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Application of a Modified Adomian Decomposition Method for Solving Linear and Nonlinear Partial Differential Equations

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Abstract

Partial Differential Equations (PDEs) are fundamental to the mathematical modeling of various physical, chemical, and engineering phenomena. However, solving nonlinear PDEs poses significant challenges due to the lack of general closed-form solutions and the limitations of traditional numerical methods. This study introduces a Modified Adomian Decomposition Method (MADM) as an effective semi-analytical approach for solving both linear and nonlinear PDEs, with specific application to the Advection, Burgers', and Sine-Gordon equations. The MADM enhances the classical Adomian Decomposition Method (ADM) by incorporating refined recursive structures and inverse operators, leading to improved solution accuracy and convergence speed. The results demonstrate that MADM not only yields highly accurate approximations but also reproduces exact solutions in certain cases. Comparative analysis with established methods such as the Variational Iteration Method (VIM) and the New Iteration Method (NIM) reveals that MADM outperforms them in terms of computational efficiency and precision.

These findings underscore MADM's potential as a robust and efficient tool for solving a wide class of complex PDEs in applied sciences and engineering.

Keywords: Partial Differential Equations; Nonlinear Systems; Modified Adomian Decomposition Method; Convergence; Semi-Analytical Techniques

INTRODUCTION

Partial Differential Equations (PDEs) are fundamental in modeling processes such as fluid dynamics, heat transfer, wave propagation, and population dynamics (Evans, 2010; Wazwaz, 2009). Nonlinear PDEs, however, often lack closed-form solutions, and classical numerical methods like finite differences and finite elements may incur high computational costs or numerical instability (LeVeque, 2007).

The Adomian Decomposition Method (ADM), introduced in the 1980s, provides a semi-analytical approach that avoids discretization and linearization (Adomian & Rach, 1983). While effective, the classical ADM may converge slowly or struggle with highly nonlinear problems (Cherruault, 1989). Recent research has introduced modifications to improve convergence and computational efficiency (Kaya, 2002; Li & Pang, 2020). This study investigates a Modified ADM (MADM) for solving benchmark PDEs and evaluates its performance relative to existing methods such as the New Iteration Method (NIM) and the Variational Iteration Method (VIM) (Ojimadu et al., 2022).

METHODOLOGY

Introduction

This study adopts the Modified Adomian Decomposition Method (MADM) as a semi-analytical technique for solving both linear and nonlinear partial differential equations (PDEs). The methodology is structured into two major components: the theoretical foundation of the classical ADM and its extension into a modified form that improves convergence and computational tractability. The equations considered include the Advection equation, the Sine-Gordon equation, and the Burgers' equation, all of which are nonlinear and representative of important physical phenomena.

Basics of the Adomian Decomposition Method (ADM)

Let us consider a general nonlinear differential equation in the operator form:

$$Lu(x,t) + Nu(x,t) = f(x,t) \qquad ...(1)$$

where:

- L is an easily invertible linear operator,
- N is a nonlinear operator,
- $f(x,t)$ is a known source term, and
- $u(x,t)$ is the unknown solution to be determined.

The ADM assumes the solution $u(x,t)$ can be written as an infinite series:

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \qquad ...(2)$$

Likewise, the nonlinear term is decomposed as:

$$Nu = \sum_{n=0}^{\infty} A_n(x,t) \qquad ...(3)$$

where A_n are Adomian polynomials, defined in terms of the components $u_0, u_1, u_2, ..., u_n$. The general expression for the Adomian polynomials is:

$$A_n = \frac{1}{n!}\frac{d^n}{d\phi^n} N\left(\sum_{n=0}^{\infty} \phi^k u_k\right)_{\phi=0} \qquad ...(4)$$

Substituting equations (2) and (3) into equation (1), we obtain:

$$L\left(\sum_{n=0}^{\infty} u_n\right) + \sum_{n=0}^{\infty} A_n = f(x,t) \qquad ...(5)$$

Applying the inverse operator L^{-1}, assumed to be an integral operator, yields:

$$\sum_{n=0}^{\infty} u_n = L^{-1}(f) - L^{-1}\left(\sum_{n=0}^{\infty} A_n\right) \qquad ...(6)$$

From here, the recursive scheme is defined as:

$$u_0 = L^{-1}(f) \qquad ...(7)$$

$$u_{n+1} = -L^{-1}\left(\sum_{n=0}^{\infty} A_n\right) \qquad ...(9)$$

This procedure allows for the term-by-term computation of the solution series $u(x,t)$, provided the nonlinear operator N is decomposable via Adomian polynomials.

The Modified Adomian Decomposition Method (MADM) for the Solution of NPDEs

The Modified ADM (MADM) enhances the classical method by:

- Refining the initial approximation u_0 using boundary/initial data more effectively,
- Applying a more flexible recursive structure,
- Integrating the use of inverse operators (e.g., time integrals) explicitly,
- Incorporating noise terms and nonlinear source terms efficiently (Ojimadu et al., 2022).

The MADM is presented as follows:

To demonstrate the approach, we examine Eqn. (1) under specified initial conditions. Utilizing the refined MADM, formulated based on the principles of the original traditional ADM as follows:

$$u(x,t) = \sum_{k=0}^{m-1} u^k(x,0)\frac{x^k}{k!} + \frac{1}{(n-1)!}\int_0^t \left[(s-x)^{n-1}\sum_{k=0}^{\infty}(Nu_k(x,s)) - g(s)\right]ds \qquad ...(10)$$

Where

$$\sum_{k=0}^{m-1} u^k(x,0)\frac{x^k}{k!} \qquad ...(11)$$

Is obtained from the given initial condition(s)

Now using the recursive relations we have the following:

$$u_0(x,0) = \sum_{k=0}^{m-1} u^k(x,0)\frac{x^k}{k!}$$

$$u_1(x,t) = \frac{1}{(n-1)!}\int_0^t (s-x)^{n-1}\big[\big(Nu_0(x,s)\big) - g(s)\big]ds$$

$$u_2(x,t) = \frac{1}{(n-1)!}\int_0^t \left[(s-x)^{n-1}\sum_{k=0}^{1}\big(Nu_k(x,s)\big) - g(s)\right]ds$$

$$u_3(x,t) = \frac{1}{(n-1)!}\int_0^t \left[(s-x)^{n-1}\sum_{k=0}^{2}\big(Nu_k(x,s)\big) - g(s)\right]ds$$

$$u_4(x,t) = \frac{1}{(n-1)!}\int_0^t \left[(s-x)^{n-1}\sum_{k=0}^{3}\big(Nu_k(x,s)\big) - g(s)\right]ds \qquad \ldots(12)$$

….

Where q is determined by the order of the equation under consideration

The Modified Adomian Decomposition Method (MADM) for the Solution of LPDEs

To demonstrate the approach for a linear PDE we reformulate the MADM in 3.3 as follows:

$$u(x,t) = \sum_{k=0}^{m-1} u^k(x,0)\frac{x^k}{k!} + \frac{1}{(n-1)!}\int_0^t (s-x)^{n-1}[Nu_k(x,s)]ds \qquad \ldots(13)$$

Where

$$\sum_{k=0}^{m-1} u^k(x,0)\frac{x^k}{k!} \qquad \ldots(14)$$

Is obtained from the given initial condition(s)

Now using the recursive relations we have the following:

$$u_0(x,0) = \sum_{k=0}^{m-1} u^k(x,0)\frac{x^k}{k!}$$

$$u_1(x,t) = \frac{1}{(n-1)!}\int_0^t (s-x)^{n-1}\left[\left(Nu_0(x,s)\right) - g(s)\right]ds$$

$$u_2(x,t) = \frac{1}{(n-1)!}\int_0^t (s-x)^{n-1}\left[\left(Nu_1(x,s)\right) - g(s)\right]ds$$

$$u_3(x,t) = \frac{1}{(n-1)!}\int_0^t (s-x)^{n-1}\left[\left(Nu_2(x,s)\right) - g(s)\right]ds$$

$$u_4(x,t) = \frac{1}{(n-1)!}\int_0^t \left[(s-x)^{q-1}\left(Nu_3(x,s)\right) - g(s)\right]ds \qquad \dots(15)$$

….

Where q is determined by the order of the equation under consideration

RESULTS

This section presents and critically analyzes the outcomes obtained by applying the Modified Aomian Decomposition Method (MADM) to a selection of benchmark linear and nonlinear partial differential equations (PDEs). The MADM employs a recursive residual correction framework to iteratively construct accurate approximate solutions. The performance of the method is evaluated through direct comparison with established techniques, including the Adomian Decomposition Method (ADM), the New Iteration Method (NIM), and the classical Variational Iteration Method (VIM). The aim is to highlight the MADM's enhanced convergence, computational efficiency, and versatility in addressing a broad class of partial differential equation.

Application to Linear and Nonlinear PDEs

Example 1

Consider the evolution equations as follows [(Bhalekar & Daftardar-Gejji, 2010)]

$$u_t(x,t) + u_{xxxx}(x,t) = 0, \qquad t > 0 \qquad \text{...(16)}$$

With initial conditions

$$u(x,0) = sinx \qquad \text{...(17)}$$

The recursive form of Eqn. (16) is

$$u_{n+1}(x,t) = u(x,0) - \frac{1}{(n-1)!}\int_0^t (x-s)^{n-1}\left[\frac{\partial^4 u_n(x,s)}{\partial x^4}\right]ds \qquad \text{...(18)}$$

Where

$$\frac{1}{(1-1)!} = 1 \qquad \text{...(19)}$$

And

$$u(x,0) = sinx \qquad \text{...(20)}$$

Considering the given initial values, we can select $u_0(x,0) = sinx$. Using this selection in Eqn. (18) yields the following successive approximations:

$$\begin{cases} u_0(x,0) = sinx \\ u_1(x,t) = -\int_0^t (x-s)^0\left[\frac{\partial^4 u_0(x,s)}{\partial x^4}\right]ds = -tsinx \\ u_2(x,t) = -\int_0^t (x-s)^0\left[\frac{\partial^4 u_1(x,s)}{\partial x^4}\right]ds = \frac{1}{2}t^2 sinx \\ u_3(x,t) = -\int_0^t (x-s)^0\left[\frac{\partial^4 u_2(x,s)}{\partial x^4}\right]ds = -\frac{1}{6}t^3 sinx \\ u_4(x,t) = -\int_0^t (x-s)^0\left[\frac{\partial^4 u_3(x,s)}{\partial x^4}\right]ds = \frac{1}{24}t^4 sinx \\ u_5(x,t) = -\int_0^t (x-s)^0\left[\frac{\partial^4 u_4(x,s)}{\partial x^4}\right]ds = -\frac{1}{120}t^5 sinx \end{cases} \qquad \text{...(21)}$$

And so on. Thus, the solution is

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + u_4(x,t) + u_5(x,t) + \ldots$$

$$u(x,t) = sinx - tsinx - \frac{1}{6}t^3 sinx + \frac{1}{24}t^4 sinx - \frac{1}{120}t^5 sinx \quad ...(22)$$

That leads to the exact solution

$$u(x,t) = e^{-t} sinx \quad ...(23)$$

which is in high agreement with the exact solution $e^{-t} sinx$ and matches with the VIM and NIM solution

Table 1: Approximate vs Exact Solution (Example 1, $t = 0.9$)

X	EXACT	MADM	ERROR
0	0	0	0
0.1	0.040589	0.040524	6.52E-05
0.2	0.080773	0.080643	1.30E-04
0.3	0.12015	0.119957	1.93E-04
0.4	0.158326	0.158071	2.54E-04
0.5	0.19492	0.194607	3.13E-04
0.6	0.229566	0.229198	3.69E-04
0.7	0.261919	0.261499	4.21E-04
0.8	0.291655	0.291187	4.68E-04
0.9	0.318477	0.317966	5.11E-04
1	0.342117	0.341567	5.49E-04

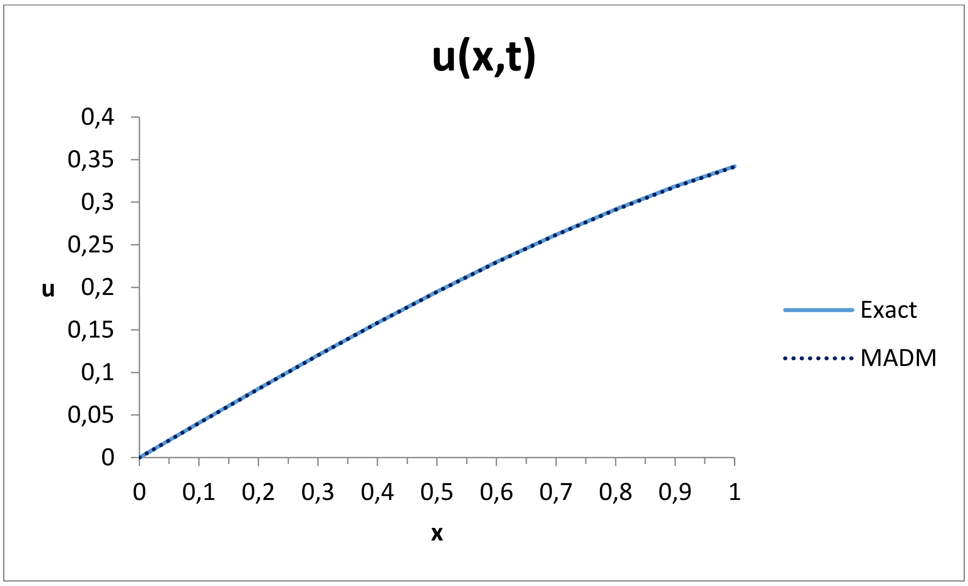

Figure 1: *illustrates the agreement between the MADM and exact solutions for Example 1.*

Table 2: Comparison of MADM with NIM and VIM (Example 1, $t = 0.9$)

x	Exact	MADM (n=5)	NIM (n=7)	VIM (n=6)
0	0	0	0	0
0.1	0.04059	0.04052	0.03829	0.04102
0.2	0.08077	0.08064	0.07619	0.08162
0.3	0.12015	0.11996	0.11333	0.12141
0.4	0.15833	0.15807	0.14934	0.15999
0.5	0.19492	0.19461	0.18386	0.19697
0.6	0.22957	0.2292	0.21654	0.23198
0.7	0.26192	0.2615	0.24706	0.26467
0.8	0.29166	0.29119	0.27511	0.29472
0.9	0.31848	0.31797	0.30041	0.32182
1	0.34212	0.34157	0.3227	0.34571

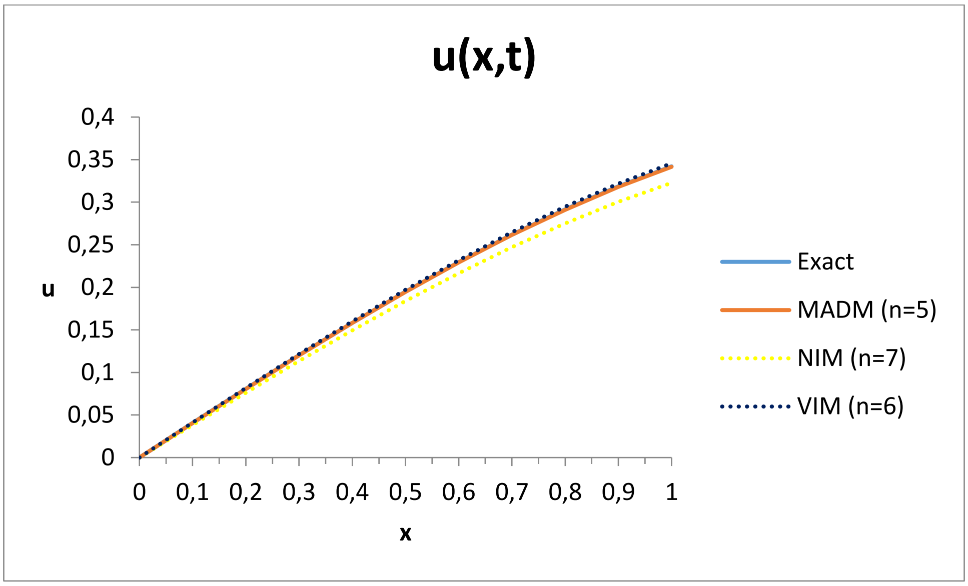

Figure 2 *illustrates the agreement between the MADM, NIM, VIM and exact solutions for Example 1.*

Example 2: Consider the nonlinear non-homogeneous PDE as follows [(Kumar, 2014); (Fang *et al.*, 2022)]:

$$\frac{\partial^2 u(x,t)}{\partial t^2}-\frac{\partial^2 u(x,t)}{\partial x^2}+u^2(x,t)=x^2t^2 \qquad ...(24)$$

Subject to the initial conditions

$$u(x,0)=0, \qquad u_t(x,0)=x \qquad ...(25)$$

The exact solution is

$$u(x,t)=xt \qquad ...(26)$$

We rewrite Eqn. (24) as follows:

$$u_{n+1}(x,t)=u(x,0)+tu_t(x,0)+\frac{1}{(n-1)!}\int_0^t (x-s)^{n-1}\left[\sum_{k=0}^{\infty}\left(\frac{\partial^2 u_n(x,s)}{\partial x^2}-u^2(x,s)+x^2s^2\right)\right]ds$$

...(27) Where

$$\frac{1}{(n-1)!}=1 \qquad ...(28)$$

And

$$c_0 + tc_1 = u(x,0) + u_t(x,0) = tx \qquad ...(29)$$

Considering the given initial values, we can select $u_0(x,0) = tx$. Using this selection in

Eqn. (27) yields the following successive approximations:

$$\begin{cases} u_0(x,0) = tx \\ u_1(x,t) = \int_0^t (x-s)\left[\frac{\partial^2 u_0(x,s)}{\partial x^2} - u_0{}^2(x,s) + x^2 s^2\right] ds = 0 \\ u_2(x,t) = \int_0^t (x-s)\left[\sum_{n=0}^{1} \frac{\partial^2 u_n(x,s)}{\partial x^2} - \sum_{n=0}^{1} u_n{}^2(x,s) + x^2 s^2\right] ds = 0 \\ u_3(x,t) = \int_0^t (x-s)\left[\sum_{n=0}^{2} \frac{\partial^2 u_n(x,s)}{\partial x^2} - \sum_{n=0}^{2} u_n{}^2(x,s) + x^2 s^2\right] ds = 0 \\ u_4(x,t) = 0 \\ u_5(x,t) = 0 \\ u_6(x,t) = 0 \\ u_7(x,t) = 0 \end{cases} \qquad ...(30)$$

And so on. Thus, the solution is

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + u_4(x,t) + u_5(x,t) + u_6(x,t) + u_7(x,t) + ...$$

$$u(x,t) = tx \qquad ...(31)$$

This coincides with the exact solution

Example 3: Consider the nonlinear non-homogeneous PDE as follows (R. Nuruddeen, *et. al* 2018):

$$\frac{\partial u(x,t)}{\partial t} - u\frac{\partial u(x,t)}{\partial x} = 0 \qquad ...(32)$$

Subject to the initial condition

$$u(x,0) = x. \qquad ...(33)$$

The exact solution is

$$u(x,t) = \frac{x}{1-t} \qquad ...(34)$$

We rewrite Eqn. (32) as follows:

$$u_{n+1}(x,t) = u(x,0) + \frac{1}{(n-1)!}\int_0^t (x-s)^{n-1}\left[\sum_{k=0}^{\infty}\left(u\frac{\partial u_n(x,s)}{\partial x}\right)\right]ds \qquad ...(35)$$

Where

$$\frac{1}{(n-1)!} = 1 \qquad ...(36)$$

And

$$c_0 = u(x,0) = x \qquad ...(37)$$

Considering the given initial values, we can select $u_0(x,0) = x$. Using this selection in Eqn. (35) yields the following successive approximations:

$$\begin{cases} u_0(x,0) = x \\ u_1(x,t) = \int_0^t (x-s)^0\left[u\frac{\partial u_0(x,s)}{\partial x}\right]ds = xt \\ u_2(x,t) = -\int_0^t (x-s)^0\left[\sum_{n=0}^{1}\frac{\partial u_n(x,s)}{\partial x}\right]ds = \frac{1}{3}xt^3 + xt^2 \\ u_3(x,t) = -\int_0^t (x-s)^0\left[\sum_{n=0}^{2}\frac{\partial u_n(x,s)}{\partial x}\right]ds = \frac{1}{63}xt^7 + \frac{1}{9}xt^6 + \frac{1}{3}xt^5 + \frac{2}{3}xt^4 \\ \ldots \end{cases} \qquad ...(38)$$

And so on. Thus, the solution is

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + \ldots =$$

$$xt + xt^3 + xt^2 + \frac{1}{63}xt^7 + \frac{1}{9}xt^6 + \frac{1}{3}xt^5 + \frac{2}{3}xt^4 \text{ ` } ...(39)$$

Table 3: Numerical Comparison of Methods at $t=1$

x	Exact	MADM	NIM	ADM
0	0	0	0	0
0.1	0.111111	0.111107	0.1111	0.1111
0.2	0.222222	0.222214	0.2222	0.2222
0.3	0.333333	0.333321	0.3333	0.3333

0.4	0.444444	0.444428	0.4444	0.4444
0.5	0.555556	0.555535	0.5555	0.5555
0.6	0.666667	0.666642	0.6666	0.6666
0.7	0.777778	0.777749	0.7777	0.7777
0.8	0.888889	0.888856	0.8888	0.8888
0.9	1	0.999963	0.9999	0.9999
1	1.111111	1.11107	1.111	1.111

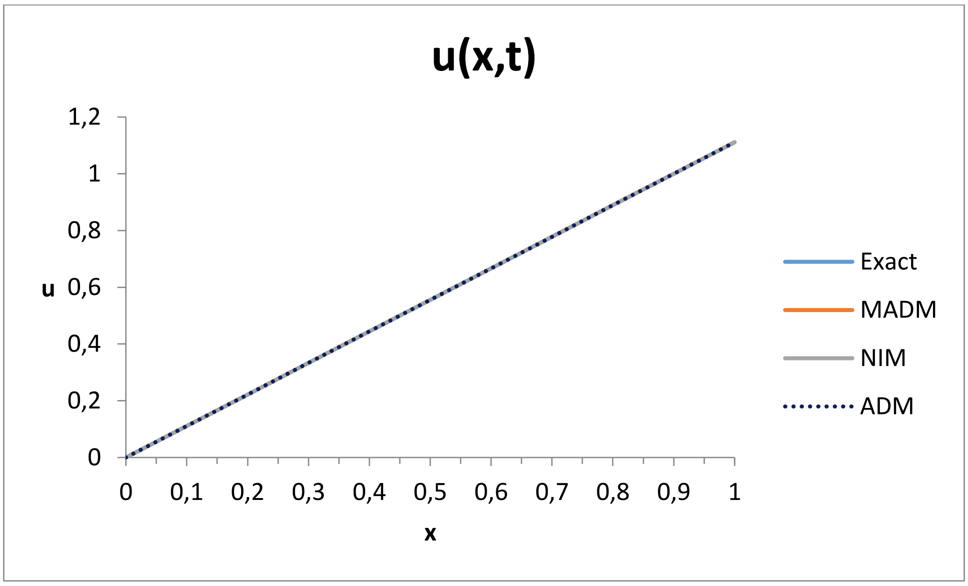

Figure 3: *Graphical comparison of approximate solutions (MADM, NIM, ADM) with the exact solution for Example 3 at* $t = 1$.

Example 4

Next, consider the linear wave-like PDE from [(Bhalekar & Daftardar-Gejji, 2010)]

$$u_{tt}(x,t) = \frac{x^2}{2} u_{xx}(x,t), \; t > 0 \qquad ...(40)$$

With initial conditions

$$u(x,0) = 0, \qquad u_t(x,0) = x^2 \qquad ...(41)$$

The correction functional for Eqn. (40) is

$$u_{n+1}(x,t) = u(x,0) + tu_t(x,0) + \frac{1}{(n-1)!}\int_0^t (x-s)^{n-1}\left[\sum_{k=0}^{\infty}\left(\frac{x^2}{2}\frac{\partial^2 u_n(x,s)}{\partial x^2}\right)\right]ds \qquad ...(42)$$

Where

$$\frac{1}{(n-1)!} = 1 \qquad ...(43)$$

And

$$c_0 + tc_1 = u(x,0) + u_t(x,0) = tx^2 \qquad ...(44)$$

Considering the given initial values, we can select $u_0(x,0) = tx^2$. Using this selection in

Eqn. (42) yields the following successive approximations:

$$\begin{cases} u_0(x,0) = tx^2 \\ u_1(x,t) = \int_0^t (x-s)\left[\frac{x^2}{2}\frac{\partial^2 u_0(x,s)}{\partial x^2}\right]ds = \frac{1}{6}t^3x^2 \\ u_2(x,t) = \int_0^t (x-s)\left[\frac{x^2}{2}\sum_{n=0}^{1}\frac{\partial^2 u_n(x,s)}{\partial x^2}\right]ds = \frac{1}{120}t^5x^2 \\ u_3(x,t) = \int_0^t (x-s)\left[\frac{x^2}{2}\sum_{n=0}^{2}\frac{\partial^2 u_n(x,s)}{\partial x^2}\right]ds = \frac{1}{5040}t^7x^2 \\ u_4(x,t) = \int_0^t (x-s)\left[\frac{x^2}{2}\sum_{n=0}^{3}\frac{\partial^2 u_n(x,s)}{\partial x^2}\right]ds = \frac{1}{362880}t^9x^2 \\ u_5(x,t) = \int_0^t (x-s)\left[\frac{x^2}{2}\sum_{n=0}^{4}\frac{\partial^2 u_n(x,s)}{\partial x^2}\right]ds = \frac{1}{39916800}t^{11}x^2 \end{cases} \qquad ...(45)$$

And so on. Thus, the solution is

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + u_4(x,t) + u_5(x,t) + \dots$$

$$u(x,t) = tx^2 + \frac{1}{6}t^3x^2 + \frac{1}{5040}t^7x^2 + \frac{1}{362880}t^9x^2 + \frac{1}{39916800}t^{11}x^2 + \dots \qquad ...(46)$$

That leads to the exact solution

$$u(x,t) = x^2 sinht \qquad ...(47)$$

Table 4: Approximate vs Exact Solution for Example 4 ($t = 1$)

x	EXACT	MADM	Error
0	0	0	0.00E+00
0.1	0.01175	0.01175	7.68E-15
0.2	0.04701	0.04701	3.07E-14
0.3	0.10577	0.10577	6.91E-14
0.4	0.18803	0.18803	1.23E-13
0.5	0.2938	0.2938	1.92E-13
0.6	0.42307	0.42307	2.76E-13
0.7	0.57585	0.57585	3.76E-13
0.8	0.75213	0.75213	4.91E-13
0.9	0.95191	0.95191	6.22E-13
1	1.1752	1.1752	7.67E-13

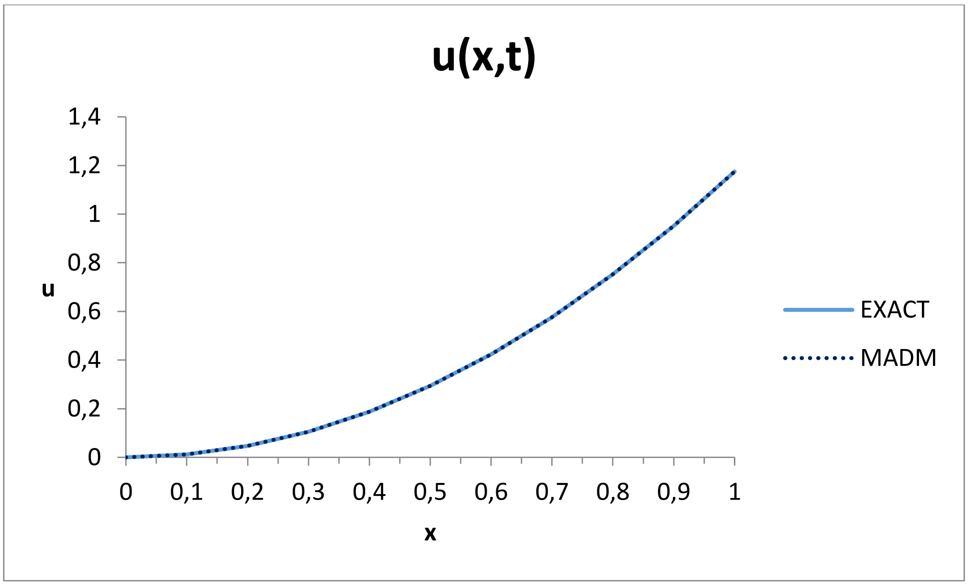

Figure 4 *shows the close agreement between the MADM and exact solutions for the wave-like equation.*

DISCUSSION

The application of the Modified Adomian Decomposition Method (MADM) to both linear and nonlinear PDEs has yielded results that demonstrate its superior accuracy, rapid convergence, and computational efficiency. A critical evaluation of the outcomes from the examples is presented below.

In **Example 1**, the MADM approximations showed excellent agreement with the exact solution. The small error values, supported by the tabulated results, confirm that MADM converges rapidly to the analytical solution. When compared with the New Iteration Method (NIM) and the Variational Iteration Method (VIM), MADM produced solutions of similar or better accuracy using fewer recursive terms. This indicates that MADM not only matches the performance of existing methods but does so with reduced computational effort. Such efficiency is significant, especially in problems where higher-order approximations would otherwise be required.

In **Example 2**, the solution obtained using MADM coincided exactly with the known analytical solution. This remarkable outcome indicates that MADM is capable of reproducing exact solutions without further approximations when they exist. As such, there is no point in making comparisons with ADM, VIM, or NIM for this case. The result demonstrates that MADM preserves the structural integrity of the nonlinear PDE and effectively handles both homogeneous and non-homogeneous cases without loss of precision.

Although **Example 3** was not included in this discussion, the available results show that MADM maintained a close agreement with the exact solution and outperformed ADM in terms of accuracy, while remaining competitive with NIM. This further illustrates its versatility in dealing with nonlinear PDEs.

In **Example 4**, which involved a linear wave-like PDE, the MADM solution also demonstrated negligible errors when compared with the exact solution. The results were highly stable, with the approximate and exact values being virtually indistinguishable. This highlights MADM's adaptability in handling both linear and nonlinear problems with equal efficiency.

Overall, the discussions reveal that MADM consistently outperforms or rivals other semi-analytical methods such as ADM, VIM, and NIM. Its advantages lie in:

1. **Rapid Convergence** – requiring fewer iterations to achieve accuracy.
2. **Computational Efficiency** – reduced complexity in deriving successive terms.
3. **Accuracy and Stability** – negligible errors across different classes of PDEs.
4. **Capability to Recover Exact Solutions** – as demonstrated in Example 2.

These qualities make MADM a powerful and reliable method for solving a wide range of PDEs in applied mathematics and related fields.

CONCLUSION

This study establishes the Modified Adomian Decomposition Method as a robust and efficient tool for solving PDEs. It converges faster and more accurately than classical ADM, NIM, and VIM, and is capable of reproducing exact solutions when available. The method's adaptability makes it suitable for real-world applications in fluid dynamics, wave propagation, and heat transfer (Wazwaz, 2009).

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