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Modification of Picard's Iterative Method for the Solution of Fractional Differential Equations

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Abstract

A robust algorithm is introduced in the development of the Modified Picard's Iterative Method (MPIM) to effectively address both linear and nonlinear Fractional Differential Equation (FDE) and other types of fractional order differential equations. The method's efficacy is demonstrated through numerical examples, showcasing its ability to solve these equations without resorting to linearization or small perturbations. The results affirm the method's strength, accuracy, and simplicity in comparison to alternative approaches.

Keywords: Picard's Iterative Method; Fractional Differential Equations

INTRODUCTION

The application of Fractional Differential equations, in numerous diverse fields of science and engineering and particularly in the modeling of various physical phenomena, very accurately, has made its appearance so frequent in these different areas of study. Many physical processes also appear to exhibit fractional order behavior that may vary with time or space Podlubny (1999). These lines subsequently necessitated the need to seek for its solution is it analytical or numerical unfortunately most fractional differential equations do not have exact solution, so approximate and numerical techniques thus required. Significant efforts have been made by many authors and current researchers studying the analytical and approximate solution of these problems in this very particular class.

The Riccati differential equation, named after the Italian nobleman Count Jacapo Francesco Riccati (1676-1754), has diverse applications in various fields including pattern formation in dynamic games, linear systems with Markovian jumps, river flows, econometric models, stochastic systems, control theory, and diffusion problems [Arikoglu & Ozkol, 2007; Kharrat & Toma 2019]. It forms the basis of fundamental theories in random processes, optimal control, and diffusion problems. The numerical treatment of the Fractional Riccati Quadratic equation has been a subject of interest due to its widespread appearance in dynamic games, linear systems with Markovian jumps, river flows, econometric models, stochastic control theory, diffusion problems, and related fields.

Several approximate numerical solutions have been proposed for the Riccati differential equation. Sweilam *et al*., (2012) applied the pade-VIM, while Khodadadi *et al*., (2015) utilized the variational iteration method to solve this problem. Momani & Odibat, (2007) employed the Adomian decomposition method for their solution approach. Additionally, Odibat, (2019) developed a decomposition algorithm for tackling this equation.

The Picard iteration method, also known as the successive approximations method, is a straightforward and effective technique for solving differential equations. This method proceeds by iteratively refining successive approximations to the solution, starting from an initial approximation. Scholars have employed the Picard iteration method to solve a wide range of problems, including both linear and non-linear differential, integral, and integro-differential equations. Notable researchers who have applied this method include Wazwaz, (2011), Chen & Duan, (2015), Palani & Usachev, (2020), Carothers *et al*., (2004) and Yu, (2022), among others. The efficacy of this method in traditional calculus has inspired researchers to adapt and delve into its potential for approximating fractional Riccati differential equations. In our study, we aim to refine Picard's iterative approach for solving not only fractional Riccati differential equations but also other types of fractional differential equations.

METHODOLOGY

Basic Idea of the Picard's Iterative Method (PIM)

Here, we consider

$$D^{\alpha} y(x) = A(x) + B(x)y(x) + C(x)y^{2}(x) \quad ...(1)$$

$x \in R,\ 0 < \alpha \leq 1,\ x > 0$.

Subject to the initial condition

$$y^{k}(0) = y_{k},\ k = 0,1,2,...n-1 \quad ...(2)$$

Where α is the order of the fractional derivatives, x is an integers, (x),(x) and $C(x)$ are known functions, and y_k is a constant. The derivative is the Caputo-type derivatives. J^{α} is introduced to both sides of (1) and simplified to obtain

$$y(x) = y_{0}(x) + J^{\alpha}\{A(x) + B(x)y(x) + C(x)y^{2}(x)\} \quad ...(3)$$

Picard's iteration scheme suggests that

$$y_{n}(x) = y_{0}(x) + J^{\alpha}\{A(x) + B(x)y_{n-1}(x) + C(x)y_{n-1}^{2}(x)\} \quad ...(4)$$

Where the zeroth approximation y_0 is our initial condition (2) several successive approximations y_k , $k \geq 1$ is determined as

$$y_{1}(x) = y_{0}(x) + J^{\alpha}\{A(x) + B(x)y_{0}(x) + C(x)y_{0}^{2}(x)\} \quad ...(5)$$

$$y_{2}(x) = y_{0}(x) + J^{\alpha}\{A(x) + B(x)y_{1}(x) + C(x)y_{1}^{2}(x)\} \quad ...(6)$$

$$y_{3}(x) = y_{0}(x) + J^{\alpha}\{A(x) + B(x)y_{2}(x) + C(x)y_{2}^{2}(x)\} \quad ...(7)$$

$$y_{n}(x) = y_{0}(x) + J^{\alpha}\{A(x) + B(x)y_{n-1}(x) + C(x)y_{n-1}^{2}(x)\} \quad ...(8)$$

In this work, our focus is on reconstructing the Picard's Method by leveraging the properties of fractional derivatives and the DJM iterates to solve fractional differential equations voids of noise and unwanted terms.

The Modified PIM Algorithm for Linear and Nonlinear FDE

Consider the following nonlinear fractional differential equations:

$$D^{\alpha}[u(t)] + L[u(t)] + N[u(t)] = g(t), \quad t > 0 \qquad \text{... (9)}$$

$where\ L$ is a linear operator, N represent a nonlinear operator, $g(t)$ is the source term, and D^{α} is the Caputo fractional derivative of order α with $m-1 < \alpha < m.$

We extend a reliable modification of the Picard's method by using the DJM iterate to reconstruct the PIM for the solutions of nonlinear fractional differential equations.

The Eqn. (9) can be approximately expressed as follows by applying the integral operator and the *n-fold* integral formula to both sides of Eqn. (9), we obtained the following:

$$u(x) = \sum_{k=0}^{m-1} u^k(0)\frac{x^k}{k!} + (-1)^q \frac{1}{(q-1)!}\int_0^x [(t-x)^{q-1}(D^{\alpha}[u(t)] + L[u(t)] + N[u(t)] - g(t))]dt \qquad \text{... (10)}$$

Now, we rewrite Eqn. (10) as follows:

$$u_n(x) = \sum_{k=0}^{m-1} u^k(0)\frac{x^k}{k!} + (-1)^q \frac{1}{(q-1)!}\int_0^x \left[(t-x)^{q-1}\sum_{k=0}^{\infty}\left(D^{\alpha}[u_k(t)] + Lu_k(t) + Nu_k(t)\right) - g(t)\right]dt \qquad \text{... (11)}$$

$n \geq 0.$

Now, we set the following scheme:

$$u_0(x) = \sum_{k=0}^{m-1} u^k(0)\frac{x^k}{k!} \quad \dots(12)$$

$$u_1(x) = (-1)^q \frac{1}{(q-1)!}\int_0^x [(t-x)^{q-1}(D^\alpha[u_0(t)] + Lu_0(t) + Nu_0(t) - g(t)]dt \quad \dots(13)$$

$$u_2(x) = (-1)^q \frac{1}{(q-1)!}\int_0^x \left[(t-x)^{q-1}\sum_{k=0}^{1}\big(D^\alpha[u_k(t)] + Lu_k(t) + Nu_k(t)\big) - g(t)\right]dt \quad \dots(14)$$

$$u_3(x) = (-1)^q \frac{1}{(q-1)!}\int_0^x \left[(t-x)^{q-1}\sum_{k=0}^{2}\big(D^\alpha[u_k(t)] + Lu_k(t) + Nu_k(t)\big) - g(t)\right]dt \quad \dots(15)$$

$$u(x) = \sum_{k=0}^{m-1} u^k(0)\frac{x^k}{k!} + u_1(x) + u_2(x) + u_3(x) + u_4(x) + \cdots \quad \dots(16)$$

RESULTS

Numerical Application

This section presents the results obtained through the application of the **Modified PIM** to solve the FDEs. The numerical simulations and experiments conducted to evaluate the effectiveness of the modified method are outlined in this section. The discussion will focus on interpreting the results and their significance in the context of the research objectives. We will apply the **Modified PIM** as follows:

Linear First Order Fractional Differential Equation.

The effectiveness of the **Modified PIM** for solving linear FDEs can be illustrated as follows:

Example 1: Consider the first order fractional homogeneous ODE

$$D^{\alpha}u(x) - 2xu(x) = 0, \qquad x > 0, \quad 0 < \alpha < 1 \qquad \dots (17)$$

Subject to the initial condition

$$u(0) = 1. \qquad \dots (18)$$

With exact solution $u(x) = e^{x^2}$ ... (19)

In view of Eqn. (10), Eqn. (17) is approximately expressed as follows:

$$u(x) = \sum_{k=0}^{m-1} u^k(0)\frac{x^k}{k!} + (-1)^q \frac{1}{(q-1)!}\int_0^x [(t-x)^{q-1}(D^{\alpha}u(t) - 2tu(t))]dt \qquad \dots (20)$$

Now, we rewrite Eqn. (20) as follows:

$$u_n(x) = \sum_{k=0}^{m-1} u^k(0)\frac{x^k}{k!} + (-1)^q \frac{1}{(q-1)!}\int_0^x \left[(t-x)^{q-1}\sum_{k=0}^{\infty}\left(D^{\alpha}u_k(t) - 2tu_k(t)\right)\right]dt \qquad \dots (21)$$

$n \geq 0$.

Now, we set the following scheme:

$$u_0(x) = \sum_{k=0}^{m-1} u^k(0)\frac{x^k}{k!} = 1 \qquad \dots (22)$$

$$u_1(x) = (-1)^q \frac{1}{(q-1)!}\int_0^x [(t-x)^{q-1}(D^\alpha[u_0(t)] - 2tu_0(t)]dt$$

$$= x^2 \qquad \dots (23)$$

$$u_2(x) = (-1)^q \frac{1}{(q-1)!}\int_0^x \left[(t-x)^{q-1}\sum_{k=0}^{1}\left(D^\alpha[u_k(t)] - 2tu_k(t)\right)\right]dt$$

$$= x^2 + \frac{x^4}{2} - \frac{2x^{3-\alpha}}{\Gamma(4-\alpha)} \qquad \dots (24)$$

$$u_3(x) = (-1)^q \frac{1}{(q-1)!}\int_0^x \left[(t-x)^{q-1}\sum_{k=0}^{2}\left(D^\alpha[u_k(t)] - 2tu_k(t)\right)\right]dt$$

$$= x^2 + x^4 + \frac{x^6}{6} - \frac{4\Gamma(5-\alpha)x^{5-\alpha}}{\Gamma(4-\alpha)\Gamma(6-\alpha)} - \frac{4x^{3-\alpha}}{\Gamma(4-\alpha)} - \frac{12x^{5-\alpha}}{\Gamma(6-\alpha)} + \frac{2x^{4-2\alpha}}{\Gamma(5-2\alpha)} \qquad \dots (25)$$

$$u(x) = u_0(x) + u_2(x) + u_3(x) + \cdots$$

$$u(x) \cong 1 + 3x^2 + \frac{3x^4}{2} + \frac{x^6}{6} - \frac{6x^{3-\alpha}}{\Gamma(4-\alpha)} - \frac{4\Gamma(5-\alpha)x^{5-\alpha}}{\Gamma(4-\alpha)\Gamma(6-\alpha)} - \frac{12x^{5-\alpha}}{\Gamma(6-\alpha)} + \frac{2x^{4-2\alpha}}{\Gamma(5-2\alpha)} \qquad \dots (26)$$

Table 1: Numerical values when ∝=0.9, 0.8, 0.7, 0.5 and 1.0 for Eqn. (17)

X	EXACT	APPRX SOLN (∝=1, n=3)	APPRX SOLN (∝=0.9, n=3)	APPRX SOLN (∝=0.8, n=3)	APPRX SOLN (∝=0.7, n=3)	APPRX SOLN (∝=0.5, n=3)
0	1	1	1	1	1	1
0.1	1.01005	1.010050167	1.013599648	1.017154936	1.020262319	1.024758595
0.2	1.040811	1.040810667	1.052156535	1.063862959	1.074765635	1.092425809
0.3	1.094174	1.0941715	1.116396376	1.139624459	1.161822003	1.200070174
0.4	1.17351	1.173482667	1.209316204	1.247530721	1.284125866	1.349679274
0.5	1.284017	1.283854167	1.335532699	1.393326398	1.447468463	1.546914629
0.6	1.433276	1.432576	1.501076956	1.585421201	1.660593066	1.800864524
0.7	1.63206	1.629658167	1.713282934	1.835204259	1.935411256	2.124157544
0.8	1.895481	1.888490667	1.980712841	2.157551463	2.287438401	2.533303148
0.9	2.24456	2.2266235	2.313101285	2.571490095	2.736400755	3.049211092
1	2.708333	2.666666667	2.721310515	3.101005557	3.306993531	3.697867688

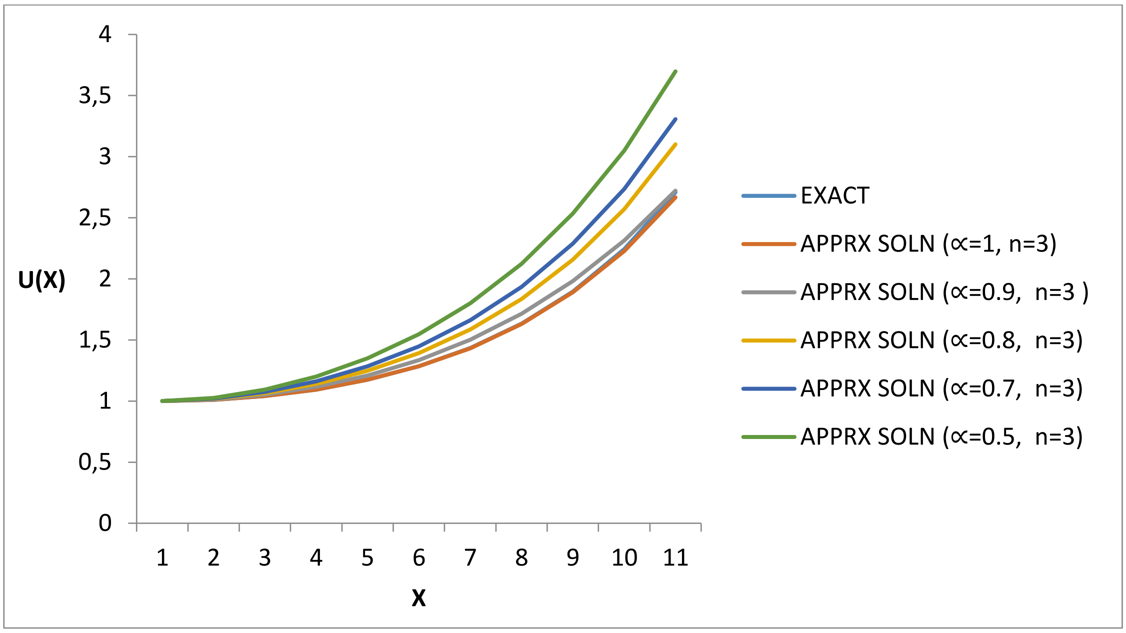

Figure 1: Comparison of approximate solutions obtained by the Modified PIM for **α=1, 0.9, 0.8, 0.7** and **0.5** with exact solution for example 1

Table 2: Approximate solution of Example 1

X	EXACT	APPRX SOLN (∝=1, n=3)	ODM (∝=1, n=7)	ADM (∝=1, n=5)	VIM (∝=1, n=15)
0	1	1	1	1	1
0.1	1.01005	1.010050167	1.010050	1.010050167	1.01005025
0.2	1.040811	1.040810667	1.040800	1.040810667	1.040816107
0.3	1.094174	1.0941715	1.094050	1.0941715	1.094234984
0.4	1.17351	1.173482667	1.172800	1.173482667	1.173851307
0.5	1.284017	1.283854167	1.281250	1.283854167	1.28531901
0.6	1.433276	1.432576	1.42480	1.432576	1.43716384
0.7	1.63206	1.629658167	1.610050	1.629658167	1.64186425
0.8	1.895481	1.888490667	1.84480	1.888490667	1.917326507
0.9	2.24456	2.2266235	2.138050	2.2266235	2.288846384
1	2.708333	2.666666667	2.500000	2.666666667	2.791666667

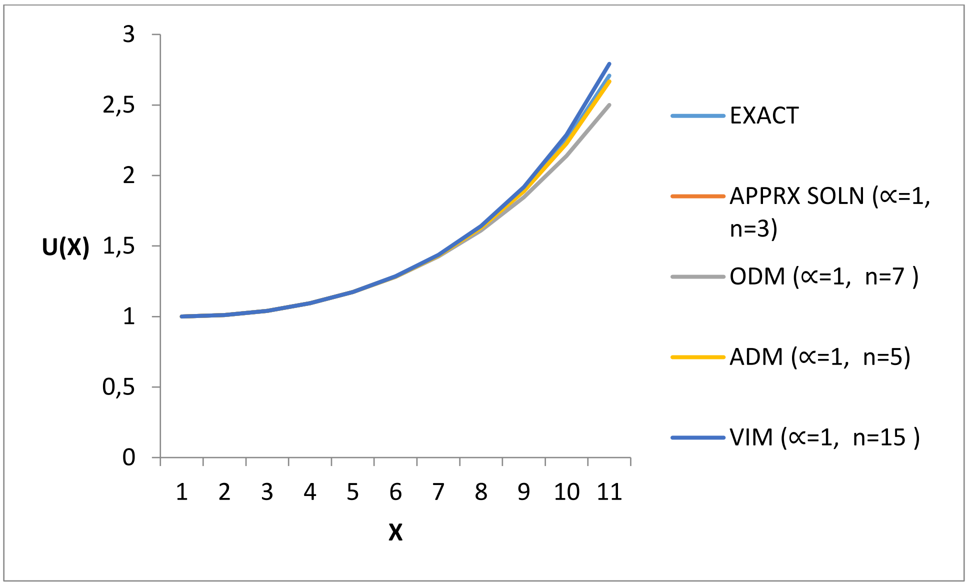

Figure 2: Comparison of approximate solutions obtained by the Modified PIM, ODM, ADM, VIM and exact solution when **∝=1** for Eqn. (17)

Nonlinear First Order Fractional Differential Equation.

Example 2:

$$D^{\alpha}u(x) + u^2(x) = 1, \qquad x \geq 0,$$
$$0 < \alpha \leq 1 \qquad \dots (27)$$

Subject to the initial condition

$$u(0) = 0. \qquad \dots (25)$$

With exact solution $u(x) = \tanh(x)$

In view of Eqn. (10), Eqn. (27) is approximately expressed as follows:

$$u(x) = \sum_{k=0}^{m-1} u^k(0)\frac{x^k}{k!} + (-1)^q \frac{1}{(q-1)!}\int_0^x [(t-x)^{q-1}(D^{\alpha}u(t) + u^2(t) - 1)]dt \qquad \dots (26)$$

Now, we rewrite Eqn. (26) in recursive relations as follows:

$$u_{n+1}(x) = \sum_{k=0}^{m-1} u^k(0)\frac{x^k}{k!} + (-1)^q \frac{1}{(q-1)!}\int_0^x \left[(t-x)^{q-1}\sum_{k=0}^{\infty}(D^\alpha u_k(t) + u^2{}_k - 1)\right] dt \quad \ldots(27)$$

$n \geq 0.$

Now, we set the following scheme:

$$u_0(x) = \sum_{k=0}^{m-1} u^k(0)\frac{x^k}{k!} = 0 \quad \ldots(28)$$

$$u_1(x) = (-1)^q \frac{1}{(q-1)!}\int_0^x \left[(t-x)^{q-1}(D^\alpha[u_0(t)] + u_0^2(t) - 1\right] dt = x \quad \ldots(29)$$

$$u_2(x) = (-1)^q \frac{1}{(q-1)!}\int_0^x \left[(t-x)^{q-1}\sum_{k=0}^{1}\left(D^\alpha[u_k(t)] + u_k^2(t) - 1\right)\right] dt = x - \frac{x^3}{3} - \frac{x^{2-\alpha}}{\Gamma(3-\alpha)} \quad \ldots(30)$$

$$u_3(x) = (-1)^q \frac{1}{(q-1)!}\int_0^x \left[(t-x)^{q-1}\sum_{k=0}^{2}\left(D^\alpha[u_k(t)] + u_k^2(t) - 1\right)\right] dt$$
$$= x - \frac{4x^3}{3} + \frac{4x^5}{15} - \frac{x^7}{63} + \frac{4\Gamma(4-\alpha)x^{4-\alpha}}{\Gamma(3-\alpha)\Gamma(5-\alpha)} - \frac{2\Gamma(6-\alpha)x^{6-\alpha}}{3\Gamma(3-\alpha)\Gamma(7-\alpha)} - \frac{\Gamma(5-2\alpha)x^{5-2\alpha}}{\Gamma(3-\alpha)\Gamma(6-2\alpha)} - \frac{2x^{2-\alpha}}{\Gamma(3-\alpha)} + \frac{2x^{4-\alpha}}{\Gamma(5-\alpha)} + \frac{x^{3-2\alpha}}{\Gamma(4-2\alpha)} \quad \ldots(31)$$

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So that the analytic solution is

$$u(x) = u_0(x) + u_2(x) + u_3(x) + \cdots$$

$$u(x) \cong 3x - \frac{5x^3}{3} + \frac{4x^5}{15} - \frac{x^7}{63} + \frac{x^{3-2\alpha}}{\Gamma(4-2\alpha)} - \frac{3x^{2-\alpha}}{\Gamma(3-\alpha)} + \frac{4\Gamma(4-\alpha)x^{4-\alpha}}{\Gamma(3-\alpha)\Gamma(5-\alpha)} - \frac{2\Gamma(6-\alpha)x^{6-\alpha}}{3\Gamma(3-\alpha)\Gamma(7-\alpha)} - \frac{4x^3}{3} + \frac{4x^5}{15} - \frac{x^7}{63} + \frac{4\Gamma(4-\alpha)x^{4-\alpha}}{\Gamma(3-\alpha)\Gamma(5-\alpha)} - \frac{2\Gamma(6-\alpha)x^{6-\alpha}}{3\Gamma(3-\alpha)\Gamma(7-\alpha)} + \frac{2x^{4-\alpha}}{\Gamma(5-\alpha)} - \frac{\Gamma(5-2\alpha)x^{5-2\alpha}}{\Gamma(3-\alpha)\Gamma(6-2\alpha)} \quad \ldots (32)$$

Table 3: Approximate solution of Example 2

X	EXACT SOLN	APPRX SOLN (α=1, n=7)	ODM (α=1, n=9)	ADM (α=1, n=12)
0	0	0	0	0
0.1	0.099668	0.099667995	0.0996680	0.099667997
0.2	0.197375	0.19737532	0.1973760	0.197375553
0.3	0.291313	0.291312612	0.2913240	0.291316363
0.4	0.379949	0.379948962	0.3800320	0.379974786
0.5	0.462117	0.462117157	0.4625000	0.462227183
0.6	0.53705	0.537049567	0.5383680	0.537390446
0.7	0.604368	0.604367777	0.6080760	0.605200136
0.8	0.664037	0.664036783	0.6730240	0.665700612
0.9	0.716298	0.716298153	0.7357320	0.719029569
1	0.761594	0.761598747	0.8000000	0.765079365

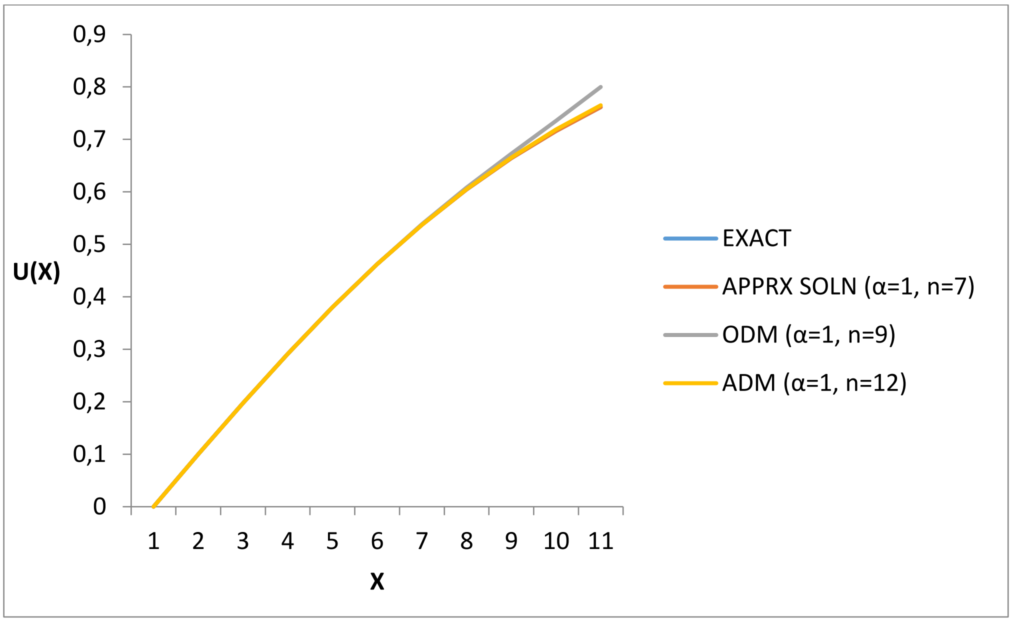

Figure 3: graph of Example 1

Table 4: Error Table of Example 2

ERROR OF APPRX SOLN	ERROR OF ODM	ERROR OF ADM
0	0	0
8.32667E-17	5.37504E-09	1.88298E-09
0	6.79775E-07	2.32791E-07
1.66533E-16	1.13875E-05	3.75041E-06
1.66533E-16	8.30377E-05	2.58238E-05
4.35207E-14	0.000382843	0.000110025
5.76239E-12	0.001318433	0.000340879
3.53712E-10	0.003708223	0.000832358
1.23864E-08	0.00898723	0.001663842
2.82443E-07	0.01943413	0.002731698
4.59102E-06	0.038405844	0.003485209

Table 5: Numerical values when **α=0.9, 0.8, 0.7** and **1.0** for Eqn. (27)

X	EXACT	APPRX SOLN (α=1)	APPRX SOLN (α=0.9)	APPRX SOLN (α=0.8)	APPRX SOLN (α=0.7)
0	0	0	0	0	0
0.1	0.099668	0.099667995	0.104824794	0.116634828	0.130573859
0.2	0.197375	0.19737532	0.201613799	0.215233465	0.234069
0.3	0.291313	0.291312612	0.291564035	0.302108771	0.320118738
0.4	0.379949	0.379948962	0.37353468	0.377189139	0.389728765
0.5	0.462117	0.462117157	0.446696128	0.440211885	0.443279055
0.6	0.53705	0.537049567	0.510816554	0.491410326	0.481538605
0.7	0.604368	0.604367777	0.56636346	0.531718609	0.505969675
0.8	0.664037	0.664036783	0.614530946	0.562824804	0.518811394
0.9	0.716298	0.716298153	0.657207765	0.587136433	0.523047622
1	0.761594	0.761598747	0.696884582	0.607672164	0.52228684

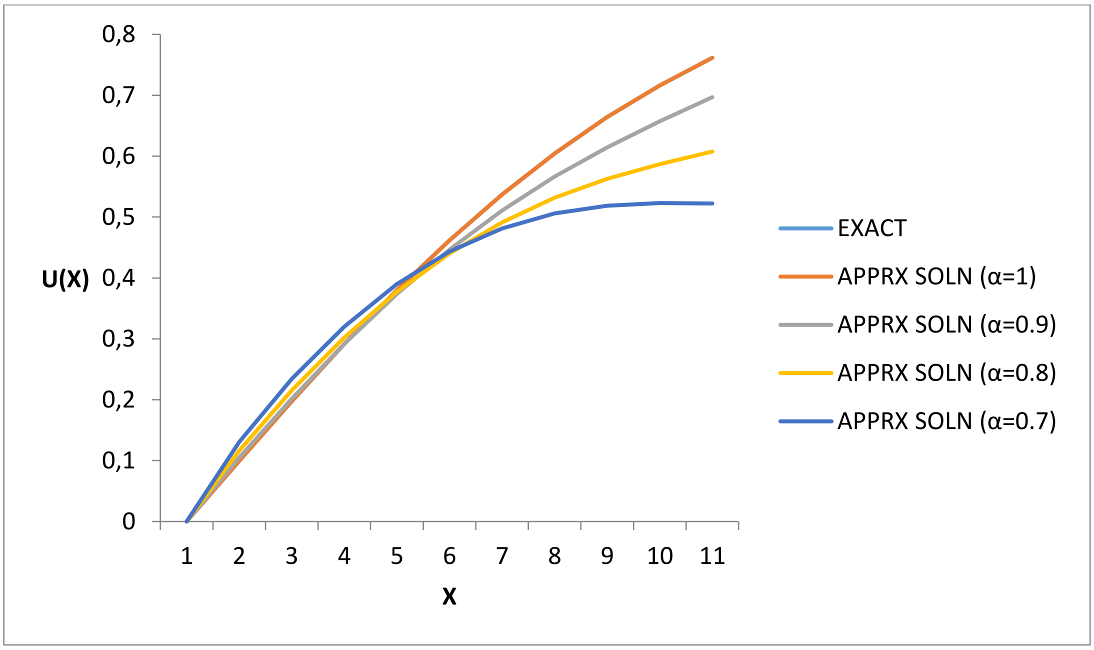

Figure 4: Comparison of approximate solutions obtained by the Modified PIM with exact solution when **α=1, 0.9, 0.8,** and **0.7** of example 2

DISCUSSION

Based on the numerical experiments performed, we observed that the method converges well with the exact solution as we approach the integer order under consideration (see Tables and graphs there in). The method facilitates the computational work and gives the solution rapidly when compared with ADM, VIM and ODM. For nonlinear problems where exact solution does not exist, a few numbers of approximations can be used for numerical purposes. However, for concrete problems, such as nonlinear fractional order differential equations problem, a good number of iterations are needed to get a reasonable accuracy level as shown in example 2 there in.

CONCLUSION

In this seminar, a MPIM has been presented for solving linear and non-linear fractional differential equations. Comparing with the other method for solving fractional differential equation by a modified MPIM, the results for numerical examples demonstrate that the present method can give more accurate results. This is also the main advantage of the present method. Therefore, the MPIM can overcome the restriction of the application area of the other methods, and then expand its scope of application.

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