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Some Studies on the Topology of Power Set

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Abstract

This paper investigates the topological structure of the power set of an infinite set XX, focusing on properties such as extreme and total disconnectedness, as well as the hierarchy of separation axioms τ0,τ1,τ2,τ3,τ4,τ5,τ6\tau_0, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6. By defining a topology on the power set, the study explores how classical topological concepts manifest in this context and introduces a novel perspective that bridges the power set with a Universal Topological Space. The analysis contributes to a deeper understanding of how separation properties and disconnectedness behave in non-traditional topological constructions, offering a foundational approach for further exploration in abstract topological frameworks.

Keywords: Topological Spaces; Separation Axioms; Power Set; Extremely Disconnected; Totally Disconnected

Introduction

Given that the set X is a finite set, the power set of such set will definitely be finite, so the number of topologies defined on such set must be finite. In this paper, we consider set X to be infinite so that we can have infinite power set and the number of topologies on the set. Throughout this paper, we use $\mathcal{P}(X)$ to stand for power set of infinite set X and $(X, \tau_{\mathcal{P}(X)})$ will be a topological space, called universal topological space (Amulya, 2019).

Definition of some important terms

Definition 1 (Amulya, 2019): Let X be an infinite set and $\mathcal{P}(X)$ be power set of X. The space $(X, \tau_{\mathcal{P}(X)})$ is called the universal topological space, where $\tau_{\mathcal{P}(X)}$ is the topology on the power set $\mathcal{P}(X)$.

Definition 2 (Kiltho and Morawo, 2020): Let $(X, \tau_{\mathcal{P}(X)})$ be a topological space, it is extremely disconnected if the closure of every open subset of $(X, \tau_{\mathcal{P}(X)})$ is open.

Definition 3 (Willard, 2004): A topological space X is locally compact if every point $x \in X$ has a compact neighborhood of X.

Definition 4 (Marcoux, 2022): A topological space X is $\tau_o - space$ (Kolmogorov space) if for every $x_1, x_2 \in X$, such that $x_1 \neq x_2$, either there is a neighborhood N_{x_1} of x_1 with $x_2 \notin N_{x_1}$ or there is a neighborhood N_{x_2} of x_2 with $x_1 \notin N_{x_2}$.

Definition 5 (Roydon, 2000): A topological space X is $\tau_1 - space$ (Tychonoff or Frechet space), if given two points $x_1, x_2 \in X$, there is an open set $A \in X$ such that $x_1 \in A$, $x_2 \notin A$ or $x_2 \in A, x_1 \notin A$.

Definition 6 (Paige, 2022): A topological space X is Hausdorff (τ_2) if for $A, B \in X, A \cap B = \emptyset$.

Definition 7 (Roydon, 2000): A $\tau_1 - space$ (X, τ) is said to be regular ($\tau_3 - space$), if given any closed set $P \subseteq X$ and $x \in X \backslash P$, there exist $A, B \in \tau$ such that $x \in A$, $cl(P \subseteq B)$ and $cl(A \cap B) = \emptyset$.

Definition 8 (Roydon, 2000): A $\tau_1 - space$ (X, τ) is said to be normal ($\tau_4 - space$), if given any two disjoint closed sets $P, Q \subseteq X$, there exist $A, B \in \tau$ such that $P \subseteq A, Q \subseteq B$ and $A \cap B = \emptyset$.

Definition 9 (Micheal, 2016): A $\tau_1 - space$ (X, τ) is said to be hereditary normal ($\tau_5 - space$ or completely normal), if all its subspaces including itself are normal.

Definition 10 (Micheal, 2016): A $\tau_1 - space$ (X, τ) is perfectly normal ($\tau_6 - space$) if given any two disjoint closed sets $P, Q \subseteq X$, there is a function $f \in C(X, [0,1])$ such that $f^{-1}(0) = P$ and $f^{-1}(1) = Q$.

Methodology

According to (Micheal, 2016), Since the open sets separate points from closed sets then, open sets separate the closed sets. Some Authors differentiate between τ_3 and regular spaces, between τ_4 and normal spaces by defining one of the two terms as including $\tau_1 - space$ but no other separation axioms. Some Authors define $\tau_3 - space$ as $\tau_3 =$ regular $+ \tau_1$, some define it as regular $= \tau_3 + \tau_1$, which may be arbitrary, provided that $\tau_3 \nRightarrow \tau_2$. Provided that we assume all $\tau_i - axioms$ to include τ_1 as stated in some of the definitions above, singleton sets are closed therefore, $\tau_6 \Rightarrow \tau_5 \Rightarrow \tau_4 \Rightarrow \tau_3 \Rightarrow \tau_2 \Rightarrow \tau_1 \Rightarrow \tau_o$. Note that converse of any of this implication is not true.

Lemma 1 (Micheal, 2016): For a $\tau_1 - space$ X, then, the following are equivalent:

(i). X is $\tau_4 - space$

(ii) For every pair $P, Q \subseteq X$ of open sets satisfying $P \cup Q = X$, there are closed sets $A, B \subseteq X$ such that $A \subseteq P$, $B \subseteq Q$ and $A \cup B = X$.

(iii) Whenever $A \subseteq P$ with A closed and P open, there is $Q \in \tau$ such that $A \subseteq Q \subseteq cl(Q) \subseteq P$.

Lemma 2 (Micheal, 2016): If the $\tau_1 - space$ (X, τ) is normal and $f: (X, \tau) \to (Y, d)$ is surjective, continuous and closed. Then, Y is normal.

Lemma 3 (Amulya, S, 2019): If a set X is finite, then the collection of all subsets of X must also be finite. Also, if a set X is infinite, the collection of all subsets of X must also be infinite.

Lemma 4 (Amulya, S, 2019): If a set X is finite, then the topology on X is finite.

Lemma 5 (Amulya, S, 2019): If a set X is infinite, then the topology on X must be infinite and hence, it is discrete topology.

Lemma 6 (Simmons, 1983): let X be a τ_1 such that for $A \subseteq O \subseteq X$, where A is closed and O is open, there is a countable family $\{F_n\}, n \in \mathbb{N}$, of open sets such that $A \subseteq \bigcup_n F_n$ and $cl(F_n) \subseteq O, \forall\, n$. Then X is normal.

Lemma 7 (Micheal, 2016): $\tau_4 \Rightarrow \tau_{3\frac{1}{2}} \Rightarrow \tau_3$.

Results and Discussion

In the following results, since the topological space is infinite, then, we are going to use **Lemma 3** and **Lemma 5** above to achieve our aim.

Theorem 1: The topological space $(X, \tau_{\mathcal{P}(X)})$ is both totally and extremely disconnected.

Proof: The proof of the theorem will be in two stages: **stage I** for totally disconnectedness and **stage II** for extremely disconnectedness.

Stage I: Given that topological space $(X, \tau_{\mathcal{P}(X)})$ is infinite such that we have various topologies as shown below:

Let $\tau_1 = \{X, \emptyset, t_1\}$, $\tau_2 = \{X, \emptyset, t_1, t_2\}$, $\tau_3 = \{X, \emptyset, t_1, t_2, t_3\}$, $\tau_4 = \{X, \emptyset, t_1, t_2, t_3, t_4\}$,… $\tau_k = \{X, \emptyset, t_1, t_2, t_3, \dots, t_k\}$, where $t_1 \neq t_2 \neq t_3 \neq t_4$. Provided that the set X is discrete, so the space $(X, \tau_{\mathcal{P}(X)})$ is discrete topological space. Therefore, the separation axioms above are the corresponding discrete topologies defined on the set X. Since these separation axioms called topologies are discrete and contain multiple points, therefore, the space $(X, \tau_{\mathcal{P}(X)})$ is totally disconnected, due to the fact that every discrete space is totally disconnected, that is $(X, \tau_{\mathcal{P}(X)}) = \bigcup_{k=1}^{\infty} \tau_k$.

Stage II: To prove for the extremely disconnectedness of topological space $(X, \tau_{\mathcal{P}(X)})$. Given two non disjoint subspaces M and N of $Y \subset (X, \tau_{\mathcal{P}(X)})$. If $M = A \cap Y$, where $A \subset Y$, and there is neighborhood of M, O_M, say, such that $P \subset O_M$, then, every neighborhood of N, O_N, say, of P must not be disjoint with an open set A, that is $O_N \cap A \neq \emptyset$. Sine M and N are disjoint, $P \subset cl_{(X, \tau_{\mathcal{P}(X)})}(A)$.

Conversely, if there is K and $O_{Y \setminus M} \cap A = \emptyset$, then $cl_{(X, \tau_{\mathcal{P}(X)})}(A) = O_M$. Therefore, the topological space $(X, \tau_{\mathcal{P}(X)})$ is extremely disconnected. Since $cl_{(X, \tau_{\mathcal{P}(X)})}(A) = O_M$, then $cl_{(X, \tau_{\mathcal{P}(X)})}(O_M) = O_M$. This shows that the topological space $(X, \tau_{\mathcal{P}(X)})$ is zero dimensional. In fact, if $Y = M \cup N,\ M \cap N \neq \emptyset$, then, the set $O_M \cup O_N = \left(X, \tau_{\mathcal{P}(X)}\right)$ and $O_M \cap O_N = \emptyset$.

Since discrete topological space is locally compact, then we use Lemma 5 above in achieving our result in **Theorem 2** below.

Theorem 2: Topological space $\left(X, \tau_{\mathcal{P}(X)}\right)$ is locally compact.

Proof: Since the set X is discrete, it implies that the topological space $\left(X, \tau_{\mathcal{P}(X)}\right)$ is discrete. As $\left(X, \tau_{\mathcal{P}(X)}\right)$ is discrete, then the singleton set $\{x\}$ is a compact neighborhood of each point $x \in \left(X, \tau_{\mathcal{P}(X)}\right)$. Therefore, the topological space $\left(X, \tau_{\mathcal{P}(X)}\right)$ is locally compact.

In the next result, we are going to make use of **Theorem 2** above to establish **Theorem 3** as shown below;

Theorem 3: Since the topological space $\left(X, \tau_{\mathcal{P}(X)}\right)$ is locally compact as shown in **Theorem 2** above, the following notions must satisfy;

(i) For each $x \in \left(X, \tau_{\mathcal{P}(X)}\right)$ and each neighborhood N_x , there exist relatively compact open set A such that $x \in A \subset cl(A) \subset N$.

(ii) For each compact sets P and open set N containing P, there exist a relatively compact set A such that $P \subset A \subset cl(A) \subset N$.

(iii) $\mathcal{B}$ is a basis of topological space $\left(X, \tau_{\mathcal{P}(X)}\right)$ consisting of relatively compact open set.

Proof: (i) Provided that the topological space $\left(X, \tau_{\mathcal{P}(X)}\right)$ is locally compact as shown in **Theorem 2** above, there exists some open set A such that $x \in A \subset cl(A)$,where A is also compact. Provided that $cl(A)$ is a regular space and $cl(A) \cap N$ is a neighborhood of point $x \in cl(A)$, then, there exist a set S that is open in $cl(A)$ such that $x \in S \subset cl_{cl(K)}(S) \subset cl(A) \cap N$. So, $S = E \cap A$, where E is open subset of topological space $\left(X, \tau_{\mathcal{P}(X)}\right)$, so the desired neighborhood of point $x \in \left(X, \tau_{\mathcal{P}(X)}\right)$ is $A = E \cap A$.

(ii) For each $p \in P$, we have relatively compact set $A(p)$ such that $cl(Ap) \subset N$. Since N is compact, finitely many of neighborhood of N cover P, therefore, the union of P has a compact closure.

(iii) Let $\mathcal{B}$ be the collection of all relatively compact open set in $(X, \tau_{\mathcal{P}(X)})$, since each $x \in (X, \tau_{\mathcal{P}(X)})$, by (ii), $\mathcal{B}$ is a basis of topological space $(X, \tau_{\mathcal{P}(X)})$.

Corollary 1: The topological space $(X, \tau_{\mathcal{P}(X)})$ is $\tau_o -$ space (Kolmogorov space).

Proof: This proof is short of argument since we know that every topological space is $\tau_o -$ space, if not, is just ordinary set. So, being $\tau_o -$ space of topological space $(X, \tau_{\mathcal{P}(X)})$ is trivial.

Theorem 4: The topological space $(X, \tau_{\mathcal{P}(X)})$ is Hausdorff space and has a convergent sequence with unique limit.

Proof: In the **Theorem 1** above, we proved that the topological space $(X, \tau_{\mathcal{P}(X)})$ is totally disconnected and by the fact that every totally disconnected space is Hausdorff space, this shows that the topological space $(X, \tau_{\mathcal{P}(X)})$ is Hausdorff space. Next, suppose there is a convergent sequence (x_k) in $(X, \tau_{\mathcal{P}(X)})$ that converges to two distinct limits x_1 and x_2 in $(X, \tau_{\mathcal{P}(X)})$. From the property of Hausdorff Space, there exists two disjoint open sets X_1 and X_2 such that $x_1 \in X_1$ and $x_2 \in X_2$. We have three cases to complete this proof as shown below;

Case 1: Provided that the sequence $(x_k) \to x_1, \exists$ an integer $\mathbb{Z}$ such that $(x_k) \in X_1$ for $z > \mathbb{Z}_1$.

Case II: Provided that the sequence $(x_k) \to x_2, \exists$ an integer $\mathbb{Z}$ such that $(x_k) \in X_2$ for $z > \mathbb{Z}_2$.

Case III: Suppose n is an arbitrary integer such that $n > \mathbb{Z}_1, \mathbb{Z}_2$ then the sequence (x_n) must be in both X_1 and X_2, which is contradicting the hypothesis that the two open sets are disjoint. Thus, they have unique limit. This ends the proof.

Corollary 2: The Universal topological space $(X, \tau_{\mathcal{P}(X)})$ is Fr$\acute{e}$chet space ($\tau_1 -$ space).

Proof: From **Theorem 4** above, we see that the space $(X, \tau_{\mathcal{P}(X)})$ is Hausdorff space and since every Hausdorff space is Fr$\acute{e}$chet, hence, the space $(X, \tau_{\mathcal{P}(X)})$ is Fr$\acute{e}$chet space. This shows that $\tau_2 \Longrightarrow \tau_1$.

Theorem 5: The Universal topological space $\left(X, \tau_{\mathcal{P}(X)}\right)$ is completely regular ($\tau_{3\frac{1}{2}}$).

Proof: Suppose A is a subspace of topological space $\left(X, \tau_{\mathcal{P}(X)}\right)$ which is closed in $\left(X, \tau_{\mathcal{P}(X)}\right)$ such that for each $x \in \left(X, \tau_{\mathcal{P}(X)}\right)$, $x \in \{\left(X, \tau_{\mathcal{P}(X)}\right) - A\}$. If φ is a map which map the universal topological space $\left(X, \tau_{\mathcal{P}(X)}\right)$ to closed and bounded interval $[0,1]$, then since the topological space $\left(X, \tau_{\mathcal{P}(X)}\right)$ is Hausdorff as shown in the **Theorem 4** above, also, since the compact interval $[0,1]$ is Hausdorff under subspace topology, there exist open sets X_1 and X_2 in compact interval $[0,1]$ such that $0 \in X_1$, $1 \in X_2$ and $X_1 \cap X_2 \neq \emptyset$. Furthermore, for $x \in \left(X, \tau_{\mathcal{P}(X)}\right)$ and $y \in A$, $x \in X_1$ and $y \in X_2$, then the map $\varphi(x) = 0 \in X_1$, $\varphi(y) = 1 \in X_2$, furthermore, $\varphi(A) = 1 \in X_2$, for every $y \in A$. Since $\varphi(x) = 0 \in X_1$ and $\varphi(y) = 1 \in X_2$, this implies that $x \in \varphi^{-1}(X_1)$ and $y \in \varphi^{-1}(X_2)$. Provided that $x \neq y$, this implies that $\varphi^{-1}(X_1) \cap \varphi^{-1}(X_2) = \emptyset$, which is shows both $\varphi^{-1}(X_1)$ and $\varphi^{-1}(X_2)$ are open subsets of universal topological space $\left(X, \tau_{\mathcal{P}(X)}\right)$ and the map φ is continuous. Thus, the universal topological space $\left(X, \tau_{\mathcal{P}(X)}\right)$ is completely regular.

Theorem 6: The topological space $\left(X, \tau_{\mathcal{P}(X)}\right)$ is regular space.

Proof: Since every completely regular space is regular, and since the space $\left(X, \tau_{\mathcal{P}(X)}\right)$ is completely regular as shown in **Theorem 5 above**, the space $\left(X, \tau_{\mathcal{P}(X)}\right)$ is regular. From **Lemma 7** above $\tau_{3\frac{1}{2}} \Longrightarrow \tau_3$.

Corollary 3: The topological space $\left(X, \tau_{\mathcal{P}(X)}\right)$ is Tychonoff's space.

Proof: From the **Theorem 4 and Theorem 5 above**, where we proved that the space $\left(X, \tau_{\mathcal{P}(X)}\right)$ is Hausdorff and completely regular respectively. From the fact that every completely regular space is Tychonoff's space. Therefore, the space $\left(X, \tau_{\mathcal{P}(X)}\right)$ is Tychonoff's space.

Theorem 7: The topological space $\left(X, \tau_{\mathcal{P}(X)}\right)$ is normal space.

Proof: Recall from **Theorem 1** above that the topological space $\left(X, \tau_{\mathcal{P}(X)}\right)$ is totally disconnected, for this, every pair of disjoint sets can be separated by disconnection of topological space X. Suppose $S_1 \cap S_2 = \emptyset$ and $x_1 \in S_1$, $x_2 \in S_2$ such that the neighborhood N_p of $p \in A$ of the topological space $\left(X, \tau_{\mathcal{P}(X)}\right)$ which consists of the open set B containing p, such that $p \in A \cap B$. Also, $q \in S_2$ implies that its neighborhood is a subset $K \subset \left(X, \tau_{\mathcal{P}(X)}\right)$ which consists of an open set C containing q such that $q \in K \cap C$. Next, we need to prove that $B \subset K$ by assuming that if $B \subset K$, so either q contains p or $p = q$. But since the topological space $\left(X, \tau_{\mathcal{P}(X)}\right)$ is totally disconnected as shown in the **Theorem 1** above and there are no disjoint sets which have points in common, so, neither p is contains in q nor $p = q$. Therefore, the neighborhood B of the set S_1 and neighborhood K of the set S_2 are disjoint. So, the topological space $\left(X, \tau_{\mathcal{P}(X)}\right)$ is normal space or $\tau_4 - space$.

Theorem 8: The topological space $\left(X, \tau_{\mathcal{P}(X)}\right)$ is completely normal space ($\tau_4 - space$).

Proof: From **Theorem 7 and Corollary 2** above, it is shown that the topological space $\left(X, \tau_{\mathcal{P}(X)}\right)$ is normal and $\tau_1 - space$ and since any topological space which is normal and $\tau_1 - space$ is $\tau_5 - space$. Therefore, the space $\left(X, \tau_{\mathcal{P}(X)}\right)$ is completely normal.

Corollary 4: The topological space $\left(X, \tau_{\mathcal{P}(X)}\right)$ is a τ_5 space.

Proof: **Consider Theorem 8 and Corollary 2** above, where we have proved that the topological space $\left(X, \tau_{\mathcal{P}(X)}\right)$ is completely normal and τ_1 space, but provided that any topological space which is completely normal and τ_1 is τ_5 space. Hence, the space $\left(X, \tau_{\mathcal{P}(X)}\right)$ is τ_5 space, which is also known as completely normal.

In the next result, we are going to use **Lemma 6** above to prove that the topological space $\left(X, \tau_{\mathcal{P}(X)}\right)$ perfectly normal ($\tau_6 - space$).

Theorem 9: A τ_1 − space $\left(X, \tau_{\mathcal{P}(X)}\right)$ is perfectly normal ($\tau_6 - space$) if and only if for every open set O, there is a countable family $\{F_n\}, n \in \mathbb{N}$, of open sets such that $cl(F_n) \subseteq O, \forall\, n$ and $O \subseteq \bigcup_n F_n$.

Proof: Let $\left(X, \tau_{\mathcal{P}(X)}\right)$ be $\tau_6 - space$, then by the fact that for a $\tau_1 - space$, every closed set is a zero – set, then, there is $f \in C\left(\left(X, \tau_{\mathcal{P}(X)}\right), [0,1]\right)$ such that $O = f^{-1}((0,1])$. Then $F_n = f^{-1}\left([\frac{1}{n}, 1)\right)$ is open for each $n \in \mathbb{N}$ which satisfies that $cl(F_n) \subseteq f^{-1}\left(\left[\frac{1}{n}, 1\right)\right) \subseteq O$, clearly, $\bigcup_n F_n = 0$.

Conversely, given that $\{F_n\}, n \in \mathbb{N}$, is a countable family of open sets, we have $O = \bigcup_n F_n \subseteq \bigcup_n cl(F_n) \subseteq O$, thus, $O = \bigcup_n F_n$. Thus, every open set is F_σ, which is equivalent to every closed set being G_σ. Also, if $A \subseteq O$ is closed, then, we can trivially have $A \subseteq O = \bigcup_n F_n$. Therefore, by **Lemma 1** above, the topological space $(X, \tau_{\mathcal{P}(X)})$ is normal and $\tau_6 - space$, by the fact that for a $\tau_1 - space\ X, X$ being perfectly normal is equivalent.

Theorem 10: If $\sim$ is an equivalence relation and is in the same connected component with topological space $(X, \tau_{\mathcal{P}(X)})$. Then the following statements are satisfied:

(i) The quotient space $(X, \tau_{\mathcal{P}(X)})/\sim$ is τ_1 and totally disconnected.

(ii) The map $f: (X, \tau_{\mathcal{P}(X)}) \to (X, \tau_{\mathcal{P}(X)})/\sim$ is homeomorphism.

Proof: Here, we have to begin the proof by showing that the space $(X, \tau_{\mathcal{P}(X)})/\sim$ has no connected subspaces with more than one point. Let $f: (X, \tau_{\mathcal{P}(X)})/\sim$ be the quotient map and let $M \subseteq (X, \tau_{\mathcal{P}(X)})/\sim$ with car $(M) \geq 2$. Then $f^{-1}(M) \subseteq (X, \tau_{\mathcal{P}(X)})$ is the union of at least two connected components of the space $(X, \tau_{\mathcal{P}(X)})$ and hence not connected. Thus, there is clopen set $N \subseteq f^{-1}(M)$ with $N \neq \emptyset \neq f^{-1}(M)$. Therefore, N is a union of connected component of the space $(X, \tau_{\mathcal{P}(X)})$. (Assume $Y \subseteq (X, \tau_{\mathcal{P}(X)})$ is connected with $Y \cap N \neq \emptyset$ and $\{Y - N\} \neq \emptyset$. Then, Y is the union of two non – empty clopen subsets, which contradicts connectedness). Thus, $N = f^{-1}(f(N))$. From the definition of Quotient topology, the subset $M \subseteq (X, \tau_{\mathcal{P}(X)})/\sim$ is clopen $iff\ f^{-1}(M) \subseteq (X, \tau_{\mathcal{P}(X)})$ is clopen. In view of $N = f^{-1}(f(N))$ and with the fact that N is clopen, then $f(N)$ is clopen. It is clear that $f(N) \subseteq M$ with $f(N) \neq \emptyset \neq$M. Therefore, the space $(X, \tau_{\mathcal{P}(X)})/\sim$ is totally disconnected or not connected.

Also, the space $(X, \tau_{\mathcal{P}(X)})/\sim$ is τ_1 if, and only if is induced with equivalent class $\sim$. In this case, the connected components are closed (respectively open). By the fact that connected components are always open and openness connected components is equivalent to local connectedness, then the space $(X, \tau_{\mathcal{P}(X)})/\sim$ is τ_1.

(ii) From **Theorem 1** above, totally disconnectedness of the space $(X, \tau_{\mathcal{P}(X)})$ shows that the equivalent relation $\sim$ is trivial. Thus, the quotient map $f: (X, \tau_{\mathcal{P}(X)})/\sim$ is a homeomorphism. By (i), the space $(X, \tau_{\mathcal{P}(X)})/\sim$ is totally disconnected, since the quotient map $f: (X, \tau_{\mathcal{P}(X)})/\sim$ is a homeomorphism, therefore, the topological space $(X, \tau_{\mathcal{P}(X)})$ is totally disconnected.

Conclusion

In this paper, we proved that some topological properties such as separation axioms are satisfied on topology of power set. Also, we disagree with Theorem 2 in (Amulya, 2019), where he stated and proved that topology of power set is not compact for an infinite topological space. We saw that compactness of infinite topological space does not make any sense but rather, we therefore checked for locally compactness instead.

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