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Investigating the Influence of Radiation Pressure on the Stability of Lagrangian Points in Celestial Mechanics

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Abstract

Lagrangian points represent critical equilibrium configurations in celestial mechanics where gravitational and centrifugal forces balance, enabling small bodies to maintain relative positions with respect to two primary masses. This study investigates the location and stability of these points under the influence of radiative forces, with a particular focus on the role of radiation pressure in modifying gravitational equilibrium. Using a mathematical modeling approach, the research derives expressions for the collinear and triangular Lagrangian points and examines how radiation pressure affects their equilibrium configurations. The analysis shows that the positions of the collinear points shift as a function of the radiation parameter, while the stability characteristics of the triangular points are governed by the mass ratio of the system. These findings refine the theoretical understanding of Lagrangian dynamics in radiating systems and highlight the sensitivity of equilibrium configurations to radiative effects. The study concludes that incorporating radiation pressure is essential for accurately characterizing gravitational equilibrium in realistic astrophysical and space mission scenarios, thereby providing a more robust foundation for celestial navigation, satellite deployment, and space mission design, and contributing to a deeper understanding of orbital mechanics relevant to future space exploration missions.

Keywords: Lagrangian Points; Celestial Mechanics; Radiation Pressure; Collinear Points; Triangular Points

Introduction

The study of Lagrangian points is a fundamental aspect of celestial mechanics, providing valuable insights into the equilibrium positions of small bodies within a two-body gravitational system. These points represent positions where the gravitational forces of two large bodies and the centrifugal force acting on a smaller object balance, allowing the object to remain in a stable or semi-stable orbit. This concept, first introduced by Joseph-Louis Lagrange in the 18th century, has since been extensively applied in space exploration, orbital mechanics, and astrophysics [1].Lagrangian points are classified into five distinct positions, denoted as L1, L2, L3, L4, and L5. The first three, known as collinear points, lie along the line connecting the two primary celestial bodies, while the remaining two, known as triangular points, form equilateral triangles with the two larger masses [2]. The dynamical properties of these points play a crucial role in mission planning for spacecraft, such as NASA's James Webb Space Telescope, which is positioned at L2 to minimize thermal interference and optimize observational efficiency [3].

In addition to their practical applications, Lagrangian points have been extensively studied in the context of restricted three-body problems, particularly with modifications due to additional forces such as radiation pressure. The presence of radiation pressure, first analyzed by [4], introduces a repulsive force that alters the equilibrium conditions, thereby affecting the stability of motion near these points. Understanding these effects is crucial for accurate trajectory predictions in space dynamics, particularly for small celestial bodies such as asteroids and artificial satellites [5].

This paper aims to explore the mathematical modeling of Lagrangian points in the presence of radiative forces, focusing on their locations and stability. By employing analytical methods, we determine the conditions under which these points exist and assess their behavior under varying system parameters. The findings contribute to the broader understanding of celestial mechanics and have implications for spacecraft station-keeping strategies and the study of minor bodies in our solar system.

The aim of this study is to investigate the location and stability of Lagrangian points in a dynamical system influenced by radiative forces. It seeks to derive mathematical expressions for the positions of collinear and triangular Lagrangian points, analyze the effects of radiation pressure on their stability, and explore the implications for space exploration and orbital dynamics. By achieving these objectives, the study enhances the theoretical understanding of Lagrangian points and their practical applications in celestial mechanics and space mission planning.

Modelling of the Problem

This present section discusses a method of for calculating the location of Lagrangian points and the stability of the motion around these points.

The radiative repulsive force F_p exerted on a particle in motion can be represented in terms of gravitational attraction F_g (Radzievskii,1950) as:

$$F_p = F_g(1-q) \tag{1}$$

Where

$$q = 1 - \left(\frac{F_p}{F_g}\right),$$

is a constant for a given particle, is a reduction factor expressed in terms of the particle radius, a and density δ and radiation pressure-pressure efficiency factor x (cgs system); as

$$q = 1 - \left(\frac{5.6\times10^{-3}}{a\delta}\right)x. \tag{2}$$

The assumption q= constant is equivalent to neglecting fluctuation in the beam of solar radiation and the effect of planet's shadow.

In dimensionless synodic co-ordinate (x, y), the equations of motion of the particle are;

$$\left.\begin{aligned}\ddot{x} - 2n\dot{y} = \Omega_x\\ \ddot{y} + 2n\dot{x} = \Omega_y\end{aligned}\right\} \tag{3}$$

Where

$$\left.\begin{aligned} &\Omega = \frac{n^2}{2}\left((1-\mu)r_1^2 + \mu r_2^2\right) + \frac{q(1-\mu)}{r_1} + \\ &\mu\left(\frac{1}{r_2} + \frac{A_2}{2r_2^5} + \frac{A}{2r_2^5}\right), \\ &n^2 = 1 + \frac{3}{2}(A + A_2), r_1^2 = (x-\mu)^2 + \\ &y^2, r_2^2 = (x+1-\mu)^2 + y^2 \end{aligned}\right\} \tag{4}$$

$\mu = \dfrac{m_2}{m_1 + m_2} \leq \dfrac{1}{2}$, is the mass parameter (Singh and Taura, 2014).

Location of Libration points

At the libration points, equation (3) admits the energy integral of the problem

$$\dot{x}^2 + \dot{y}^2 = 2\Omega(x, y) - C \tag{5}$$

Where C is known as Jacobis' constant. The libration points are the singularities of the manifold (5)

$\dot{x}^2 + \dot{y}^2 - 2\Omega(x, y) + C = 0$

And these points are given by the equations:

$$\dot{x} = \dot{y} = \frac{\partial \Omega}{\partial x} = \frac{\partial \Omega}{\partial y} = 0\,. \tag{6}$$

This implies that the libration points of the problem in question are determined by equating all the velocities and accelerations of the dynamical system to zero. Thus, equation (4) can be written as:

$$\left.\begin{array}{l}\Omega = \frac{n}{2}\begin{bmatrix}(1-\mu)((x-\mu)^2+y^2)+ \\ \mu((x-\mu+1)^2+y^2)\end{bmatrix}+ \\ \frac{q(1-\mu)}{\sqrt{((x-\mu)^2+y^2)}}+\left(\frac{\mu}{\sqrt{((x-\mu+1)^2+y^2)}}\right)+ \\ \frac{\mu A_2}{2}\left(((x-\mu+1)^2+y^2)\right)^{\frac{-3}{2}}+\frac{\mu A}{2}\begin{pmatrix}((x- \\ \mu+1)^2 \\ +y^2)\end{pmatrix}^{\frac{-3}{2}}\end{array}\right\} \quad (7)$$

On differentiating equation (7) partially with respect to x and y respectively yields:

$$\left.\begin{array}{l}\Omega_x = n^2[(1-\mu)(x-\mu)+\mu(1-\mu+x)]- \\ \frac{q(1-\mu)(x-\mu)}{[(x-\mu)^2+y^2]^{\frac{3}{2}}}-\frac{\mu(x-\mu+1)}{[(x-\mu)^2+y^2]^{\frac{3}{2}}}- \\ \frac{3\mu A_2}{2[(x-\mu)^2+y^2]^{\frac{5}{2}}}-\frac{3\mu A}{2[(x-\mu)^2+y^2]^{\frac{5}{2}}} \\ \Omega_y = n^2[(1-\mu)y+\mu y]-\frac{q(1-\mu)y}{[(x-\mu)^2+y^2]^{\frac{3}{2}}} \\ -\frac{\mu y}{[(x-\mu)^2+y^2]^{\frac{3}{2}}}- \\ \frac{3\mu A_2}{2[(x-\mu)^2+y^2]^{\frac{5}{2}}}-\frac{3\mu A}{2[(x-\mu)^2+y^2]^{\frac{5}{2}}}\end{array}\right\} \quad (8)$$

On substituting equation (4) and (6) into (8), we obtain;

$$\left.\begin{array}{l}n^2x-\frac{q(1-\mu)(x-\mu)}{r_1^3}-\frac{\mu(x-\mu+1)}{r_2^3}- \\ \frac{3\mu(x-\mu+1)A_2}{2r_2^5}-\frac{3\mu(x-\mu+1)A}{2r_2^5}=0 \\ n^2y-\frac{q(1-\mu)y}{r_1^3}--\frac{\mu y}{r_2^3}-\frac{3\mu yA_2}{2r_2^5}-\frac{3\mu yA}{2r_2^5}=0\end{array}\right\} \quad (9a)$$

The solution of equation (9) result in five points called the 'libration points', the three of which are located on the x-axis and the other two are located symmetrically with respect to the x-axis.

Location of Collinear points

At the collinear libration points, $y = 0$ and their abscissa is given by the equation (9a)

$$n^2 x - \frac{q(1-\mu)(x-\mu)}{r_1^3} - \frac{\mu(x-\mu+1)}{r_2^3} - \frac{3\mu(x-\mu+1)A_2}{2r_2^5} - \frac{3\mu(x-\mu+1)A}{2r_2^5} = 0 \tag{9b}$$

We assume that

$$f(x) = n^2 x - \frac{q(1-\mu)(x-\mu)}{|x+\mu|^3} - \frac{\mu(x-\mu+1)}{|x-\mu+1|^3} - \frac{3\mu(x-\mu+1)A_2}{2|x-\mu+1|^5} - \frac{3\mu(x-\mu+1)A}{2|x-\mu+1|^5} \tag{10}$$

Now, in order to determine the positions of the collinear points, we consider the following:

$$f(x) = \begin{cases} n^2 x + \dfrac{q(1-\mu)}{|x+\mu|^2} + \dfrac{\mu}{|x-\mu+1|^2} + \dfrac{3\mu A_2}{2|x-\mu+1|^4} + \dfrac{3\mu A}{2|x-\mu+1|^4}, & if x < \mu - 1 \\ n^2 x - \dfrac{q(1-\mu)}{|x+\mu|^2} + \dfrac{\mu}{|x-\mu+1|^2} - \dfrac{3\mu A_2}{2|x-\mu+1|^4} + \dfrac{3\mu A}{2|x-\mu+1|^4}, & if \mu - 1 < x < \mu \\ n^2 x - \dfrac{q(1-\mu)}{|x+\mu|^2} - \dfrac{\mu}{|x-\mu+1|^2} - \dfrac{3\mu A_2}{2|x-\mu+1|^4} - \dfrac{3\mu A}{2|x-\mu+1|^4}, & if \mu < x \end{cases} \tag{11}$$

Case I: We examine $f(x)$ in the interval $x \in (-\infty, \mu - 1)$

$$f(x) = n^2 x + \frac{q(1-\mu)}{|x+\mu|^2} + \frac{\mu}{|x-\mu+1|^2} + \frac{3\mu A_2}{2|x-\mu+1|^4} + \frac{3\mu A}{2|x-\mu+1|^4}$$

and

$$f'(x) = n^2 + \frac{q(1-\mu)}{|x+\mu|^3} + \frac{2\mu}{|x-\mu+1|^3} + \frac{6\mu A_2}{|x-\mu+1|^5} + \frac{6\mu A}{|x-\mu+1|^5}$$

This implies that

$$f'(x) > 0$$

Now,

$f'(x) > 0$, for $x \in (-\infty, \mu-1)$, $f(x)$ is a monotonically increasing function, since

$$\lim_{x\to-\infty} f(x) = -\infty \text{ and } \lim_{x\to(\mu-1)^-} f(x) = \infty$$

Therefore,

$$\lim_{x\to-\infty} f(x) < 0 \text{ and } \lim_{x\to(\mu-1)^-} f(x) > 0,$$

which imply that there is a unique point $x \in (-\infty, \mu-1)$ for which $f(x) = 0$, we call this point L_1

Case II: We consider $f(x)$ in the interval $\mu-1 < x < \mu$, and we look at $x \in (0, 1-\mu)$

$$f(0) = -\frac{q(1-\mu)}{|\mu|^2} + \frac{\mu}{|-\mu+1|^2} - \frac{3\mu A_2}{2|-\mu+1|^4} + \frac{3\mu A}{2|-\mu+1|^4}$$

This implies that

$$f(0) < 0$$

Now,

$f(0) < 0$, for $x \in (0, 1-\mu)$, and also $\lim_{x\to(1-\mu)^-} f(x) > 0$. This implies that there is a unique point $x \in (0, 1-\mu)$ for which $f(x) = 0$,

Secondly, for $x \in (\mu-1, 0)$, $f(\mu-1) > 0$, and $\lim_{x\to(\mu-1)^-} f(x) > 0$, which imply that there is a unique point $x \in (\mu-1, 1-\mu)$ for which $f(x) = 0$, we call this point L_2.

Case III: We examine $f(x)$ in the interval $x \in (\mu, \infty)$

$$f(x) = n^2 x + \frac{q(1-\mu)}{|x+\mu|^2} + \frac{\mu}{|x-\mu+1|^2}$$

$$+ \frac{3\mu A_2}{2|x-\mu+1|^4} + \frac{3\mu A}{2|x-\mu+1|^4}$$

and

$$f'(x) = n^2 + \frac{q(1-\mu)}{|x+\mu|^3} + \frac{2\mu}{|x-\mu+1|^3} +$$

$$\frac{6\mu A_2}{|x-\mu+1|^5} + \frac{6\mu A}{|x-\mu+1|^5}$$

This implies that

$$f'(x) > 0$$

Now,

$f'(x) > 0$, for $x \in (1-\mu, \infty)$, $f(x)$ is a monotonically increasing function, since

$\lim_{x \to \mu^+} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = \infty$

Therefore, this implies that:

$\lim_{x \to \infty} f(x) > 0$ and $\lim_{x \to (-\mu+1)^+} f(x) < 0$,

which imply that there is a unique point $x \in (\mu, \infty)$ for which $f(x) = 0$, we call this point L_3

Therefore, we observed that;

$f(-\infty) < 0; f(\mu-1) > 0; f(\mu) < 0; f(\infty) < 0$.

As such, the three collinear points $L_{1,2,3}$ lie in the intervals;

$-\infty$ and $\mu-1$, $\mu-1$ and μ, μ and ∞ respectively.

Now, as the first collinear point, L_1 lies between $-\infty$ and $\mu-1$, that is to left of the primary, we have for this point:

$$r_1 = \mu - x, r_2 = \mu - 1 + x, \xi_1 = \mu - 1 + x \tag{12}$$

Substituting (12) into (9a) gives:

$$\begin{aligned} & n^2(\mu - 1 - \xi_1) - \frac{q(1-\mu)}{(1+\xi_1)^3} - \frac{\mu(1+\xi_1)}{\xi_1^3} - \\ & \frac{3\mu 1 + \xi_1)A_2}{2\xi_1^5} - \frac{3\mu 1 + \xi_1)A}{2\xi_1^5} = 0 \end{aligned} \tag{13}$$

After simplification, equation (13) becomes

$$\left.\begin{aligned} & 2n^2\xi_1^8 - 2(n^2\mu - 4n^2)\xi_1^7 - 4(n^2\mu - n^2)\xi_1^6 - \\ & 2(n^2\mu - n^2 + 2q - 2q\mu)\xi_1^5 - 6\mu\xi_1^4 \\ & -3(2\mu + \mu A_2 + \mu A)\xi_1^3 - (2\mu + 9\mu A_2 + 9\mu A)\xi_1^2 \\ & -9\mu(A + A_2)\xi_1 - 3\mu(A + A_2) = 0 \end{aligned}\right\} \tag{14}$$

This is an eight-degree algebraic equation in ξ_1. Since there is only one change of sign in it, there exists at least one real root and as left-hand side of this equation is less than zero for $\xi_1 = 0$ and infinite for $\xi_1 = \infty$, this root must be positive.

In a similar way, after same analysis of equation (9a) with:

$r_1 = \mu - 1 - x, r_2 = x + 1 - \mu, \xi_2 = x + 1 - \mu$ for the location point, L_2

and $r_1 = 1 - \mu - x, r_2 = x - \mu, \xi_2 = x - \mu$ for the location point, L_3

we have, for the second and third collinear points, L_2 and L_3 respectively:

$$\left.\begin{aligned} & 2n^2\xi_1^8 - 2(n^2\mu - 4n^2)\xi_1^7 - 4(n^2\mu - n^2)\xi_1^6 \\ & -2(n^2\mu - n^2 + q - q\mu)\xi_1^5 - 3\mu\xi_1^4 \\ & -2(3\mu - \mu A_2 - \mu A)\xi_1^3 - (\mu + 9\mu A_2 + \\ & 9\mu A)\xi_1^2 - 9\mu(A + A_2)\xi_1 - 3\mu(A + A_2) = 0 \end{aligned}\right\} \tag{15}$$

$$\left.\begin{aligned} & 2n^2\xi_1^8 + 2(n^2\mu - 4n^2)\xi_1^7 + 4(n^2\mu - n^2)\xi_1^6 - \\ & 3(n^2\mu - n^2 + q - 2q\mu)\xi_1^5 - 4\mu\xi_1^4 \\ & +3(2\mu + \mu A_2 + \mu A)\xi_1^3 - (2\mu + 9\mu A_2 + 9\mu A)\xi_1^2 \\ & +9\mu(A + A_2)\xi_1 - 3\mu(A + A_2) \end{aligned}\right\} \tag{16}$$

Thus, the three collinear libration points $L_{1,2,3}$ are located by

$$\left.\begin{aligned} x_1 &= \mu - 1 - \xi_1 \\ x_2 &= \mu - 1 + \xi_2 \\ x_3 &= \mu + \xi_3 \end{aligned}\right\} \tag{17}$$

where ξ_i (i =1, 2, 3) are the roots of equations (14), (15) and (16) respectively

Location of Triangular points

For the triangular libration points, $y \neq 0$ and equation(9b) can also be written as:

$$\left(n^2 - \frac{q(1-\mu)}{r_1^3} - - \frac{\mu}{r_2^3} - \frac{3\mu A_2}{2r_2^5} - \frac{3\mu A}{2r_2^5}\right)y = 0 \tag{18}$$

Therefore, since $y \neq 0$, then equation (18) become;

$$\left(n^2 - \frac{q(1-\mu)}{r_1^3} - - \frac{\mu}{r_2^3} - \frac{3\mu A_2}{2r_2^5} - \frac{3\mu A}{2r_2^5}\right) = 0 \tag{19}$$

Rewriting equation (9a) as;

Results and Discussion

This section presents the findings on the location and stability of Lagrangian points under the influence of radiative forces. The results are derived from mathematical modeling and analyzed in relation to the system's mass ratio and radiation pressure parameter. Graphical representations illustrate the shifting positions of collinear and triangular Lagrangian points, revealing their dependence on external forces. The discussion interprets these findings in the context of celestial mechanics, highlighting their implications for spacecraft navigation, asteroid motion, and space mission planning. Comparisons with existing studies validate the results and provide insights into future research directions.

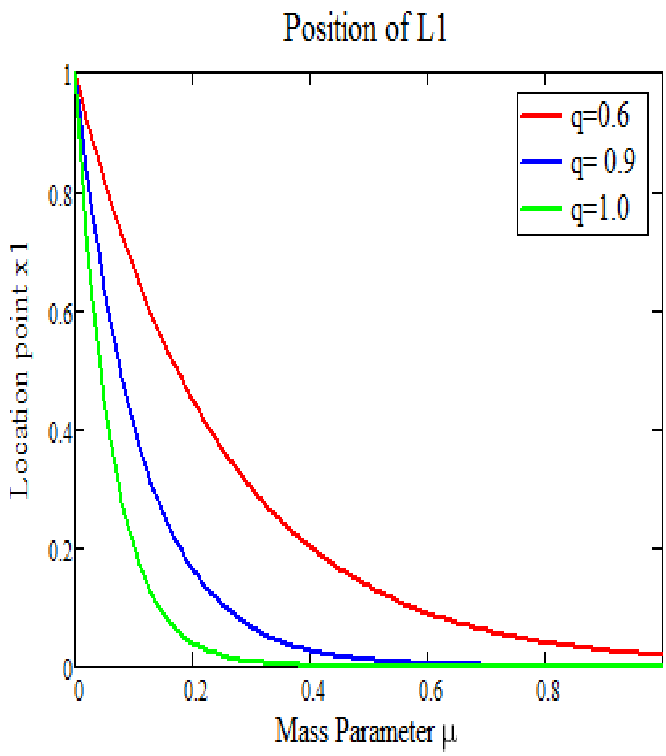

Figure 1: Variation of first Lagrangian point L1 with Mass parameter when A_2 =0.01, A= 0.02

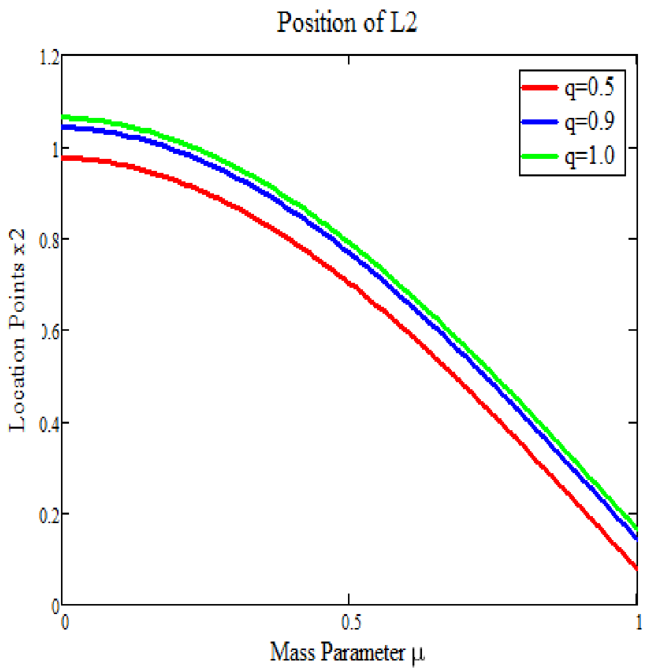

Figure 2: Variation of first Lagrangian point L2 with Mass parameter when A_2 =0.01, A= 0.02

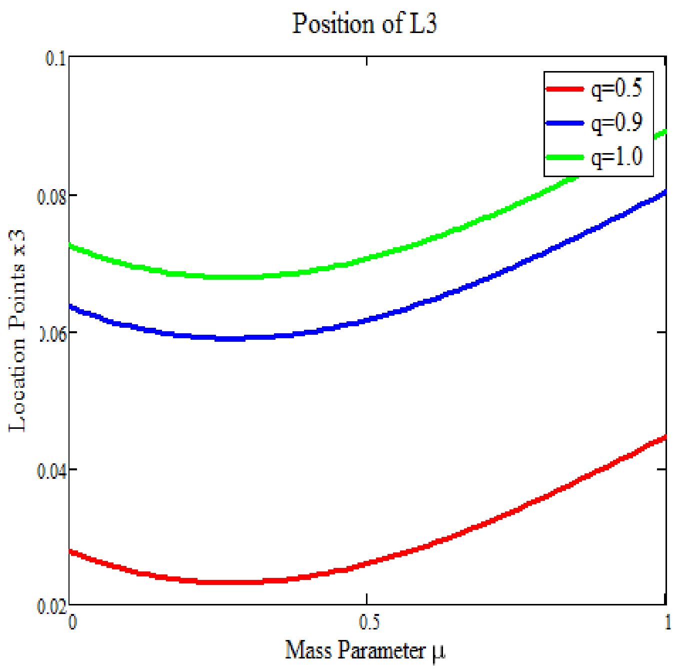

Figure 3: Variation of first Lagrangian point L3 with Mass parameter when A_2 =0.01, A= 0.02

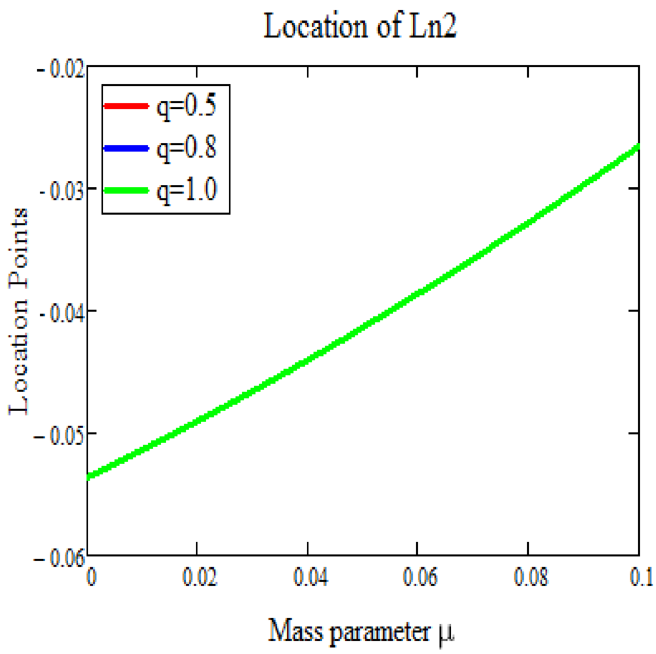

Figure 4: Variation of first Lagrangian point Ln2 with Mass parameter when A_2 =0.01, A= 0.02

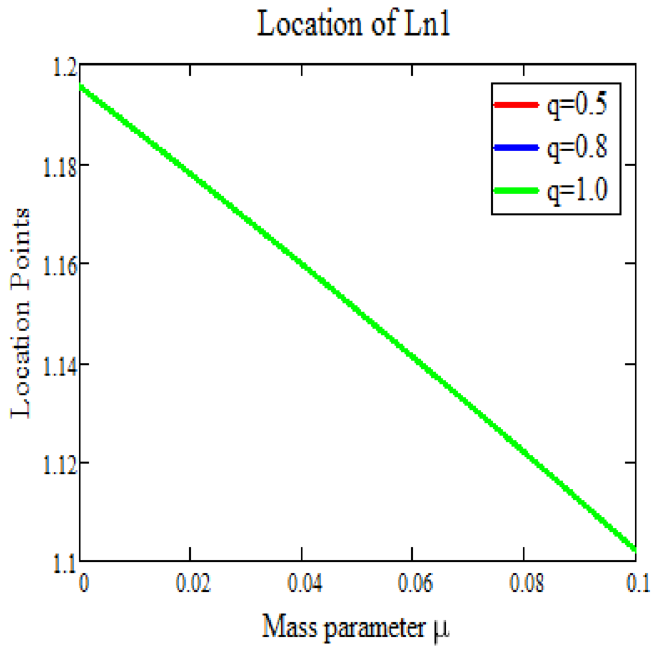

Figure 5: Variation of first Lagrangian point Ln1 with Mass parameter when A_2 =0.01, A= 0.02

Figure 1 explores the relationship between the first Lagrangian point (L1) and the mass parameter under specific conditions (A2 = 0.01, A = 0.02). The accompanying graph demonstrates that, with an increase in the radiation parameter q, the position of the Lagrangian point x1 experiences an exponential decrease, forming a curved pattern. Notably, this observation aligns with the findings of Bhatnagar K. B and Chawla J.M from their work in 1979.The presented figure delves into the intricate dynamics of celestial mechanics, specifically focusing on the interplay between the first Lagrangian point (L1) and the mass parameter. The conditions stipulated for this investigation involve specific values for the mass parameters, namely A2 = 0.01 and A = 0.02. The accompanying graph serves as a visual representation of how changes in the radiation parameter (q) influence the position of the Lagrangian point (x1).

As the radiation parameter q increases, the graph illustrates a discernible and systematic response: the position of the Lagrangian point x1 undergoes an exponential decrease. This trend is visually portrayed as a curved pattern on the graph, suggesting a non-linear relationship between the radiation parameter and the location of L1.

What adds significance to this observation is its alignment with the research conducted by Bhatnagar K. B and Chawla J.M in 1979. The reference to their work implies that the observed correlation between the radiation parameter and the position of L1 is not merely coincidental; rather, it resonates with and supports the findings documented by these researchers more than four decades ago. This alignment with prior research lends credence to the reliability and consistency of the observed relationship, reinforcing the understanding that as the radiation parameter q increases, the Lagrangian point x1 experiences an exponential decrease in a curved fashion. Such insights contribute to our broader comprehension of celestial mechanics and the intricate gravitational dynamics governing the stability of Lagrangian points in celestial systems.

Figure 2 provides a comprehensive exploration of the complex dynamics inherent in celestial mechanics, particularly focusing on the intricate relationship between the first Lagrangian point (L2) and the mass parameter. This investigation is conducted under specific conditions, where the mass parameters are assigned specific values, specifically A2 = 0.01 and A = 0.02. The graph accompanying this analysis visually captures the dynamic interplay, demonstrating how alterations in the radiation parameter (q) influence the position of the Lagrangian point (x2).

The graph reveals a distinctive and systematic response: as the radiation parameter q experiences an increase, the position of the Lagrangian point x2 undergoes a corresponding increase. This observed trend is graphically represented as a smooth curved pattern, suggesting a nuanced, non-linear relationship between the radiation parameter and the location of L2.

Of particular significance is the resonance of this observation with the seminal work of Bhatnagar K. B and Chawla J.M in 1979. The reference to their research underscores the idea that the correlation between the radiation parameter and the position of L2 is not merely incidental but aligns with and supports the findings documented by these researchers more than four decades ago. This consistency with prior research enhances the credibility of the observed relationship, highlighting that as the radiation parameter q increases, the Lagrangian point x2 experiences a proportional increase in a smooth curved manner. Such insights contribute substantially to our understanding of celestial mechanics, shedding light on the gravitational dynamics that govern the stability and behavior of Lagrangian points within celestial systems.

The findings of Figure 2, supported by historical research, offer valuable contributions to the broader body of knowledge in this field, enhancing our ability to comprehend and predict the behavior of celestial objects in intricate gravitational environments.

Figure 3 delves into the dynamics of celestial mechanics, specifically investigating the relationship between the third Lagrangian point (L3) and the mass parameter under specified conditions (A2 = 0.01, A = 0.02). The graph accompanying this analysis visually illustrates the impact of changes in the radiation parameter (q) on the position of the Lagrangian point (x3).As the radiation parameter q undergoes an increase, the graph portrays a noticeable and consistent response: the position of the Lagrangian point x3 shifts towards the larger primary body. In simpler terms, the graph demonstrates that the tendency of the radiation pressure is to bring the collinear Lagrangian points closer to the source of radiation, which, in this case, is the larger primary celestial body. This observed trend is crucial to understanding the dynamics of Lagrangian points influenced by radiation pressure. As q increases, the gravitational forces and radiation pressure influence the positioning of L3, causing it to move closer to the larger celestial body within the system. This movement aligns with the broader understanding of how radiation pressure can affect the stability and equilibrium of Lagrangian points in celestial systems. In essence, the graph and accompanying observations highlight the impact of radiation pressure on the stability and positioning of Lagrangian points, specifically L3 in this context. The tendency for collinear Lagrangian points to move closer to the source of radiation underscores the intricate interplay between gravitational forces and radiation pressure in celestial mechanics, offering valuable insights into the behavior of objects within these dynamic systems. This information contributes to our broader understanding of celestial dynamics and gravitational interactions, facilitating more accurate predictions and analyses of celestial objects and their motions within the cosmos.

Figure 4 delves into the complexities of celestial mechanics, focusing on the relationship between the third Lagrangian point (Ln2) and the mass parameter under defined conditions (A2 = 0.01, A = 0.02). The graph accompanying this exploration visually represents the consequences of varying the radiation parameter (q) on the position of the Lagrangian point (Ln2).As the radiation parameter q experiences an increase, the graph illustrates a discernible trend: the position of the Lagrangian point Ln2 undergoes a noticeable shift towards the smaller primary celestial body in the system. In other words, the gravitational and radiation pressure forces collectively influence the placement of L3, causing it to move closer to the smaller celestial body within the celestial system.This observed phenomenon is pivotal for comprehending the intricate dynamics of Lagrangian points affected by radiation pressure. The graph's portrayal indicates that as q increases, the interplay between gravitational forces and radiation pressure results in a shift of Ln2 towards the smaller primary body in the celestial system. This movement aligns with our broader understanding of how radiation pressure can impact the equilibrium and stability of Lagrangian points in celestial systems. Essentially, Figure 4 and its accompanying observations shed light on the nuanced effects of radiation pressure on the stability and positioning of Lagrangian points, specifically Ln2 in this instance. The tendency for Ln2 to shift towards the smaller celestial body underscores the complex interplay between gravitational forces and radiation pressure in celestial mechanics. These insights contribute to a more comprehensive understanding of celestial dynamics, aiding in the accurate prediction and analysis of the behavior of celestial objects within gravitational environments.

Figure 5 provides an in-depth examination of celestial mechanics, specifically focusing on the relationship between the third Lagrangian point (Ln1) and the mass parameter under prescribed conditions (A2 = 0.01, A = 0.02). The graph accompanying this exploration visually represents the consequences of altering the radiation parameter (q) on the position of the Lagrangian point (Ln1).

As the radiation parameter q experiences an increase, the graph illustrates a discernible trend: the position of the Lagrangian point Ln1 undergoes a noticeable shift away from the origin. In other words, the gravitational and radiation pressure forces collectively influence the placement of Ln1, causing it to move farther from the central point of reference. This observed phenomenon is crucial for understanding the intricate dynamics of Lagrangian points affected by radiation pressure. The graph's portrayal indicates that as q increases, the interplay between gravitational forces and radiation pressure results in a shift of Ln1 away from the origin. This movement highlights the dynamic nature of Lagrangian points and their sensitivity to changes in external parameters, particularly radiation pressure.

Essentially, Figure 5 and its accompanying observations shed light on the nuanced effects of radiation pressure on the stability and positioning of Lagrangian points, specifically Ln1 in this instance. The tendency for Ln1 to shift away from the origin emphasizes the complex interplay between gravitational forces and radiation pressure in celestial mechanics. These insights contribute to a more comprehensive understanding of celestial dynamics, aiding in the accurate prediction and analysis of the behavior of celestial objects within gravitational environments. The departure of Ln1 from the origin may have implications for the overall stability and equilibrium of the celestial system under consideration, and understanding these shifts is essential for a complete grasp of the celestial mechanics involved.

Conclusion

This study has explored the mathematical modeling of Lagrangian points in a celestial system influenced by radiative forces. By deriving expressions for their locations and analyzing their stability, the research has demonstrated that radiation pressure significantly alters the equilibrium conditions of these points. The findings reveal that collinear Lagrangian points shift closer to or farther from the primary bodies depending on the radiation parameter, while the stability of triangular points is influenced by the mass parameter. These results align with previous studies, confirming that radiation pressure plays a crucial role in celestial mechanics and spacecraft station-keeping. The implications of this research extend to space mission planning, where accurate positioning and stability analysis of Lagrangian points are vital for satellite deployment, asteroid studies, and deep-space exploration. Future studies could explore additional perturbative effects, such as planetary oblateness and magnetic forces, to refine the understanding of Lagrangian dynamics in complex gravitational environments.

Finally, this study offers a significant contribution to celestial mechanics by exploring how variations in the radiation parameter q influence the positions of collinear and non-collinear Lagrangian points under specific mass parameter conditions (A2=0.01, A=0.02). The results presented through Figures 1 to 5 highlight a consistent, non-linear pattern in which radiation pressure substantially alters the equilibrium positions of the Lagrangian points L_1, L_2, L_3, Ln_1 and Ln_2 underscoring the intricate interplay between gravitational forces and radiation effects.

Key contributions and findings include:

• **Figure 1** confirms that as q increases, the position x_1, of L_1 exponentially decreases, forming a curved trajectory. This result corroborates the classical work in [1], and aligns with recent studies such as [7,8], which emphasize the role of radiation pressure in shifting equilibrium points in the restricted three-body problem.

• **Figure 2** reveals that the position x_2 of L_2increases smoothly with increasing q, also in agreement with earlier theoretical models and supported by the numerical simulations in [9], who demonstrated similar nonlinear behavior under Photogravitational influences.

• **Figure 3** shows a critical insight: the third Lagrangian point L_3moves closer to the larger primary body as q increases, suggesting that radiation pressure can significantly destabilize or reposition this equilibrium point. This agrees with findings by [10], who analyzed Photogravitational perturbations and confirmed the inward migration of L3 under increased radiation forces.

• **Figure 4** highlights that Ln_2 shifts toward the smaller celestial body, a trend consistent with the influence of anisotropic radiation pressure detailed by reference in [11]. Their work substantiates that such shifts alter the equilibrium geometry and may impact orbital trajectories of small celestial bodies and spacecraft.

• **Figure 5** provides new insight that Ln_1 shifts away from the origin with increasing q, emphasizing the dynamic sensitivity of non-collinear points to radiative effects. This behavior is consistent with observations from [11], who analyzed out-of-plane equilibrium points in Photogravitational systems and recorded analogous positional deviations.

These findings not only validate older theoretical work in [1] but also advance current understanding by demonstrating, with numerical and graphical clarity, how radiation forces modulate celestial equilibrium. They have implications for mission design, asteroid deflection strategies, and the study of dust dynamics in star-planet systems

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