 < \alpha < m$.

Definition 2 (Riemann-Liouville fractional integral):

$$D^{\alpha} f(x) = \frac{d^m}{dx^m}\left(I^{m-\alpha} f(x)\right) \qquad ...(4)$$

Definition 3 (Absolute error):

$$\text{Absolute Error} = |y(t) - y_n(t)| \quad a \le t \le b,$$

Methodology: MTDM for FLDE

Consider the FLDE in Caputo form:

$$^{C}D_t^{\alpha} y(t) = ry(t) - \frac{r}{k} y^2(t), \qquad 0 < \alpha \le 1, \qquad y(0) = y_0 \qquad ...(5)$$

Applying J^{α} (Riemann-Liouville integral) and using initial condition:

$$y(t) = y_0 + rJ^{\alpha}[y(t)] - \frac{r}{k} J^{\alpha}[y^2(t)], \qquad ...(6)$$

Assume a series solution $y(t) = \sum_{r=0}^{\infty} y_r(t)$ and decompose the nonlinear

term $N(y) = \frac{r}{k} y^2(t) = \sum_{r=0}^{\infty} N_r Y(t)$ with recursive definition:

$$N_0 = N(Y_0), \qquad N_r = N\left(\sum_{K=0}^{r} y_k\right) - N\left(\sum_{K=0}^{r-1} y_k\right), \; r \ge 1.$$

Substituting and comparing gives the iteration scheme:

$$\begin{cases} y_0(t) = y_0 \\ y_{r+1}(t) = rJ^{\alpha}[y_r(t)] - \frac{r}{k} J^{\alpha}\left[\sum_{r=0}^{\infty} N_r Y(t)\right], \end{cases} \qquad ...(7)$$

The approximate solution is $y(t) \approx \sum_{r=0}^{m} y_r(t)$.

Numerical Examples

In this section, numerical applications of the fractional logistic differential equations are presented. We present three examples to show the efficiency of the TDM. Comparison with the exact solution of when $a = 1$ is reported.

Example 1: Consider the following fractional logistic equation (Sachin B. &Varsha D. G. 2006)

$$ {}^{C}D_t^{\alpha} y(t) = \frac{1}{4^{\alpha}} y(t)\left(1 - y(t)\right), \quad t > 0 \quad 0 < \alpha \le 1, \qquad y(0) = \frac{1}{3} \qquad ...(8)$$

The exact solution is

$$ y(t) = \frac{e^{\frac{1}{4}t}}{2 + e^{\frac{1}{4}t}} \qquad ...(9)$$

Operating J^{α} on both sides of Eqn. (8) and using the given initial condition, we get:

$$ y(t) = \frac{1}{3} + \frac{1}{4^{\alpha}} J^{\alpha}\left[y(t)\right] - \frac{1}{4^{\alpha}} J^{\alpha}\left[y^2(t)\right], \qquad ...(10)$$

In view of the MTDM

$$ y_0(t) = \frac{1}{3}, \qquad ...(11)$$

and the nonlinear term as

$$ N(y) = \frac{1}{4^{\alpha}} y^2(t) = \sum_{r=0}^{\infty} N_r Y(t) \qquad ...(12)$$

Substituting Eqn. 11 and 12 into Eqn. 10, we obtain

$$ \sum_{r=0}^{\infty} y_r(t) = \frac{1}{3} + \frac{1}{4^{\alpha}} J^{\alpha}\left[\sum_{r=0}^{\infty} y_r(t)\right] - \frac{1}{4^{\alpha}} J^{\alpha}\left[\sum_{r=0}^{\infty} N_r Y(t)\right], \qquad ...(13)$$

Comparing both sides of Eqn. 13, the iterations are defined by the recursive relations

$$ \begin{cases} y_0(t) = \dfrac{1}{3} \\ y_{r+1}(t) = \dfrac{1}{4^{\alpha}} J^{\alpha}\left[y_r(t)\right] - \dfrac{1}{4^{\alpha}} J^{\alpha}\left[\displaystyle\sum_{r=0}^{\infty} N_r Y(t)\right] \quad r \ge 0, \end{cases} \qquad ...(14)$$

Where N_r can be calculated by

$$\begin{cases} N_0 y(t) = N y_0(t) \\ N_r y(t) = NY_r(t) - NY_{r-1}(t), \qquad r = 1,2,3,\ldots \end{cases} \qquad \ldots(15)$$

Using the iteration formula, Eqn. (14), we obtain

$$\begin{cases} y_0(t) = \dfrac{1}{3} \\ y_1(t) = \dfrac{1}{4^\alpha} J^\alpha\left[y_0(t)\right] - \dfrac{1}{4^\alpha} J^\alpha\left[N_0 Y(t)\right] = \dfrac{t^\alpha}{18\Gamma(1+\alpha)} \\ y_2(t) = \dfrac{1}{4^\alpha} J^\alpha\left[y_1(t)\right] - \dfrac{1}{4^\alpha} J^\alpha\left[\sum_{r=0}^{1} N_1 Y(t)\right] = \dfrac{t^{2\alpha}}{216\Gamma(1+2\alpha)} - \dfrac{\Gamma(1+2\alpha)t^{3\alpha}}{1296\Gamma^2(1+\alpha)\Gamma(1+3\alpha)} \\ \vdots \end{cases} \qquad \ldots(16)$$

So, we obtain the following approximate solution of Eqn. (8)

$$y(t) = \frac{1}{3} + \frac{t^\alpha}{18\Gamma(1+\alpha)} + \frac{t^{2\alpha}}{216\Gamma(1+2\alpha)} - \frac{\Gamma(1+2\alpha)t^{3\alpha}}{1296\Gamma^2(1+\alpha)\Gamma(1+3\alpha)} + \ldots \qquad \ldots(17)$$

Table 1: The numerical error of MTDM, TDM, and NIM when $\alpha = 1$

t	EXACT	MTDM (n=2)	TDM (n=5)	NIM (n=4)	MTDM ERROR	TDM ERROR	NIM ERROR
0	0.333333	0.33333333	0.3333333	0.33333333	0	0	0
0.1	0.338912	0.33891178	0.3388889	0.34583333	6.22958E-08	2.2953E-05	0.006921491
0.2	0.344535	0.34453498	0.3444444	0.35833333	4.82412E-07	9.1017E-05	0.013797871
0.3	0.350203	0.35020139	0.3500000	0.37083333	1.57459E-06	0.00020296	0.020630370
0.4	0.355913	0.35590947	0.3555556	0.38333333	3.60619E-06	0.00035752	0.027420262
0.5	0.361664	0.36165766	0.3611111	0.39583333	6.79848E-06	0.00055335	0.03416887
0.6	0.367456	0.36744444	0.3666667	0.40833333	1.13276E-05	0.00078911	0.040877561
0.7	0.373286	0.37326826	0.3722222	0.42083333	1.73255E-05	0.00106336	0.047547747
0.8	0.379152	0.37912757	0.3777778	0.43333333	2.48811E-05	0.00137468	0.054180880
0.9	0.385055	0.38502083	0.3833333	0.44583333	3.40414E-05	0.00172154	0.060778459
1	0.390991	0.39094650	0.3888889	0.45833333	4.48131E-05	0.00210243	0.067342018

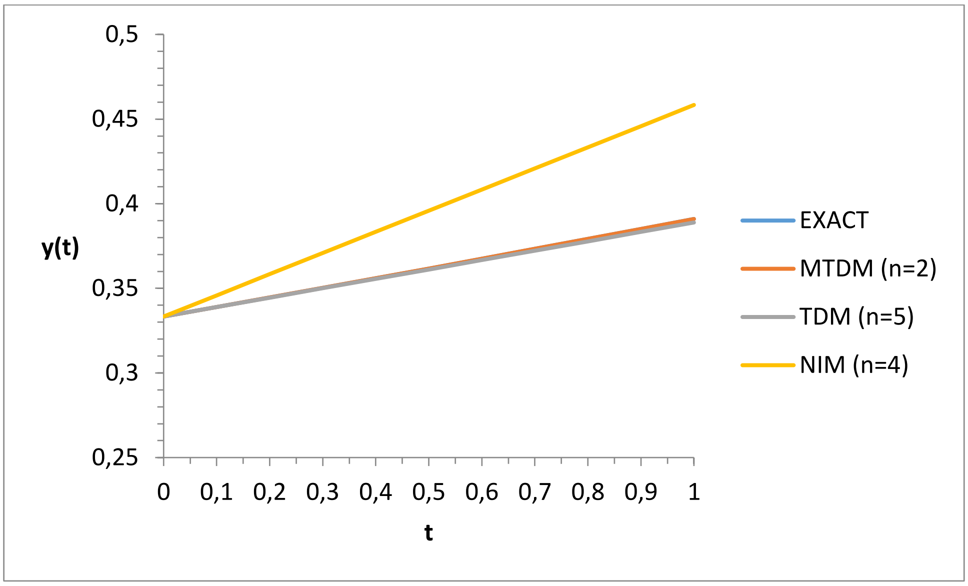

Figure 1: Graph of the MTDM, TDM, and NIM in comparison with the exact solution when $\alpha = 1$

Table 2: The Numerical solution of example 1 for different values of α

t	EXACT	MTDM (α=1)	MTDM (α=0.9)	MTDM (α=0.8)	MTDM (α=0.7)
0	0.333333	0.333333333	0.333333333	0.333333333	0.333333333
0.1	0.338912	0.33891178	0.340648503	0.342866614	0.345676851
0.2	0.344535	0.344534979	0.347051556	0.350030693	0.353524795
0.3	0.350203	0.350201389	0.353182965	0.356547850	0.360304092
0.4	0.355913	0.355909465	0.359158613	0.362691606	0.366483813
0.5	0.361664	0.361657665	0.365029788	0.368579788	0.372259425
0.6	0.367456	0.367444444	0.3708249	0.374276993	0.377736029
0.7	0.373286	0.373268261	0.376561641	0.379823445	0.382978277
0.8	0.379152	0.379127572	0.382251848	0.385246281	0.388029457
0.9	0.385055	0.385020833	0.387903836	0.390564856	0.392920316
1	0.390991	0.390946502	0.393523637	0.395793543	0.397673656

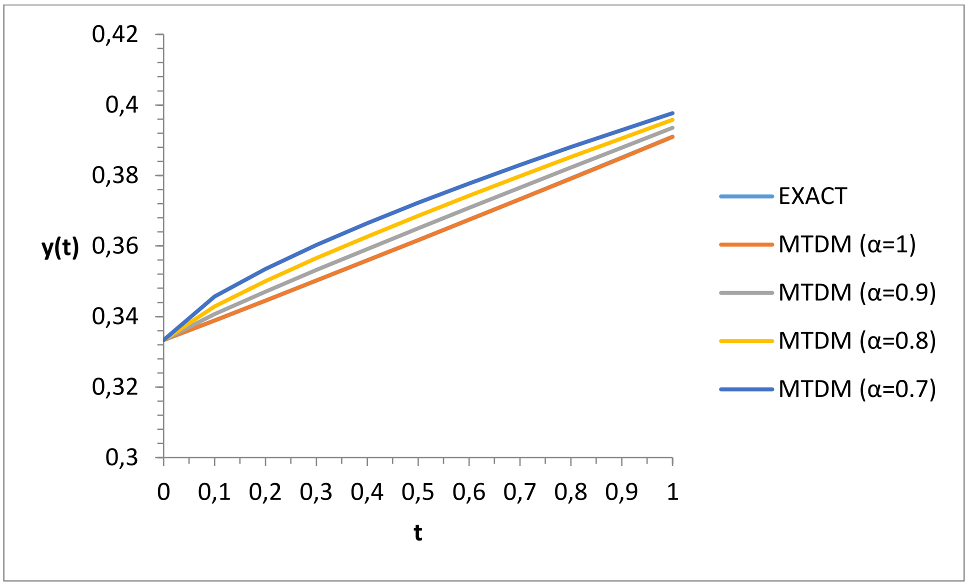

Figure 2: The approximate solution plots of example 2 for different values of α

Example 2: Consider a fractional-order logistic equation (Sachin B. &Varsha D. G. 2006; Patcharee D. *et. al.* 2020)

$$ {}^{C}D_t^{\alpha} y(t) = \frac{1}{2} y(t)\left(1 - y(t)\right), \quad t > 0 \quad 0 < \alpha \leq 1, \quad y(0) = \frac{1}{2} \quad ...(18)$$

The exact solution is

$$y(t) = \frac{e^{\frac{1}{2}t}}{1 + e^{\frac{1}{2}t}} \quad ...(19)$$

Operating J^{α} on both sides of Eqn. (18) and using the given initial condition, we obtain:

$$y(t) = \frac{1}{2} + \frac{1}{2} J^{\alpha}\left[y(t)\right] - \frac{1}{2} J^{\alpha}\left[y^2(t)\right], \quad ...(20)$$

In view of the TDM

$$y_0(t) = \frac{1}{2}, \quad ...(21)$$

and the nonlinear term as

$$N(y) = \frac{1}{2} y^2(t) = \sum_{r=0}^{\infty} N_r Y(t) \quad ...(22)$$

Substituting Eqn. 21 and 22 into Eqn. 20, we obtain

$$\sum_{r=0}^{\infty} y_r(t) = \frac{1}{2} + \frac{1}{2} J^{\alpha}\left[\sum_{r=0}^{\infty} y_r(t)\right] - \frac{1}{2} J^{\alpha}\left[\sum_{r=0}^{\infty} N_r Y(t)\right], \qquad \text{...(23)}$$

Comparing both sides of Eqn. 23, the iterations are defined by the recursive relations

$$\begin{cases} y_0(t) = \frac{1}{2} \\ y_{r+1}(t) = \frac{1}{2} J^{\alpha}\left[y_r(t)\right] - \frac{1}{2} J^{\alpha}\left[\sum_{r=0}^{\infty} N_r Y(t)\right] \qquad r \geq 0, \end{cases} \qquad \text{...(24)}$$

Where N_r can be calculated by

$$\begin{cases} N_0 y(t) = N y_0(t) \\ N_r y(t) = NY_r(t) - NY_{r-1}(t), \quad r = 1,2,3,\ldots \end{cases} \qquad \text{...(25)}$$

Using the iteration formula, Eqn. (24), we obtain

$$\begin{cases} y_0(t) = \frac{1}{2} \\ y_1(t) = \frac{1}{2} J^{\alpha}\left[y_0(t)\right] - \frac{1}{2} J^{\alpha}\left[N_0 Y(t)\right] = \frac{t^{\alpha}}{8\Gamma(1+\alpha)} \\ y_2(t) = \frac{1}{2} J^{\alpha}\left[y_1(t)\right] - \frac{1}{2} J^{\alpha}\left[\sum_{r=0}^{1} N_1 Y(t)\right] = -\frac{\Gamma(2\alpha+1) t^{3\alpha}}{128\Gamma^2(1+\alpha)\Gamma(1+3\alpha)} \\ y_3(t) = \frac{1}{2} J^{\alpha}\left[y_2(t)\right] - \frac{1}{2} J^{\alpha}\left[\sum_{r=0}^{2} N_2 Y(t)\right] = \frac{\Gamma(4\alpha+1)\Gamma(2\alpha+1) t^{5\alpha}}{1024\Gamma^3(1+\alpha)\Gamma(1+3\alpha)\Gamma(1+5\alpha)} \\ \vdots \end{cases} \qquad \text{...(26)}$$

So, we obtain the following approximate solution of Eqn. (18)

$$y(t) = \frac{1}{2} + \frac{t^{\alpha}}{8\Gamma(1+\alpha)} - \frac{\Gamma(2\alpha+1) t^{3\alpha}}{128\Gamma^2(1+\alpha)\Gamma(1+3\alpha)} + \frac{\Gamma(4\alpha+1)\Gamma(2\alpha+1) t^{5\alpha}}{1024\Gamma^3(1+\alpha)\Gamma(1+3\alpha)\Gamma(1+5\alpha)} + \ldots \qquad \text{...(27)}$$

Table 3: The numerical error of MTDM, TDM, and RPSM when $\alpha = 1$

t	EXACT	MTDM (n=3)	TDM (n=5)	RPSM (n=3)	MTDM ERROR	TDM ERROR	RPSM ERROR
0	0.5	0.5	0.5	0.5	0	0	0
0.02	0.502500	0.502499979	0.5025	0.502499975	0	2.08331E-08	3.85823E-09
0.04	0.505000	0.504999833	0.5050	0.504999802	5.55112E-16	1.6666E-07	3.08709E-08
0.06	0.507499	0.507499438	0.5075	0.507499333	1.09912E-14	5.62449E-07	1.04217E-07
0.08	0.509999	0.509998667	0.5100	0.509998420	8.32667E-14	1.33312E-06	2.47127E-07
0.1	0.512497	0.512497396	0.5125	0.512496914	3.97127E-13	2.60352E-06	4.82904E-07

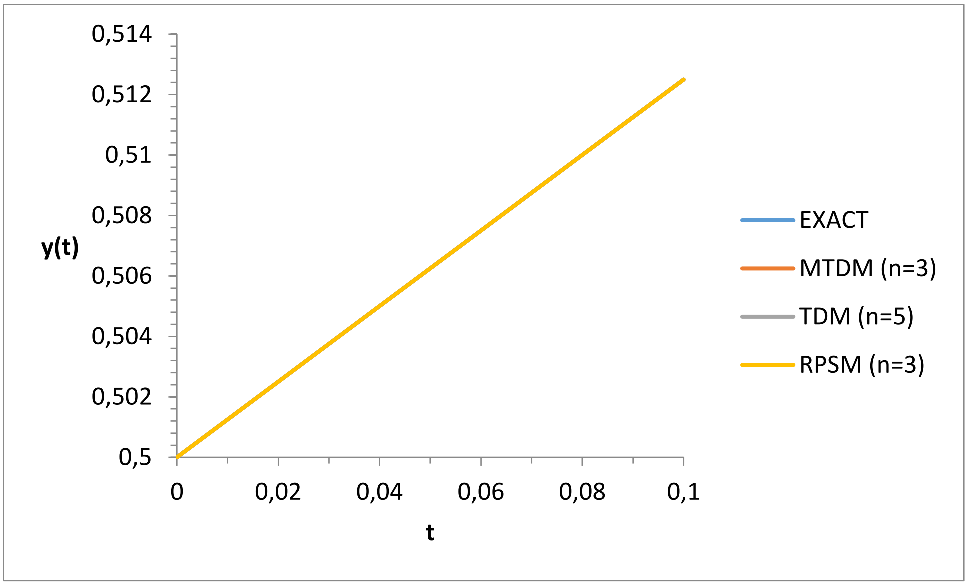

Figure 3: Graph of the MTDM, TDM, and RPSM in comparison with the exact solution when $\alpha = 1$

Table 4: The Numerical solution of example 2 for different values of α

t	EXACT (α=1)	MTDM (α=1)	MTDM (α=0.7)	MTDM (α=0.5)
0	0.5	0.5	0.5	0.5
0.02	0.502499979	0.502499979	0.508895475	0.519925985
0.04	0.504999833	0.504999833	0.514446893	0.528149820
0.06	0.507499438	0.507499438	0.519182097	0.534440001
0.08	0.509998667	0.509998667	0.523452570	0.539726061
0.1	0.512497396	0.512497396	0.527406118	0.544368485

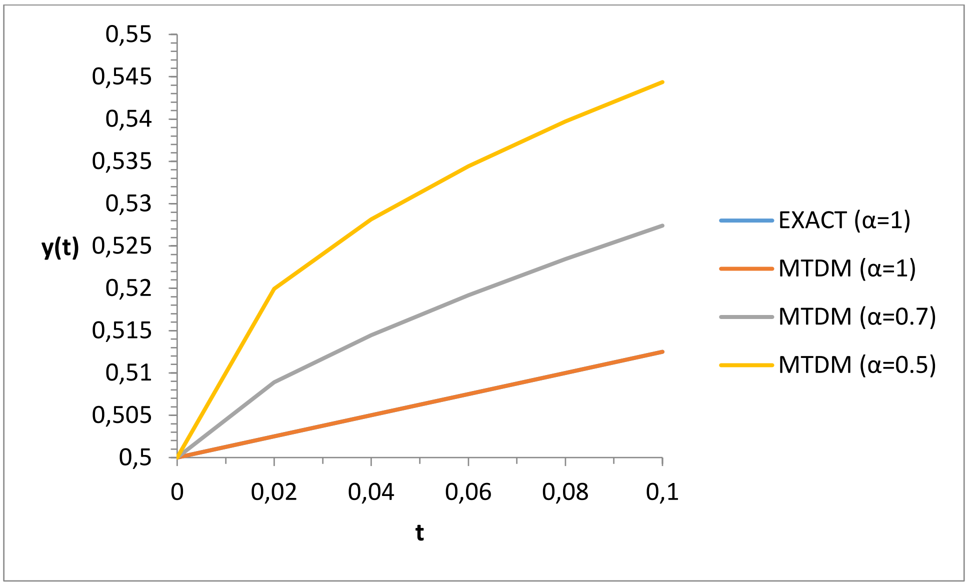

Figure 4: The approximate solution plots of example 2 for different values of α

Example 3: Consider the following nonlinear fractional differential equation (Chita F. *et. al.* 2025)

$$ {}^{C}D_t^{\alpha} y(t) = 1 - y^2(t), \quad t > 0 \quad 0 < \alpha \le 1, \qquad y(0) = 0 \qquad \text{...(28)} $$

If $\alpha = 1$ the exact solution of Eqn. (28) is

$$ y(t) = \tanh(t). \qquad \text{...(29)} $$

Operating J^{α} on both sides of Eqn. (28) and using the given initial condition, we get:

$$ y(t) = \frac{t^{\alpha}}{\Gamma(\alpha+1)} - J^{\alpha}\left[y^2(t)\right], \qquad \text{...(30)} $$

In view of the MTDM

$$ y_0(t) = \frac{t^{\alpha}}{\Gamma(\alpha+1)}, \qquad \text{...(31)} $$

and the nonlinear term as

$$ N(y) = y^2(t) = \sum_{r=0}^{\infty} N_r Y(t) \qquad \text{...(32)} $$

Substituting Eqn. 31 and 32 into Eqn. 30, we obtain

$$\sum_{r=0}^{\infty} y_r(t) = \frac{t^{\alpha}}{\Gamma(\alpha+1)} - J^{\alpha}\left[\sum_{r=0}^{\infty} N_r Y(t)\right], \qquad ...(33)$$

Comparing both sides of Eqn. 33, the iterations are defined by the recursive relations

$$\begin{cases} y_0(t) = \dfrac{t^{\alpha}}{\Gamma(\alpha+1)} \\ y_{r+1}(t) = -J^{\alpha}\left[\displaystyle\sum_{r=0}^{\infty} N_r Y(t)\right] \qquad r \geq 0, \end{cases} \qquad ...(34)$$

Where N_r can be calculated by

$$\begin{cases} N_0 y(t) = N y_0(t) \\ N_r y(t) = N Y_r(t) - N Y_{r-1}(t), \qquad r = 1,2,3,... \end{cases} \qquad ...(35)$$

Using the iteration formula, Eqn. (34), we obtain

$$\begin{cases} y_0(t) = \dfrac{t^{\alpha}}{\Gamma(\alpha+1)} \\ y_1(t) = -J^{\alpha}\left[N_0 Y(t)\right] = -\dfrac{\Gamma(2\alpha+1)t^{3\alpha}}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)} \\ y_2(t) = -J^{\alpha}\left[\displaystyle\sum_{r=0}^{1} N_1 Y(t)\right] = \dfrac{2\Gamma(2\alpha+1)\Gamma(4\alpha+1)t^{5\alpha}}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)\Gamma(5\alpha+1)} - \dfrac{\Gamma^2(2\alpha+1)\Gamma(6\alpha+1)t^{7\alpha}}{\Gamma^4(\alpha+1)\Gamma^2(3\alpha+1)\Gamma(7\alpha+1)} \\ \vdots \end{cases} \qquad ...(36)$$

So, we obtain the following approximate solution of Eqn. (28)

$$\begin{aligned} y(t) = {} & \frac{t^{\alpha}}{\Gamma(\alpha+1)} - \frac{\Gamma(2\alpha+1)t^{3\alpha}}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)} + \frac{2\Gamma(2\alpha+1)\Gamma(4\alpha+1)t^{5\alpha}}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)\Gamma(5\alpha+1)} \\ & - \frac{\Gamma^2(2\alpha+1)\Gamma(6\alpha+1)t^{7\alpha}}{\Gamma^4(\alpha+1)\Gamma^2(3\alpha+1)\Gamma(7\alpha+1)} + ... \end{aligned} \qquad ...(37)$$

Table 5: The numerical error of MTDM, ETDM, and TDM when $\alpha = 1$

t	EXACT	MTDM (n=2)	ETDM (n=5)	TDM (n=4)	MTDM ERROR	ETDM ERROR	TDM ERROR
0	0	0	0	0	0	0	0
0.02	0.019997	0.019997334	0.0199973	0.0199987	1.04083E-17	4.26598E-10	1.33291E-06
0.04	0.039979	0.03997868	0.0399787	0.0399893	6.93889E-18	1.36445E-08	1.0653E-05
0.06	0.059928	0.059928104	0.059928	0.059964	6.93889E-18	1.03529E-07	3.58965E-05
0.08	0.07983	0.079829769	0.0798293	0.0799147	6.93889E-17	4.35778E-07	8.48976E-05
0.1	0.099668	0.099667995	0.0996667	0.0998333	4.44089E-16	1.32796E-06	0.000165339

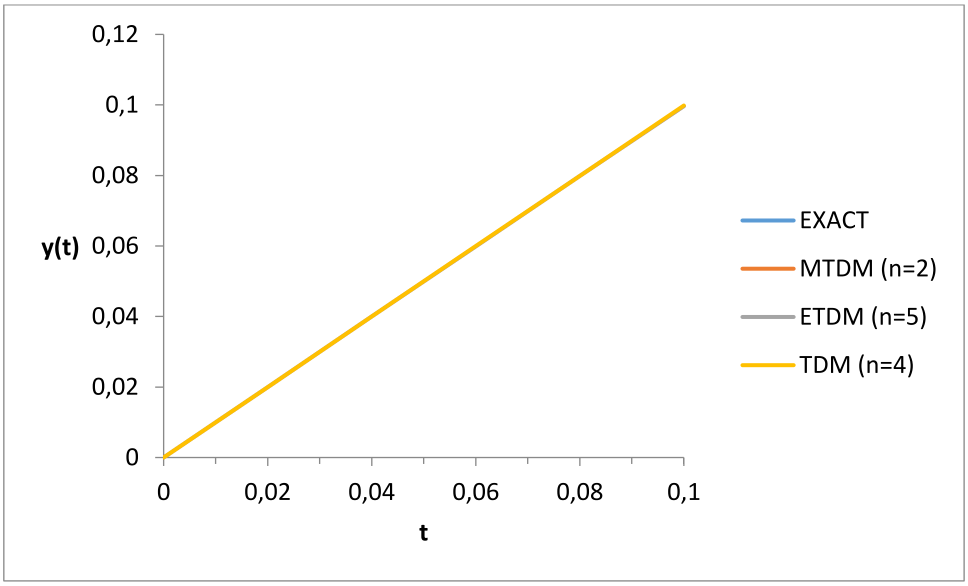

Figure 5: Graph of the MTDM, TDM, and ETDM in comparison with the exact solution when $\alpha = 1$

Table 6: The Numerical solution of example 3 for different values of α

t	EXACT	MTDM (α=1)	MTDM (α=0.9)	MTDM (α=0.8)	MTDM (α=0.7)
0	0	0	0	0	0
0.02	0.019997	0.019997334	0.030739652	0.046910298	0.070991218
0.04	0.039979	0.039978680	0.057310273	0.081513191	0.114842475
0.06	0.059928	0.059928104	0.082437106	0.112442524	0.151760795
0.08	0.079830	0.079829769	0.106609265	0.141078235	0.184560793
0.1	0.099668	0.099667995	0.130038768	0.168016098	0.214436479

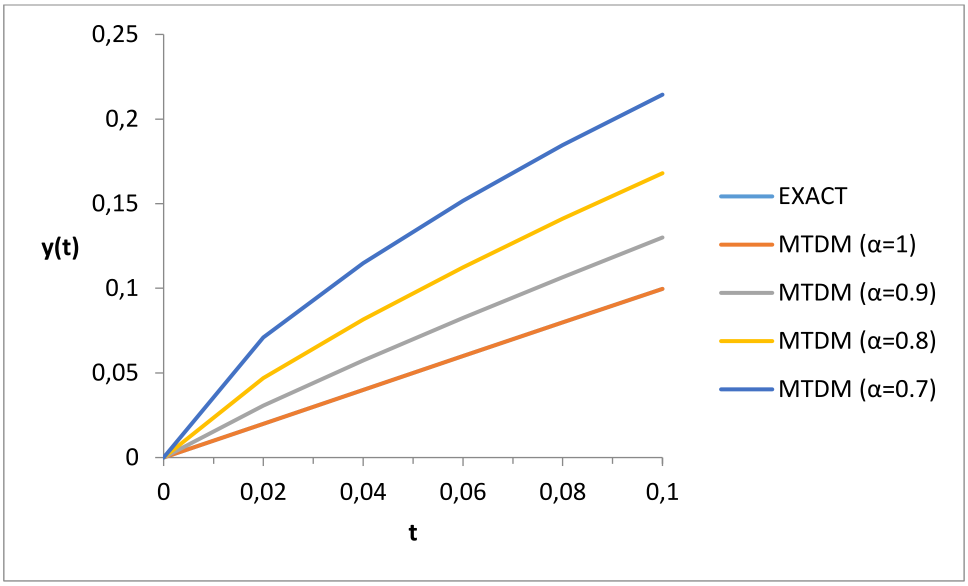

Figure 6: The approximate solution plots of example 3 for different values of α

Discussion

The numerical results demonstrate that MTDM produces highly accurate approximations with very low absolute errors, especially for $\alpha = 1$. Compared to TDM, NIM, and RPSM, MTDM converges faster and maintains accuracy over larger time domains. Graphical results (Figs. 1–6) confirm that MTDM solutions closely follow exact solutions. As α decreases, the growth rate slows, reflecting stronger memory effects consistent with fractional calculus theory.

MTDM avoids complex symbolic computations (e.g., Adomian polynomials, Lagrange multipliers), making it computationally efficient and easy to implement.

Conclusion

This study successfully applied the Multistage Telescoping Decomposition Method (MTDM) to solve fractional logistic differential equations. MTDM exhibits faster convergence, higher accuracy, and lower computational complexity than traditional methods like TDM, NIM, and RPSM. Numerical and graphical validations confirm its robustness across different fractional orders. MTDM is a valuable tool for modeling memory-dependent phenomena in population dynamics, epidemiology, and physics. Future work may extend MTDM to higher-dimensional fractional partial differential equations.

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