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Development of GeoGebra-Assisted Digital Student Worksheets Based on Discovery Learning for Analytic Geometry

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Abstract

Integrating GeoGebra into digital worksheets structured around discovery learning offers a way to support mathematical exploration. The application of this approach to analytic geometry warrants further investigation. This study aimed to develop a GeoGebra-assisted digital worksheet for analytic geometry and demonstrate how its validity, practicality, and effectiveness could be evaluated. The research and development design followed the ADDIE model and specified procedures for expert validation and classroom evaluation. An illustrative dataset comprising 30 simulated student responses and three simulated expert assessments was analyzed using descriptive statistics, normalized gain, and a paired-samples t test. The simulation yielded validity and practicality scores of 91.33% and 85.79%, respectively. Mean scores in the simulated dataset increased from 58.93 to 78.30, with a normalized gain of 0.48 and a statistically significant paired difference ($p < .001$). The proposed worksheet guides students in exploring connections between algebraic and geometric representations. The simulated analysis demonstrates how the worksheet can be evaluated, and its instructional effectiveness remains to be established with empirical classroom data.

Keywords: Analytic Geometry; Digital Student Worksheets; Discovery Learning; GeoGebra; Mathematics Education

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INTRODUCTION

Mathematics education plays an important role in developing students' ability to reason, interpret information, and solve problems across different situations (Basid et al., 2024; Chen, 2025; Naz & Qayyum, 2025). Mathematics learning, however, continues to present challenges in many countries, particularly when students are expected to move beyond routine calculations and apply their understanding to unfamiliar problems. The Programme for International Student Assessment (PISA) 2022 reported that only 18% of Indonesian students achieved at least Level 2 proficiency in mathematics, compared with the OECD average of 69% (Organisation for Economic Co-operation and Development, 2023). Although these findings do not specifically measure achievement in analytic geometry, they indicate broader difficulties in mathematical reasoning and application that deserve attention in classroom instruction. The National Council of Teachers of Mathematics (National Council of Teachers of Mathematics, 2014) emphasizes the importance of meaningful mathematical tasks, conceptual understanding, and opportunities for students to construct and communicate their reasoning. Educational technology offers possibilities for supporting these learning experiences, provided that its use responds to clear instructional needs rather than simply replacing conventional teaching materials with digital formats (UNESCO, 2023).

Analytic geometry is a branch of mathematics that requires students to understand the relationships between algebraic expressions and geometric representations (Basid et al., 2024; Chen, 2025; Naz & Qayyum, 2025). Concepts such as coordinates, distances between points, equations of straight lines, and curves involve more than the application of formulas; they require learners to interpret how changes in mathematical expressions affect geometric objects. Duval (2006) explains that mathematical understanding depends partly on the ability to recognize and coordinate different forms of representation, including symbolic expressions, diagrams, and graphical models. Arcavi (2003) similarly highlights the contribution of visualization to mathematical thinking, particularly when learners must interpret abstract relationships or examine the meaning of mathematical ideas. These perspectives suggest that learning analytic geometry should provide students with opportunities to connect calculations with visual evidence and mathematical explanations. Research by Saha et al. (2010) further demonstrates the relevance of dynamic mathematical software to coordinate geometry learning. Instructional activities that allow students to examine equations, observe graphical changes, and explain the relationships they discover

may therefore provide a more meaningful learning experience than activities focused predominantly on procedural exercises.

Digital student worksheets, commonly referred to as *Lembar Kerja Peserta Didik* (LKPD) in Indonesia, offer a practical means of organizing mathematical learning activities while accommodating interactive resources. Their educational value lies not merely in presenting information electronically but in guiding students through questions, investigations, and opportunities to reflect on their findings. Muchlis et al. (2021) developed web-based worksheets supported by GeoGebra and reported that the resulting materials met the criteria of validity, practicality, and effectiveness in plane geometry learning. Laili et al. (2023) also demonstrated the potential of digital learning materials incorporating Discovery Learning and GeoGebra in teaching geometric transformations. These studies indicate that digital worksheets can support mathematical exploration when technological features are meaningfully connected to learning activities. Plomp & Nieveen (2013) emphasize the importance of developing educational interventions that are relevant to their intended purposes, usable in practice, and capable of producing the expected learning outcomes. Consequently, the development of digital worksheets requires attention to the accuracy of mathematical content, the clarity of instructions, the organization of learning tasks, and the accessibility of interactive features.

Discovery Learning provides a pedagogical approach that encourages students to construct mathematical understanding through exploration, observation, and the examination of evidence. Bruner (1961) describes discovery as an intellectual process through which learners organize information and develop understanding through their own reasoning. This perspective is particularly relevant to analytic geometry because students can investigate geometric patterns, propose relationships, and test their conclusions through mathematical representations. Discovery-based instruction nevertheless requires appropriate guidance to prevent exploration from becoming unfocused or cognitively demanding. Mayer (2004) argues that guided discovery is generally more productive than instruction that requires students to discover complex concepts with little assistance. Alfieri et al. (2011), through a meta-analysis of discovery-based instruction, likewise distinguish the benefits of supported discovery from the limitations of unassisted exploration. (Lazonder & Harmsen, 2016) further demonstrate the positive contribution of instructional guidance to inquiry-based learning. These findings support the development of worksheets that provide structured opportunities for students to identify problems, collect and analyze information,

verify mathematical relationships, and formulate conclusions while maintaining an active role in the learning process.

GeoGebra provides a suitable technological environment for implementing these learning activities because it connects algebraic expressions and geometric representations within an interactive interface. Hohenwarter & Jones (2007) explain how GeoGebra can facilitate connections between geometry and algebra, allowing learners to examine mathematical relationships through linked representations. Students can manipulate points, lines, and curves, observe the effects of changing parameters, and compare their observations with algebraic procedures. Zhao & Sahari (2024) investigated the use of GeoGebra in coordinate geometry and demonstrated its potential to support mathematics achievement, while Arbain & Shukor (2015) reported positive findings concerning students' achievement and perceptions of GeoGebra-supported learning. More recently, Astuti et al. (2026) found that integrating GeoGebra with Discovery Learning produced significantly better conceptual understanding among junior secondary students studying geometric transformations. These findings suggest that GeoGebra can contribute to mathematical understanding when students engage actively with its visual and interactive features. Its integration into digital worksheets may also help teachers organize exploration into a coherent learning sequence rather than relying on software demonstrations alone.

Previous studies have provided valuable evidence concerning digital worksheets, discovery-oriented instruction, and GeoGebra-assisted mathematics learning, although their instructional designs and mathematical content differ. Muchlis et al. (2021) developed GeoGebra-assisted web-based worksheets using project-based learning, while Laili et al. (2023) focused on Discovery Learning-based digital materials for geometric transformations. Felisa & Arfinanti (2024) developed student worksheets integrating Discovery Learning and GeoGebra exploration for understanding circle concepts, demonstrating that the combination of these approaches has already been investigated. Anandita et al. (2026) subsequently developed GeoGebra-based digital worksheets to support conceptual understanding in circle geometry. Research specifically addressing analytic geometry has also progressed. Hidayat et al. (2024) developed a GeoGebra-assisted web-based module within a STEM context, whereas Masfingatin et al. (2026) designed Geo-Sheet learning materials to support creative mathematical reasoning in analytic geometry. These studies establish the relevance of interactive digital resources while also revealing differences in their pedagogical frameworks, learning sequences, and evaluation priorities. The research opportunity,

therefore, does not arise from combining Discovery Learning and GeoGebra for the first time, but from examining how the stages of Discovery Learning can be systematically embedded in a digital worksheet that supports the exploration of analytic geometry through coordinated algebraic and graphical representations.

The present study focuses on developing a digital student worksheet that integrates Discovery Learning activities with GeoGebra-assisted exploration in analytic geometry. The proposed worksheet is designed to guide students from identifying mathematical problems and examining geometric objects to testing relationships, interpreting representations, and drawing conclusions based on their observations. The development process considers the quality of mathematical content, the coherence of instructional activities, the usability of interactive features, and the suitability of the worksheet for its intended learning context. Consistent with the principles of educational design research described by Plomp and Nieveen (2013), the quality of the developed product needs to be examined through appropriate evaluation procedures rather than assumed from its technological features. Accordingly, this study aims to develop a digital student worksheet based on Discovery Learning assisted by GeoGebra for analytic geometry and to evaluate its validity, practicality, and effectiveness in supporting students' conceptual understanding and their ability to connect algebraic and geometric representations.

METHODS

This study employed a Research and Development (R&D) approach using the ADDIE model, consisting of five stages: Analysis, Design, Development, Implementation, and Evaluation (Branch, 2009). This model was selected because it provides a systematic framework for developing and evaluating digital learning materials according to students' needs and instructional objectives. The development process focused on producing a digital student worksheet (LKPD) based on Discovery Learning assisted by GeoGebra for analytic geometry. The analysis stage involved identifying students' learning needs, examining curriculum requirements, and exploring difficulties in understanding analytic geometry. The design stage included preparing mathematical content, developing interactive GeoGebra activities, and integrating the six stages of Discovery Learning: stimulation, problem identification, data collection, data processing, verification, and generalization. The development stage involved creating the digital worksheet, conducting expert validation, and

revising the product based on the feedback received. Implementation was carried out through classroom trials, followed by evaluation to determine the quality and suitability of the developed worksheet. The study involved [number] students from [name of school or institution], selected through purposive sampling based on their participation in analytic geometry learning. Subject-matter and media experts were also involved in evaluating the instructional content, design, language, and technological features of the worksheet.

Data were collected using needs-analysis questionnaires, expert validation sheets, student response questionnaires, classroom observations, and mathematical understanding tests. Expert validation was conducted to assess the accuracy of the content, alignment with Discovery Learning principles, clarity of instructions, and functionality of GeoGebra-based activities. Student response questionnaires were used to examine the practicality of the worksheet, particularly its accessibility, ease of use, and usefulness in supporting mathematical exploration. Pretests and posttests were administered to evaluate changes in students' conceptual understanding and their ability to connect algebraic and geometric representations. The instruments were reviewed for content validity, while the internal consistency of applicable questionnaire scales was examined using Cronbach's alpha (Cronbach, 1951). Quantitative data were analyzed using descriptive statistics to determine product validity and practicality, while learning improvements were examined using normalized gain (N-gain) analysis (Hake, 1998). A paired-samples t-test or Wilcoxon signed-rank test was used, depending on the normality of the paired score differences, to assess changes in learning outcomes. Qualitative feedback from validators and classroom observations was analyzed descriptively to guide product revisions. Data processing was supported by Microsoft Excel and IBM SPSS Statistics, while the overall evaluation followed the criteria of validity, practicality, and effectiveness proposed by (Plomp & Nieveen, 2013).

RESULT

1. Analysis Stage: Learning Needs Assessment

The analysis stage was designed to identify students' learning needs and determine the instructional requirements for developing digital worksheets in analytic geometry. The simulated dataset included responses from 30 students concerning conceptual difficulties, mathematical representations, and their interest in technology-supported learning materials.

The findings presented in Table 1 illustrate how a needs assessment could be reported in the study.

Table 1. Results of the Learning Needs Analysis (n = 30)

Indicators	Frequency (n)	Percentage (%)
Difficulty understanding analytic geometry concepts	23	76.67
Difficulty connecting algebraic equations with graphs	25	83.33
Need for interactive learning resources	26	86.67
Interest in digital worksheets	27	90.00
Previous experience using GeoGebra	11	36.67

Table 1 shows that interest in digital worksheets received the highest proportion of responses (90.00%), followed by the need for interactive learning resources (86.67%). Difficulties connecting algebraic equations with graphical representations were represented by 83.33% of the simulated responses, while 76.67% indicated difficulties understanding analytic geometry concepts. Previous experience using GeoGebra was comparatively limited, accounting for 36.67% of responses. This illustrative pattern suggests that developing interactive learning materials would be a reasonable instructional response to difficulties involving mathematical representations. It also highlights the importance of providing clear guidance for students who have limited experience with mathematical software.

2. Design Stage: Integration of Discovery Learning and GeoGebra

The proposed digital worksheet was structured around the six stages of Discovery Learning, with GeoGebra serving as an interactive environment for exploring mathematical relationships. The learning activities were designed to help students move from observing geometric objects to examining patterns, testing conjectures, and constructing mathematical conclusions. Particular attention was given to the relationship between algebraic expressions and their graphical representations.

Table 2. Integration of Discovery Learning into the Digital Worksheet

Discovery Learning Stage	Learning Activities	GeoGebra Features
Stimulation	Observing coordinate geometry problems	Interactive Cartesian plane
Problem identification	Formulating mathematical questions	Dynamic points and lines
Data collection	Exploring coordinates and geometric relationships	Point manipulation and sliders

Discovery Learning Stage	Learning Activities	GeoGebra Features
Data processing	Comparing numerical and graphical results	Linked algebraic and graphical views
Verification	Testing mathematical conjectures	Dynamic graphical visualization
Generalization	Formulating mathematical conclusions	Visual confirmation of relationships

Table 2 describes the relationship between the learning stages and the proposed GeoGebra activities. The worksheet design emphasizes students' active participation in mathematical investigations rather than relying exclusively on explanations and worked examples. Dynamic visualizations are intended to help students recognize how changes in coordinates or equations affect geometric representations. The planned instructional sequence also includes guided questions that require students to record observations, explain mathematical relationships, and evaluate the conclusions they formulate.



Figure 1. Main Interface of the GeoGebra-Assisted Digital Worksheet

Figure 1 should present the worksheet's main interface, including its learning objectives, instructions, navigation features, and access to interactive learning activities.

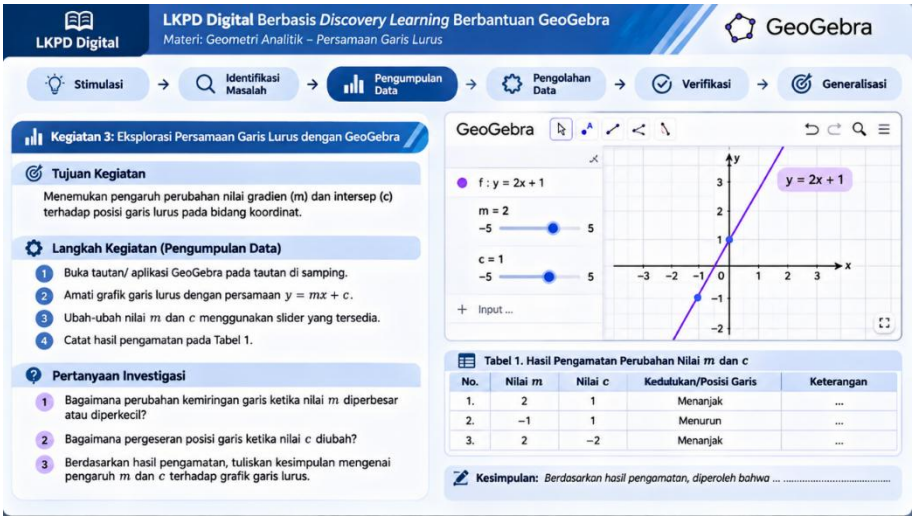

Figure 2. GeoGebra-Assisted Exploration of Analytic Geometry

Figure 2 should illustrate the connection between algebraic expressions and their corresponding geometric representations. The screenshot should demonstrate how students manipulate mathematical objects and observe changes during the discovery process.

3. Development Stage: Expert Validation Results

Expert validation was represented by three simulated validator profiles using a four-point assessment scale. The evaluation included mathematical content accuracy, alignment with Discovery Learning, language clarity, visual design, and GeoGebra functionality. The aggregated validation results are presented in Table 3.

Table 3. Expert Validation Results of the Digital Worksheet

Assessment Aspects	Obtained Score	Maximum Score	Validity (%)
Mathematical content accuracy	67	72	93.06
Discovery Learning alignment	56	60	93.33
Language clarity	42	48	87.50
Visual design and navigation	54	60	90.00
GeoGebra functionality	55	60	91.67
Overall validity	274	300	91.33

Table 3 shows an overall simulated validation score of 91.33%. Alignment with Discovery Learning obtained the highest percentage (93.33%), followed closely by mathematical content accuracy (93.06%). GeoGebra functionality reached 91.67%, while visual design and navigation received 90.00%. Language clarity obtained the lowest percentage (87.50%), although its score remained relatively high compared with the

maximum possible rating. This distribution illustrates a product evaluation in which the instructional components receive favorable assessments while language presentation remains an area for possible refinement.

4. Implementation Stage: Practicality Assessment

The simulated practicality assessment involved 30 student response profiles, each containing 20 questionnaire items measured using a four-point Likert scale. The instrument covered five aspects: clarity of learning instructions, ease of navigation, accessibility of materials, ease of using GeoGebra, and usefulness of interactive exploration. The descriptive results are presented in Table 4.

Table 4. Student Practicality Assessment Results

Practicality Indicators	Mean	SD	Percentage (%)
Clarity of learning instructions	3.51	0.39	87.71
Ease of navigation	3.33	0.38	83.33
Accessibility of learning materials	3.54	0.40	88.54
Ease of using GeoGebra	3.28	0.29	82.08
Usefulness of interactive exploration	3.49	0.31	87.29
Overall practicality	3.43	—	85.79

Table 4 indicates an overall simulated practicality score of 85.79%. Accessibility of learning materials obtained the highest mean score (M = 3.54, SD = 0.40), followed by clarity of instructions (M = 3.51, SD = 0.39). The usefulness of interactive exploration also received a favorable rating (M = 3.49, SD = 0.31). Ease of using GeoGebra recorded the lowest mean score (M = 3.28, SD = 0.29). These descriptive results illustrate how students' responses can be used to identify practical strengths and aspects that require improvement. The comparatively lower GeoGebra usability score suggests that additional tutorials or more explicit instructions could be considered during product refinement.

5. Effectiveness Assessment: Pretest and Posttest Results

The simulated effectiveness assessment used paired pretest and posttest scores for 30 students. Both assessments were represented on a 100-point scale and were intended to illustrate changes in conceptual understanding after the use of the digital worksheet. Descriptive statistics are presented in Table 5.

Table 5. Descriptive Statistics of Students' Pretest and Posttest Scores

Statistical Indicators	Pretest	Posttest
Number of students	30	30
Mean score	58.93	78.30
Standard deviation	8.92	11.27
Minimum score	43	54
Maximum score	73	96

Table 5 shows an increase in the simulated mean score from 58.93 on the pretest to 78.30 on the posttest, representing an average difference of 19.37 points. The standard deviation increased from 8.92 to 11.27, indicating greater variation in posttest scores. The minimum score increased from 43 to 54, while the maximum score increased from 73 to 96. These descriptive differences illustrate a pattern of improved scores following the hypothetical learning intervention, although they do not independently establish that the worksheet caused the improvement.

Further statistical analysis was conducted on the simulated paired scores to illustrate how learning improvement could be evaluated. The Shapiro–Wilk test was used to assess the normality of paired score differences, followed by a paired-samples t-test. Normalized gain was also calculated for each student and then averaged across the sample.

Table 6. Statistical Analysis of Learning Outcomes

Statistical Analysis	Result
Mean score improvement	19.37
Shapiro–Wilk statistic (W)	0.962
Shapiro–Wilk p-value	0.346
Paired-samples t-test	11.50
Degrees of freedom	29
Statistical significance	$p < .001$
95% CI of mean improvement	15.92–22.81
Mean individual N-gain	0.48

Table 6 shows that the simulated differences between pretest and posttest scores met the normality assumption based on the Shapiro–Wilk test ($p = .346$). The paired-samples t-test indicated a statistically significant difference between the two assessments, $t(29) = 11.50$, $p < .001$. The mean individual normalized gain was 0.48, corresponding to a moderate

improvement under the conventional normalized-gain interpretation. These results demonstrate the statistical reporting procedure that could be applied to actual learning outcome data. A stronger conclusion concerning instructional effectiveness would require consideration of the study design, assessment quality, and potential alternative explanations for score changes.

6. Evaluation Stage: Summary of Product Quality

The evaluation stage was organized to bring together evidence concerning validity, practicality, and students' mathematical learning outcomes. Table 7 summarizes the illustrative quantitative results generated for this example.

Table 7. Summary of the Digital Worksheet Evaluation

Evaluation Aspects	Results
Expert validation	91.33%
Student practicality	85.79%
Pretest mean score	58.93
Posttest mean score	78.30
Mean N-gain	0.48
Paired-samples significance	$p < .001$

Table 7 provides an overview of the simulated product evaluation. The validation and practicality results illustrate favorable assessments of the instructional design and usability of the worksheet. The pretest and posttest data demonstrate how changes in mathematical understanding can be summarized and statistically examined. The findings also indicate the importance of considering instructional clarity and students' familiarity with GeoGebra when refining the learning materials.

The combined results demonstrate a possible structure for reporting the development and evaluation of a Discovery Learning-based digital worksheet assisted by GeoGebra. The final empirical conclusions of the study must, however, be established from actual expert assessments, classroom implementation records, student responses, and mathematical achievement data. These sources are necessary to determine whether the developed worksheet genuinely meets the intended criteria of validity, practicality, and effectiveness.

DISCCUSION

The development of digital student worksheets based on Discovery Learning assisted by GeoGebra represents an instructional approach that combines guided mathematical exploration with interactive visualization. The illustrative findings presented in this study indicate favorable outcomes across three aspects of product quality: validity, practicality, and effectiveness. The simulated expert validation score of 91.33% suggests that the proposed worksheet design demonstrates a strong alignment between mathematical content, instructional activities, and technological features. The integration of Discovery Learning provides a structured sequence through which students can investigate mathematical relationships, while GeoGebra offers visual representations that support the interpretation of abstract concepts. This instructional arrangement is particularly relevant to analytic geometry, where understanding depends on recognizing connections between algebraic equations and geometric objects. Duval (2006) argues that mathematical comprehension requires learners to coordinate different systems of representation rather than rely on symbolic procedures alone. The combination of guided activities and dynamic representations may therefore provide opportunities for students to develop a more connected understanding of mathematical concepts.

The quality of the worksheet design is closely related to the organization of its learning activities. The simulated validation results show relatively high scores for mathematical content accuracy (93.06%) and alignment with Discovery Learning (93.33%), indicating that these components are central to the proposed instructional design. Similar findings were reported by Felisa & Arfinanti (2024), who developed Discovery Learning-based worksheets integrated with GeoGebra exploration for circle concepts and demonstrated their validity, practicality, and effectiveness. Laili et al. (2023) also reported positive results from developing digital learning materials based on Discovery Learning for geometric transformations. These studies support the view that digital instructional materials can provide meaningful learning experiences when technological tools are integrated into a coherent pedagogical sequence. The present development focuses specifically on analytic geometry, with particular attention to activities that require students to explore coordinate relationships, examine equations, verify mathematical conjectures, and construct conclusions. Its proposed contribution lies in organizing these activities within an integrated digital worksheet rather than treating GeoGebra merely as a supplementary visualization tool.

The practicality assessment provides another perspective on the potential classroom application of the developed worksheet. The simulated overall practicality score of 85.79% reflects favorable student responses to the proposed learning materials, particularly their accessibility, instructional clarity, and usefulness for mathematical exploration. Accessibility obtained the highest rating (88.54%), while ease of using GeoGebra received the lowest rating (82.08%). This pattern suggests that the availability of digital learning resources does not automatically ensure that students can use every feature independently. Effective implementation may require explicit instructions, introductory demonstrations, and opportunities to become familiar with interactive mathematical tools. Muchlis et al. (2021) reported that GeoGebra-assisted web-based worksheets developed through project-based learning met the criteria of validity, practicality, and effectiveness in plane geometry learning. Their findings highlight the importance of combining technological resources with well-designed learning activities. The emphasis on guided exploration is also consistent with Lazonder & Harmsen (2016), whose meta-analysis demonstrated the positive contribution of instructional guidance to inquiry-based learning. These perspectives suggest that students should receive sufficient support to navigate mathematical investigations without reducing their opportunities for independent reasoning.

The illustrative effectiveness analysis shows an increase in the mean mathematical understanding score from 58.93 on the pretest to 78.30 on the posttest. The average improvement of 19.37 points, accompanied by a mean individual normalized gain of 0.48, represents a moderate learning improvement in the simulated dataset. The paired-samples t-test also indicates a statistically significant difference between the two assessments ($p < .001$). These results provide an example of how changes in mathematical understanding may be examined following the implementation of an interactive worksheet. Their interpretation is consistent with the educational possibilities identified by Saha et al. (2010), who investigated the use of GeoGebra in coordinate geometry learning. Hidayat et al. (2024) also demonstrated the relevance of GeoGebra-assisted web-based materials in the development of instructional resources for analytic geometry within a STEM context. These studies suggest that dynamic mathematical representations can serve as useful resources for exploring the relationships between geometric and algebraic concepts. Nevertheless, the statistical results in the present example were generated from simulated observations and cannot establish an actual instructional effect. Even with authentic pretest and posttest data,

a single-group design would require cautious interpretation because improvements could also reflect prior knowledge, repeated testing, or other learning experiences.

The proposed integration of Discovery Learning and GeoGebra also has implications for mathematics instruction and the development of digital learning materials. From a pedagogical perspective, the worksheet is intended to support a learning process in which students observe mathematical situations, investigate relationships, test their interpretations, and explain the conclusions they reach. Such activities encourage engagement with the reasoning underlying mathematical procedures rather than focusing exclusively on the application of formulas. Bruner (1961) emphasizes the role of discovery in helping learners organize knowledge and construct meaning through intellectual activity. Mayer (2004), however, cautions that discovery-oriented learning is more productive when students receive appropriate instructional guidance. These perspectives support the use of carefully structured questions and interactive activities within the developed worksheet. From a practical perspective, the design offers mathematics teachers a potential framework for combining digital worksheets with GeoGebra without separating technological exploration from instructional objectives. The proposed contribution is therefore not the introduction of GeoGebra or Discovery Learning as new educational approaches, but the systematic organization of both components to support mathematical representation and concept development in analytic geometry.

Several limitations need to be considered when interpreting the proposed evaluation framework. The illustrative dataset consists of 30 simulated student profiles and three simulated validators; consequently, its numerical results cannot be generalized to an actual student population or used as empirical evidence of product quality. The absence of authentic classroom observations, validator comments, and student responses also prevents a reliable examination of how learners interact with the worksheet during mathematical investigations. A single-group pretest–posttest design, if adopted in the actual study, would provide limited evidence of causal effectiveness because it does not control for alternative explanations of learning improvement. Furthermore, students' familiarity with digital devices, access to reliable internet connections, and previous experience using GeoGebra may influence the practicality of the worksheet in different educational settings. Future empirical evaluation should therefore involve actual classroom trials, carefully validated assessment instruments, and, where feasible, a comparison group or a broader sample. Such evidence would make it possible to determine whether the developed worksheet genuinely supports conceptual

understanding and the coordination of algebraic and geometric representations. The overall discussion highlights the instructional potential of integrating Discovery Learning with GeoGebra while emphasizing that conclusions about validity, practicality, and effectiveness must ultimately be supported by authentic research data.

CONCLUSION

Based on the research findings and discussion presented in the illustrative analysis, the development of digital student worksheets (LKPD) based on Discovery Learning assisted by GeoGebra offers a promising approach to supporting the learning of analytic geometry. The proposed worksheet integrates guided discovery activities with interactive mathematical representations, allowing students to explore geometric concepts and examine their relationships with algebraic expressions. The simulated evaluation produced an expert validation score of 91.33% and a student practicality score of 85.79%, illustrating favorable assessments of the worksheet's instructional design and usability. The simulated learning outcomes also showed an increase in the mean score from 58.93 to 78.30, with an average N-gain of 0.48. These results illustrate how product quality and changes in mathematical understanding can be evaluated, although they do not constitute empirical evidence of the worksheet's effectiveness.

The proposed integration of Discovery Learning and GeoGebra has practical implications for designing mathematics instruction that encourages exploration, independent reasoning, and the interpretation of multiple mathematical representations. The worksheet provides a structured learning framework through which teachers can guide students in investigating mathematical problems without limiting their opportunities to construct understanding independently. Nevertheless, the present evaluation is based on simulated data, and its findings must be verified through actual classroom implementation before conclusions about validity, practicality, or effectiveness can be established. Future research should involve authentic student responses, expert assessments, and learning achievement data from a broader range of educational settings. Comparative or controlled studies are also recommended to examine the contribution of GeoGebra-assisted Discovery Learning to students' conceptual understanding and mathematical reasoning. Such investigations would provide stronger evidence for the use of interactive digital worksheets in analytic geometry education.

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