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From Representation to Reasoning: A Systematic Review of Students' Mathematical Representation Translation

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Abstract

Translating between mathematical representations can support reasoning. Evidence on how students perform these translations is dispersed across educational levels, topics, and theoretical frameworks. This systematic review synthesized evidence on translation types and directions, the translation process, difficulties and associated factors, instructional supports, and links to mathematical reasoning. Following PRISMA 2020, a Scopus search covering 2015–2026 identified 289 records. Two reviewers screened the records ($\varkappa$ = 0.877), and 53 studies were included after full-text assessment. Methodological quality was appraised using the 2018 Mixed Methods Appraisal Tool, and findings were synthesized descriptively and thematically. The studies spanned 21 countries and ranged from primary to university education, with 36 using qualitative designs. Symbolic representations were the target in 45 of 106 reported translation directions. Concrete representations and verbal targets received little attention. Interpretation errors were the most common, and difficulty varied by translation direction. Dynamic technology, scaffolded task design, and meaningful language were the most frequently studied instructional supports, with limited evidence of their causal effects. Structural translation was associated with explanation, generalization, and justification, suggesting that articulating the same mathematical structure across representations can support reasoning. The review maps translation directions, proposes a representation–translation–reasoning model, and identifies priorities for research on underexamined directions, experimental studies, and instruments that assess translation and reasoning together.

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Keywords: Mathematical Representations; Representation Translation; Mathematical Reasoning; Mathematics Education; Systematic Literature Review

INTRODUCTION

Mathematical representations are the main means by which students understand, communicate, and solve mathematical problems. Mathematical objects are abstract and cannot be observed directly, so students can only access them through semiotic representations (Duval, 2006). These representations take many forms, namely verbal, symbolic, graphical, tabular, pictorial or visual, and concrete (Lesh et al., 1987). Each form highlights a different aspect of a concept, so the use of several representations can deepen conceptual understanding (Ainsworth, 2006; Rau & Matthews, 2017).

However, this potential is not realized automatically. The DeFT framework explains that the benefits of multiple representations depend on students' ability to understand each representation and to connect one representation with another (Ainsworth, 2006). Without the ability to connect them, multiple representations can instead confuse students (Rau & Matthews, 2017). A meta-analysis of 46 STEM studies likewise found that the learning gains from more than two representations depend heavily on the support provided (Rexigel et al., 2024). The main challenge is therefore not merely providing many representations but helping students connect several representations of the same concept.

This ability to connect representations is what the literature calls translation between representations: the process of converting information from a source representation to a target representation while preserving equivalence of meaning (Adu-Gyamfi et al., 2012; Bossé et al., 2011). Duval calls this process conversion and distinguishes it from treatment, a transformation within a single register; in his view, conversion is the decisive factor in learning mathematics (Duval, 2006). The directions of translation studied are diverse, for example verbal to symbolic for algebraic statements (Castro et al., 2022; Molina et al., 2017), symbolic to graphical and vice versa for linear functions (Adu-Gyamfi et al., 2012), and table to graph and algebraic relation (Testa & Catena, 2022).

Although defined as a change of form, meaningful translation demands more than swapping one representation for another. Among 414 students in Grades 5–6, surface translation, which only matches numbers without explaining structure, occurred frequently

and was associated with lower conceptual understanding of multiplication (Hankeln & Prediger, 2025). In conditional probability, some students grasped the concept only when teachers explicitly connected conceptual elements across representations rather than merely translating them (Post & Prediger, 2024). Mathematically meaningful conversion also helps students move from empirical to structural generalization while explaining why the generalization holds (Yao, 2022). These findings suggest that structural translation is a bridge from representation to mathematical reasoning, that is, explaining, generalizing, and justifying. The relevance of translation spans many topics, including algebra, functions, fractions, trigonometry (Mosese & Ogbonnaya, 2021), probability, and contextual problem solving (Rahmawati et al., 2025). Translation is thus a process of connecting mathematical meaning that opens the way to reasoning, not merely a procedural skill.

Because it requires coordinating meaning across representations, translation is among the most consistently reported sources of difficulty in empirical research. In verbal-to-symbolic translation, secondary school students make errors, some of which persist to the end of compulsory education (Molina et al., 2017), and many stem from arithmetic misconceptions such as confusing multiplication with exponentiation (Castro et al., 2022). Of 65 Grade 11 students, only 3.08% translated a quadratic-function problem correctly (Rahmawati et al., 2025). In the opposite direction, lower secondary students failed to preserve semantic congruence when converting a system of equations into verbal and graphical form (Nurrahmawati et al., 2021). University mathematics students also make construction errors, such as loss of representation attributes, pseudo-thinking, and broken connections (Afriyani et al., 2019).

These difficulties are not uniform; they differ by translation direction, topic, and student characteristics. Bossé et al. (2011) showed that difficulty is influenced by fact gaps, misleading facts, and attribute density in each representation. Adu-Gyamfi et al. (2012) identified three error types, namely implementation, interpretation, and preservation, which occur at different stages of translation. Among 643 upper secondary students, algebraic representations were hardest to interpret, and the ability to move between representations was related to spatial reasoning and confidence (Testa & Catena, 2022). High-ability students managed to pass through the translation stages, whereas medium- and low-ability students tended to make interpretation errors (Rahmawati et al., 2025). Ultimately, these difficulties directly affect conceptual understanding and problem-solving success (Hankeln & Prediger, 2025). Students who stop at surface translation also lose the opportunity to reason about the

mathematical structure behind representations, so translation difficulties may hinder the development of their mathematical reasoning.

Such diverse difficulties have encouraged research on translation across educational levels. In primary school, studies have examined translation between pictorial and mathematical representations in estimation (Ainsworth et al., 2002) and between symbolic, graphical, and textual representations in multiplication (Hankeln & Prediger, 2025). In secondary school, the focus includes algebraic statements, systems of equations, functions, and linear relations (Molina et al., 2017; Nurrahmawati et al., 2021; Testa & Catena, 2022), while at university the studies target mathematics students (Afriyani et al., 2019). The foci also vary, from representation types and translation directions, to thinking processes by translation stage (Adu-Gyamfi et al., 2012), student errors, and related factors such as prior ability, spatial reasoning, and task characteristics.

Beyond mapping difficulties and their influencing factors, several studies have tested instructional supports to develop translation ability. Dynamic technology is studied most often: GeoGebra improved students' ability to connect representations of trigonometric functions (Bedada & Machaba, 2022; Mosese & Ogbonnaya, 2021), and a 20-year meta-analysis reported a medium-to-large effect ($g = 0.653$) on mathematics achievement, although not specific to translation (Zhang et al., 2025). Other supports include meaningful language for explaining structure (Hankeln & Prediger, 2025), teacher practices for connecting representations (Post & Prediger, 2024), and design principles for using multiple representations (Rau & Matthews, 2017). Research in other STEM areas, such as kinematics (Testa & Catena, 2022) and computational thinking (Moore et al., 2020), enriches understanding of translation but is treated here only as supporting literature. Translation has therefore been studied from many perspectives, but the findings remain scattered and need to be synthesized.

Efforts to synthesize this literature have begun but have not targeted translation specifically. Escudero et al. (2026) found that conversion among graphical, algebraic, tabular, and verbal representations is one of the difficulties students face in solving function problems. Castillo et al. (2025) positioned the coordination of semiotic registers as a strategy for building conceptual understanding at university, while Ruamba et al. (2025) examined visual and multimodal representations in relation to cognitive load and problem solving. Other reviews are more general, for example on trends and abilities in mathematical

representation (Anwar et al., 2025; Putra et al., 2023; Putri et al., 2025), multiple representations in ethnomathematics-based realistic mathematics education (Setianingsih et al., 2026), and the number of representations in STEM learning (Rexigel et al., 2024). These reviews treat representation as a general ability or limit translation to a single topic; as far as an initial search could determine, none has made translation between representations the main unit of analysis, let alone linked it to mathematical reasoning.

Because translation has not been a unit of analysis in its own right, several important aspects remain inadequately synthesized. First, the types and directions of translation studied most often have not been mapped comprehensively. Second, findings on the translation process have not been integrated with students' errors and difficulties, even though the Translation–Verification model shows that particular errors arise at particular stages (Adu-Gyamfi et al., 2012). Third, the pattern of factors related to translation success or failure, such as prior ability, spatial reasoning, and task characteristics, is unclear. Fourth, instructional approaches and supports for overcoming these difficulties have not been summarized. Fifth, how translation leads students to mathematical reasoning, for instance through explanation, generalization, and justification, has not been synthesized even though several studies have shown the link (Hankeln & Prediger, 2025; Yao, 2022). Sixth, possible differences in focus and findings by educational level, mathematical topic, and research method have not been examined.

The novelty of this review lies in three aspects. First, translation between representations is treated as a unit of analysis in its own right and distinguished clearly from the general use of representations. Second, translation is linked to mathematical reasoning through the distinction between surface and structural translation. Third, findings on the process, difficulties, and instructional supports are integrated into a single representation–translation–reasoning model. A systematic literature review was chosen because its procedures are explicit, transparent, traceable, and replicable, and it is reported following PRISMA 2020 (Page, McKenzie, Bossuyt, Boutron, Hoffmann, Mulrow, Shamseer, Tetzlaff, Akl, Brennan, & others, 2021).

The theoretical foundation draws on four ideas. **Mathematical representation**. Representations are understood as observable external expressions of students' internal conceptualizations. Lesh et al. (1987) distinguished five representation systems, namely real situations, manipulative models, pictures, spoken language, and written symbols, and stressed

that understanding is visible in the ability to move among them. In this review, representation forms are grouped as verbal, symbolic or algebraic, tabular, graphical, pictorial or visual, and concrete. DeFT explains that multiple representations can complement one another, constrain interpretation, and jointly build deeper understanding, provided students can connect them (Ainsworth, 2006).

Translation between representations. Translation is defined as converting information from a source to a target representation while preserving equivalence of meaning. Duval (2006) distinguished treatment, a transformation within one register, from conversion, a transformation between registers, and held that conversion is the decisive factor in learning mathematics. The Translation–Verification model details the process in four stages, namely deconstructing the source, initial coordination, building the target, and checking equivalence, and identifies three error types: implementation, interpretation, and preservation (Adu-Gyamfi et al., 2012). The difficulty of translation is also influenced by representation characteristics such as fact gaps and attribute density (Bossé et al., 2011).

From translation to reasoning. Translation can occur at a surface or a structural level. Surface translation only matches visible elements, such as numbers, without explaining the mathematical structure shared by both representations; structural translation, in contrast, articulates that structure (Hankeln & Prediger, 2025). Meaningful conversion helps students move from empirical to structural generalization accompanied by an explanation of why the generalization holds (Yao, 2022). The ability to create, interpret, translate, and connect representations meaningfully is called representational fluency (Fonger, 2019). In this review, mathematical reasoning is operationalized as explaining, generalizing, justifying, and arguing.

Conceptual framework. Based on the above, this review views translation as a bridge from representation to reasoning (Figure 1). A source representation is translated into a target representation through a series of stages (RQ1 and RQ2). Errors may arise at these stages and are influenced by student, task, and representation factors (RQ3). The quality of translation, surface or structural, determines whether students arrive at shallow understanding or at mathematical reasoning (RQ5), while instructional supports are expected to move students toward structural translation (RQ4). This framework also served as the basis for the initial categories in data coding.

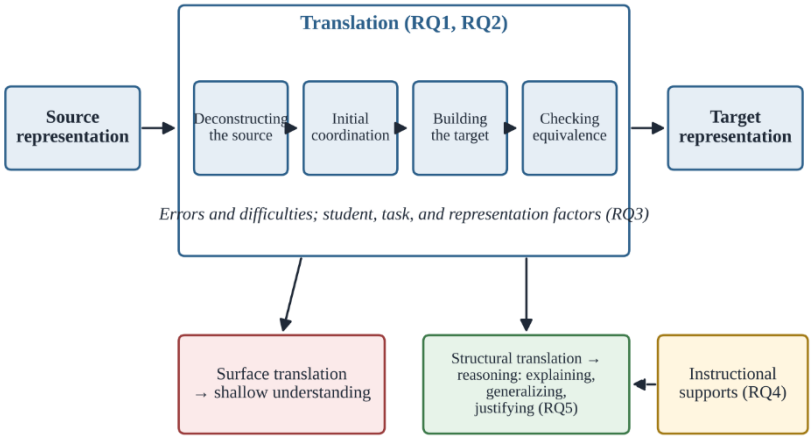


Figure 1. Conceptual framework: from representation to reasoning through translation, mapped to RQ1–RQ5

Based on these needs, this study aims to produce a systematic synthesis of the journey from representation to reasoning through translation, as a basis for research and for mathematics teaching practice. Specifically, it maps the types and directions of mathematical representation translation studied in mathematics education (RQ1), examines how students process translation between representations (RQ2), identifies students' difficulties and errors and the factors related to them (RQ3), summarizes the instructional approaches and supports used to facilitate translation (RQ4), and analyzes how students' representation translation is linked to their mathematical reasoning, such as explaining, generalizing, and justifying (RQ5).

METHODS

1. Type of Research

This study is a systematic literature review with a mixed synthesis approach: study characteristics (RQ1) were analyzed descriptively and quantitatively, whereas the process, difficulties, instructional supports, and the link to reasoning (RQ2–RQ5) were synthesized qualitatively.

2. Research Design

The review is reported following the PRISMA 2020 guideline (Page, McKenzie, Bossuyt, Boutron, Hoffmann, Mulrow, Shamseer, Tetzlaff, Akl, Brennan, Chou, et al., 2021), with reporting of the search strategy following PRISMA-S (Rethlefsen et al., 2021). The implementation steps were adapted from guidance for systematic reviews in education

(Ghamrawi et al., 2025; Newman & Gough, 2020). The review protocol was prepared before the selection stage; deviations from it are described in the section on data sources and search strategy below.

3. Participants and Sampling Technique

The unit of analysis was the peer-reviewed journal article; no human participants were involved. All documents captured by the search string and meeting the eligibility criteria were included, without sampling. The search was conducted in the Scopus database on 8 November 2026 for publications from 2015 to the search date. The search string was built from three keyword groups, namely translation process, representation, and mathematics, plus a subject group (students), combined with Boolean operators. The proximity operator W/5 was used so that translation terms and representation terms appear close together. The search was limited to English-language publications, excluding the document types Review, Book, Book chapter, and Conference review, and restricted to the subject areas Social Sciences, Mathematics, Psychology, Arts and Humanities, and Multidisciplinary. The final string was as follows.

> TITLE-ABS-KEY (("translat*" OR "conver*" OR "transform*" OR "transition*" OR "connect*") W/5 ("representation*" OR "semiotic register*")) AND TITLE-ABS-KEY ("mathemat*") AND TITLE-ABS-KEY ("student*" OR "learner*" OR "pupil*") AND PUBYEAR > 2014 AND PUBYEAR < 2027 AND LANGUAGE (english) AND NOT DOCTYPE (re) AND NOT DOCTYPE (bk) AND NOT DOCTYPE (ch) AND NOT DOCTYPE (cr) AND (LIMIT-TO (SUBJAREA , "SOCI") OR LIMIT-TO (SUBJAREA , "MATH") OR LIMIT-TO (SUBJAREA , "PSYC") OR LIMIT-TO (SUBJAREA , "ARTS") OR LIMIT-TO (SUBJAREA , "MULT"))

This search is the second version and deviates from the protocol in three ways. First, the date range starts in 2015 rather than 2010. This starting point was chosen because National Council of Teachers of Mathematics (2014) established using and connecting mathematical representations as one of eight effective teaching practices, so studies from 2015 onward fall within that framework. In addition, over the last decade digital multi-representation learning environments have become common in mathematics teaching, so this range captures translation in present-day learning contexts and matches the ten-year window common in systematic reviews. Second, the proximity operator was widened from W/3 to W/5 so that phrases such as "translating between graphs and equations" would still

be captured. Third, the subject-area restriction was added to reduce records outside education. Checking the records removed by this restriction revealed three apparently relevant studies; this is noted as a limitation.

Eligibility criteria were built with the PICOS framework and are presented in Table 1. Studies were included if they examined translation between mathematical representations among students from primary school to university as a main focus, were empirical, and were published in peer-reviewed English-language journals.

Table 1. Inclusion and exclusion criteria

Aspect	Inclusion	Exclusion
Population	Primary school to university students, including pre-service mathematics teachers	In-service teachers only; textbooks or curricula only, without student data
Phenomenon	Translation or conversion between mathematical representations as the main focus or a measured variable	Representations mentioned only in passing; treatment within one register only
Field	Mathematics learning (algebra, functions, fractions, geometry, calculus, statistics, and others)	Other science fields without clear mathematical content
Outcome	Data on ability, errors, processes, translation intervention outcomes, or links to reasoning	Student perceptions or attitudes only
Design	Empirical studies: quantitative, qualitative, or mixed methods	Theoretical articles, opinion pieces, editorials, and literature reviews
Publication	Peer-reviewed journal articles, English, published 2015–2026, full text available	Proceedings, dissertations, preprints, book chapters

4. Instruments and Data Collection

Search results were exported in CSV format with abstracts and managed in a screening worksheet. Title and abstract screening was conducted independently by two reviewers, both researchers in mathematics education. Documents judged "include" or "unsure" by both reviewers proceeded to full-text assessment. Conference papers and non-article documents that were still captured were excluded as not journal articles, in line with Table 1. Interrater agreement was calculated with Cohen's kappa (McHugh, 2012), and disagreements were resolved through discussion. Reports whose full text could not be obtained were sought via DOI, the Zotero full-text search function, and institutional access. Three decision rules were applied consistently at the full-text stage. First, studies on mathematical connections were included only if they reported data on conversion between registers (direction, quality, or errors), not merely a typology of connections. Second, studies set in physics were included only if the mathematical context was also measured directly

(three studies); purely science contexts were excluded as E2. Third, pre-service teachers were included if studied as learners rather than as instructors.

Methodological quality of the included studies was appraised with the Mixed Methods Appraisal Tool (MMAT) version 2018, which has five core criteria for each of the qualitative, randomized quantitative, non-randomized quantitative, quantitative descriptive, and mixed-methods designs (Hong et al., 2018). As its developers advise, MMAT was not used to exclude studies; the per-criterion detail was used to weigh the strength of evidence, and studies meeting only two criteria are flagged in the Appendix.

Data were extracted with a form first piloted on five articles. Extraction categories followed the research questions and the theoretical framework (Table 2). One extraction row was created for each translation direction studied.

Table 2. Data extraction categories by research question

RQ	Focus	Data extracted
RQ1	Types and directions of translation	Source; target representation, mathematical topic, level, theoretical framework; study characteristics (year, country, design)
RQ2	Translation process	Translation stage, strategy, transitional representation
RQ3	Difficulties, errors, and factors	Error type, stage at which it arises, student, task, and representation factors
RQ4	Instructional supports	Intervention type, technology, duration, outcomes, effect size where available
RQ5	Link to reasoning	Reasoning construct, form of the link, quality of translation (surface or structural)

5. Validity, Reliability, and Trustworthiness

Reliability of selection was maintained through independent screening by two reviewers, with interrater agreement computed using Cohen's kappa and disagreements resolved by discussion (McHugh, 2012). Trustworthiness was maintained through coding by two reviewers, an audit trail of every decision, and reflexive discussion among the researchers (Fromm et al., 2025). Screening, MMAT appraisal, and data extraction were assisted by a large language model (Claude, Anthropic) that produced preliminary suggestions; all final decisions were verified and made by the researchers, in line with the joint position statement of Cochrane, the Campbell Collaboration, JBI, and CEE on the use of AI in evidence synthesis (Flemyng et al., 2025).

6. Data Analysis Technique

RQ1 was analyzed descriptively through frequencies, annual trends, and a source × target translation matrix. RQ2, RQ3, and RQ5 were synthesized thematically (Thomas & Harden, 2008) with initial codes drawn from the theoretical framework and kept open to new codes, consistent with a framework synthesis approach (Brunton et al., 2020). RQ4 was synthesized narratively by intervention type; a meta-analysis would be performed only if at least five comparable experimental studies were available.

7. Ethics, Research Time, and Location

This review used secondary data from published articles and involved no human participants, so ethical approval was not required. Publication ethics were maintained by citing all sources and by openly reporting the use of a large language model. The database search was carried out online on 8 November 2026; all selection, appraisal, and synthesis steps were carried out online by the research team.

RESULTS

1. Main Findings

Types and directions of translation (RQ1). Research concentrated on translations ending in a symbolic representation: 45 of 106 directions had a symbolic/algebraic target. The most frequently studied directions were symbolic → graph (17), picture/visual → symbolic (16), graph → symbolic (13), and verbal → symbolic (11). By contrast, concrete/manipulative representations appeared in only 3 directions and verbal targets in 7 directions, all at the primary to lower secondary levels (Figure 2). The most frequently used theoretical frameworks were Duval's semiotic registers, Lesh's translation model, Prediger's framework of representation connections, and the Extended Theory of Connections.

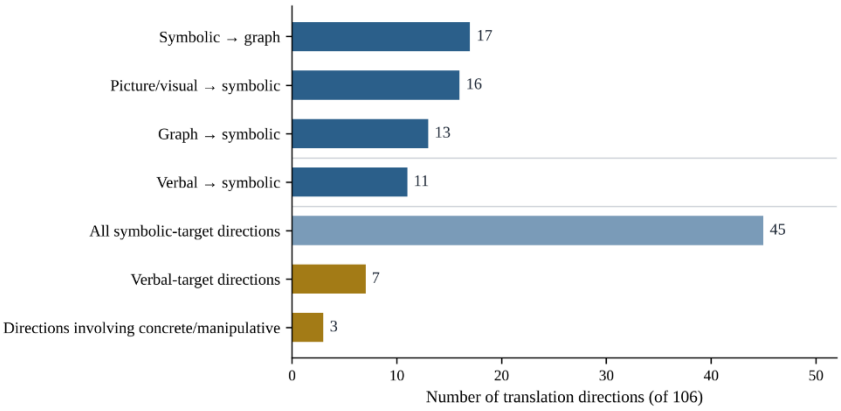


Figure 2. The most and least frequently studied translation directions (number of directions out of 106 in total)

Student translation process (RQ2). Most studies observed the final product of translation; only one study mapped all four stages explicitly (Swastika et al., 2023). Building the target was the most frequently studied stage, whereas checking equivalence was the least often performed by students (Table 3). The transitional representations that consistently helped were visual-concrete models (area models, 10×10 grids, base-ten blocks, Cuisenaire rods), meaningful language, and connected digital representations. Three studies proposed levels of process quality: surface versus structural translation (Hankeln & Prediger, 2025), SOLO levels of representational fluency (Fonger, 2019), and procedural–partial–coordinated levels (Siregar et al., 2026).

Table 3. Translation stages, indicators, and example studies

Stage	Indicator	Studies
Deconstructing the source	Making sense of the attributes and structure of the source representation	S15, S19, S41, S47, S49
Initial coordination	Planning the mapping of source elements to the target; moving back and forth between representations	S01, S49
Building the target	Producing a complete and accurate target representation	S22, S23, S29, S33, S38, S43, S49
Checking equivalence	Comparing the meaning of source and target	S17, S27, S49

Difficulties, errors, and related factors (RQ3). Interpretation errors were the most dominant, reported by 20 of the 26 studies that analyzed errors, mainly at the stages of deconstructing the source and building the target (Table 4). Translation direction was reported to determine difficulty: transitions involving formulas were hardest (Ceuppens et al., 2018), items starting from a graph were easier than those starting from an equation (Van

den Eynde et al., 2019), and the type of error depended on the direction of the connection (De Bock et al., 2015). Student factors included narrow concepts, a predominantly symbolic-procedural understanding, language, and spatial ability; task factors included context and complexity; representation factors included function type and periodicity. Surface translation occurred often and correlated with low scores (Hankeln & Prediger, 2025).

Table 4. Error types by translation stage

Error type	Deconstructing the source	Initial coordination	Building the target	Checking equivalence
Implementation			S07	
Interpretation	S08, S14, S15, S19, S20, S33, S41, S47		S05, S11, S23, S29, S37, S38, S43, S48	S27
Preservation				S17
Other			S02, S26	

Instructional supports (RQ4). Thirty-two studies tested or designed instructional supports (Table 5). Dynamic technology was studied most often (13 studies), followed by task design and scaffolding (11), meaningful language (4), teacher practices (3), and generative AI (1). Causal evidence was limited: one randomized trial (Korntreff et al., 2026) and two quasi-experiments (Ariawan et al., 2026; Mosese & Ogbonnaya, 2021).

Table 5. Interventions, contexts, and outcomes

Intervention type	Studies	Level and topic	Outcome
Dynamic technology	S02, S07, S14, S18, S25, S31, S32, S34, S39, S40, S42, S45, S46	Primary to university; functions, geometry, calculus, early algebra	Positive effects on representation connections (S45, S46); the tool may take over translation (S02) or give rise to misconceptions about dynamic properties (S14)
Task design and scaffolding	S03, S06, S16, S17, S19, S21, S23, S24, S26, S38, S41	Primary to university; functions, rational numbers, early algebra	A contextual–numerical–symbolic/graphical sequence helps; transitions do not occur automatically (S38)
Meaningful language	S04, S08, S10, S52	Primary to university; functions, probability, multiplication	Meaningful phrases were associated with understanding (S52, n = 414); language uptake predicted outcomes (S10)
Teacher practices	S09, S12, S15	Preschool to upper secondary; addition, data, probability	Prompting, unfolding dense concepts, and explicit connections helped weaker students
Generative AI	S30	University; calculus	AI responses were often epistemically irrational; only strong students could filter them

Translation and reasoning (RQ5). Thirty-two studies linked translation to reasoning, most often explaining (15 studies), generalizing (9), and justifying (9) (Table 6). Meaningful conversion was reported to produce generalizations that explain why a formula is true (Yao, 2022), and conversion and reasoning were reported to drive each other (Morais et al., 2018). Conversely, surface translation or translation taken over by a tool severed the basis for justification (Gregersen, 2024). The quality of translation was reported as structural in 11 studies, surface in 8, and mixed in 11; the evidence for this translation–reasoning link was almost entirely qualitative.

Table 6. Reasoning constructs and the form of their link with translation

Reasoning construct	Form of link with translation	Studies
Explaining	Explaining connections requires articulating the same structure in both representations	S01, S04, S05, S08, S10, S11, S12, S15, S22, S23, S26, S34, S37, S48, S52
Generalizing	Conversion supports structural generalization and variational thinking	S03, S16, S21, S24, S25, S28, S31, S38, S42
Justifying	Conversion becomes a source of justification; tool-taken-over translation weakens it	S01, S02, S06, S16, S17, S26, S28, S33, S34
Arguing	Representation transformation as a component of an argument	S14, S30
Proving	An accurate diagram from a verbal problem precedes a coherent proof	S45

2. Supporting Findings: Study Selection, Characteristics, and Quality

The search yielded 289 documents, consisting of 250 journal articles, 37 conference papers, 1 editorial, and 1 note. Before the subject-area restriction, the same search yielded 354 documents. A check of DOIs and titles found no duplicates. Because 2026 had not ended, data for that year cover publications only up to 8 November 2026. The selection flow is shown in Figure 3.

Agreement between the two reviewers at the title–abstract stage reached 92.0% ($\varkappa$ = 0.877). Of the 289 records, 129 were excluded at this stage: not a journal article (E1) 37, subjects not students (E4) 29, outside mathematics (E2) 27, translation not the focus (E3) 22, not an empirical study (E5) 12, and outcome limited to perceptions (E6) 2. Of the 160 reports sought, the full text of 73 could not be obtained. Eighty-seven reports were assessed for eligibility; 34 were excluded (E3 22, E4 6, E5 4, E2 2), leaving 53 studies for synthesis.

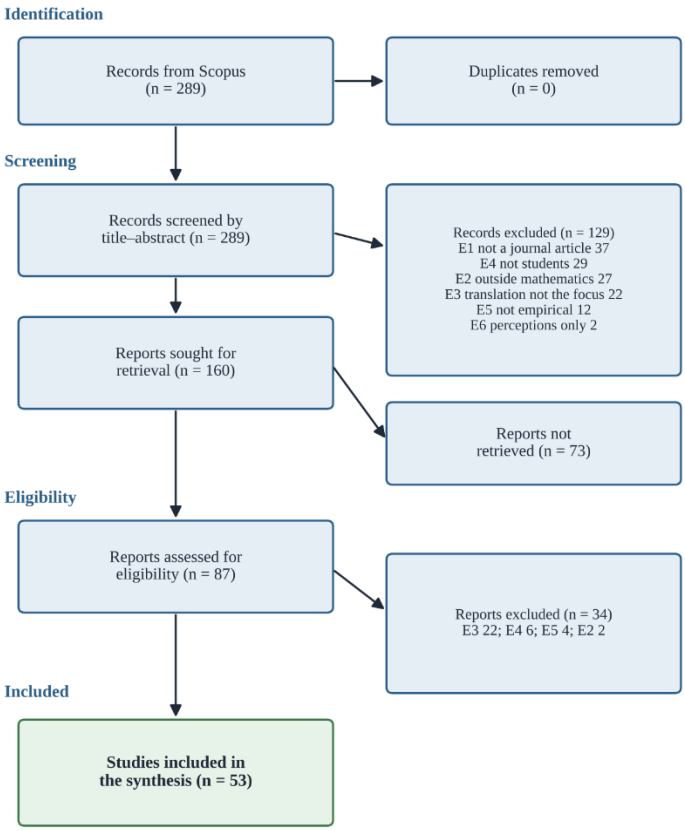


Figure 3. PRISMA 2020 flow diagram of the study selection process

In total, 53 studies from 21 countries met the criteria (Table 7). Publications increased over time: 4 studies appeared in 2015–2018, 18 in 2019–2022, 14 in 2023–2024, and 17 in 2025–2026 (Figure 4). Indonesia (9 studies) and Germany (8) contributed the most. Levels were spread across lower secondary (13), upper secondary (13), university (13), primary (9), pre-service teachers (3), and mixed (2). Qualitative designs predominated (36 studies), followed by quantitative descriptive (9), mixed methods (5), quantitative non-randomized (2), and one randomized trial. Functions and calculus were the most frequently studied topics, whereas numbers and operations dominated in primary school. MMAT appraisal showed that 23 studies met all five criteria, 12 met four, 15 met three, and 3 met two criteria (S35, S39, S51). The most frequent weaknesses were insufficiently detailed qualitative analysis procedures and unreported response rates in quantitative studies.

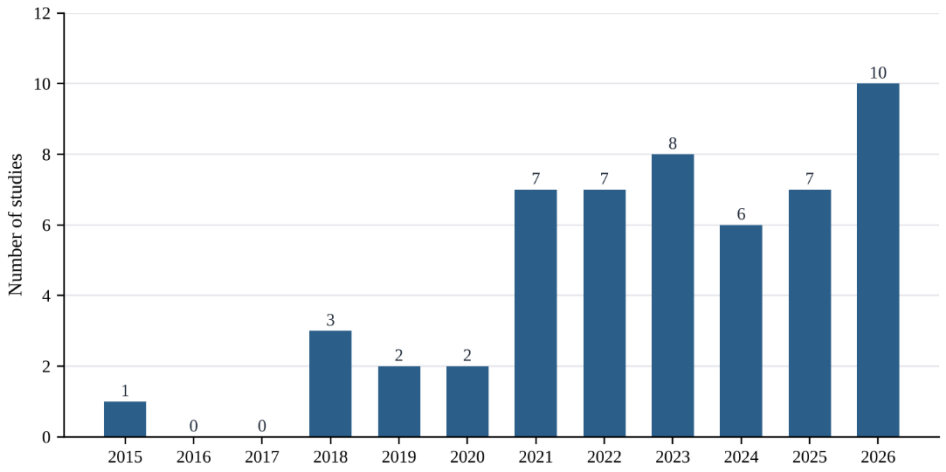


Figure 4. Number of included studies per year (2026 data up to 8 November 2026)

Table 7. Summary of characteristics of the 53 included studies (full list in the Appendix)

Characteristic	Category (number of studies)	Note
Publication year	2015–2018 (4); 2019–2022 (18); 2023–2024 (14); 2025–2026 (17)	2026 data up to 8 November 2026
Country	Indonesia (9); Germany (8); Mexico (4); Israel (4); Colombia (3); United States (3); Belgium (3); 14 other countries (1–2 studies)	21 countries
Level	Lower secondary (13); upper secondary (13); university (13); primary (9); pre-service teachers (3); mixed (2)	Includes 1 preschool study (in the mixed category)
Mathematical topic	Functions (16); numbers and operations (11); calculus (9); algebra and patterns (9); geometry and vectors (5); statistics and probability (3)	Functions include linear, quadratic, trigonometric, and covariation
Design (MMAT category)	Qualitative (36); quantitative descriptive (9); mixed methods (5); quantitative non-randomized (2); randomized trial (1)	
MMAT criteria met	5 of 5 (23); 4 of 5 (12); 3 of 5 (15); 2 of 5 (3)	Studies scoring 2: S35, S39, S51

3. Negative Data, Divergent Findings, and Anomalies

Several findings did not follow the general pattern. First, in studies of dynamic technology, digital tools were reported to take over the translation process (S02) and to give rise to misconceptions about dynamic properties (S14), although two other studies reported positive effects on representation connections (S45, S46). Second, scaffolding did not automatically produce transitions between representations (S38). Third, in the generative AI study (S30), AI responses were often epistemically irrational and only strong students could filter them. Fourth, only one of the 53 studies mapped all four translation stages explicitly (Swastika et al., 2023). Fifth, the full text of 73 of the 160 reports sought could not be

obtained, and three studies met only two of the five MMAT criteria (S35, S39, S51). The meta-analysis on GeoGebra reported a medium-to-large effect on mathematics achievement in general, not on translation (Zhang et al., 2025).

DISCUSSION

1. Analysis and Interpretation of Findings

The findings for RQ1–RQ5 can be assembled into one integrative model (Figure 5). Translation begins with a source representation whose meaning must be deconstructed; this is the stage that fails most often, especially when the direction of translation goes to or from a formula (Ceuppens et al., 2018; Van den Eynde et al., 2019). Failure at this stage produces interpretation errors and surface translation, that is, matching visible elements without a shared structure (Hankeln & Prediger, 2025).

Instructional supports shift students from surface to structural translation through three routes: visual-concrete transitional representations, meaningful language that articulates structure, and connected digital representations that show simultaneous change. A principle that appears across all types of support is connecting rather than merely translating: a support appears effective when it makes students articulate the same structure in both representations. Structural translation then opens the way to reasoning: meaningful conversion produces generalizations that explain and justifications that are well grounded (Morais et al., 2018; Yao, 2022). In other words, it is not the number of representations that matters but whether students connect, rather than merely translate. Because causal evidence for instructional supports is still limited (one randomized trial and two quasi-experiments), these routes should be read as consistent patterns in the literature, not as proven cause-and-effect relationships.

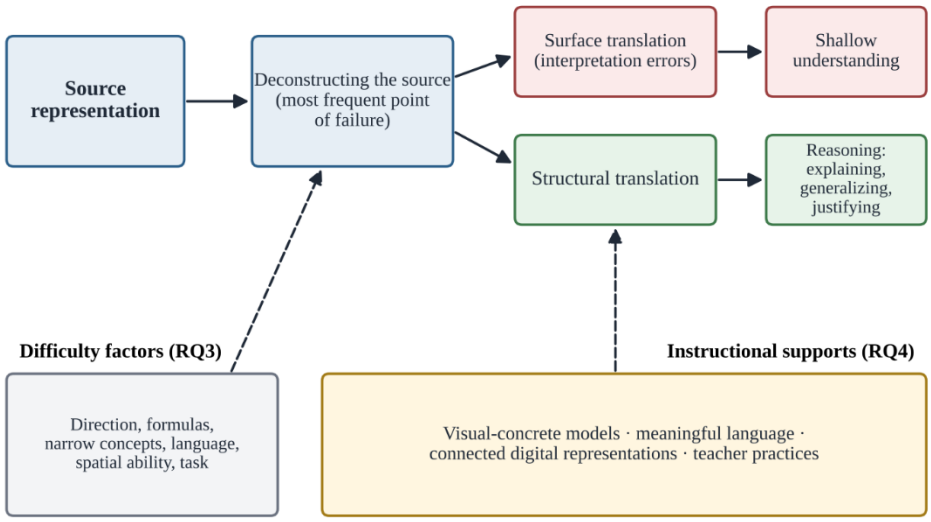


Figure 5. Integrative representation–translation–reasoning model based on evidence from 53 studies (a revision of Figure 1)

2. Comparison with Theories and Previous Literature

Compared with earlier reviews, this study makes translation the unit of analysis and links it to reasoning. Escudero et al. (2026) noted conversion between representations as one of the difficulties in function problems; this review extends that finding by showing that the difficulty depends on translation direction and appears across topics. Castillo et al. (2025) positioned register coordination as a learning strategy at university, whereas this review details the types of support and the strength of their evidence from primary school to university. More general reviews on representation ability (Anwar et al., 2025; Putra et al., 2023) and visual-multimodal representations (Ruamba et al., 2025) treat representation as a single ability. This review adds the distinction between surface and structural translation, which appears to determine whether translation leads to reasoning.

The findings reinforce Duval (2006)'s view that conversion, not treatment, is the critical point of mathematical understanding: meaningful conversion yields generalizations that explain, whereas symbolic treatment yields only correct formulas (Yao, 2022). The asymmetry of translation direction reported by studies that compared directions (Ceuppens et al., 2018; De Bock et al., 2015; Van den Eynde et al., 2019) is also consistent with Duval's principle of semantic incongruence: conversion is harder when source and target elements do not correspond one to one.

3. Research Implications

Theoretically, this review extends existing theory in two ways. First, language acts as a connecting register that enables structural connections, not merely as a means of communication (Hankeln & Prediger, 2025; Post & Prediger, 2024). Second, digital tools can take over part of the conversion process, so translation theory needs to account for the division of labor between students and tools (Bach et al., 2024; Gregersen, 2024). Conversely, the Translation–Verification model (Adu-Gyamfi et al., 2012) has rarely been tested in full; only one study mapped all four of its stages.

For teachers, there are three practical implications. First, deliberately vary translation directions, including those that start from symbols and go to graphs, words, or concrete objects, because these directions are the hardest and the least practiced. Second, ask students to explain in words how elements in one representation correspond to elements in another; meaningful phrases such as "two groups of four" were associated with understanding (Hankeln & Prediger, 2025). Third, use dynamic technology in a targeted way: pause animations or ask for predictions before the tool displays results so that translation stays with the student.

For curriculum and textbook developers, including in implementing Indonesia's Kurikulum Merdeka, tasks should include variation in conversion direction and a step for checking equivalence of meaning. The Indonesian studies in this review show that transitions between representations do not occur automatically and need scaffolding (Nasrullah et al., 2026; Rahmawati et al., 2025; Siregar et al., 2026).

Five research agendas emerge from this review (Table 8). First, neglected translation directions: concrete/manipulative representations and verbal targets appeared in only 3 and 7 of 106 directions. Second, causal evidence: of 32 studies on instructional support, only one was a randomized trial and two were quasi-experiments. Third, process: research should model the translation stages, not only the final product. Fourth, measurement: no instrument yet measures translation quality and reasoning together, so their relationship has not been tested quantitatively. Fifth, generative AI: only one study has examined AI as a translation partner, finding that its responses were often epistemically irrational (Urhan et al., 2026).

Table 8. Research agenda based on the gaps found

Agenda	Gap found	Suggested research direction
Neglected translation directions	Concrete/manipulative representations only 3 and verbal targets 7 of 106 directions	Studies of translation to and from concrete and verbal representations, especially in primary school and with contextual problems
Causal evidence for supports	1 randomized trial and 2 quasi-experiments among 32 instructional support studies	Experiments with control groups comparing types of support: dynamic technology, meaningful language, and scaffolding
Stage-by-stage process	Only 1 study mapped all four translation stages	Think-aloud or eye-tracking studies that model the stages from deconstructing the source to checking equivalence
Measuring translation and reasoning	No instrument measures both; evidence of the link is almost entirely qualitative	Develop and validate an instrument for translation quality (surface–structural) and reasoning, then test their relationship quantitatively
Generative AI	Only 1 study; AI responses often epistemically irrational	Studies of AI as a translation partner and designs of use that keep translation with the student

4. Research Limitations and Directions for Future Research

This study has several limitations. The search was conducted only in Scopus and restricted to English-language articles, so studies in other languages, including Indonesian national journals, may not have been captured. The subject-area restriction and the 2015–2026 range also excluded potentially relevant studies: an initial search covering 2010–2026 found 28 records published in 2010–2014, 16 of which both reviewers judged "include" or "unsure" at the title–abstract stage. Three apparently relevant studies were filtered out by the subject-area restriction and were not included. The full text could not be obtained for 73 of 160 reports, so the results may be biased toward easily accessible studies. A sensitivity analysis excluding the three lowest-quality studies by MMAT (S35, S39, S51) was not conducted.

Definitions of translation and reasoning vary across studies, and many studies do not report the direction, stage, or quality of translation explicitly, so coding involved interpretation. Most of the evidence comes from small-sample qualitative studies and from two productive research groups, so generalization should be cautious. Subjectivity was reduced through two reviewers, written decision rules, and an audit trail in the worksheet. Future studies are encouraged to broaden the databases (for example Web of Science and accredited national journals), include literature in Indonesian, carry out the agenda in Table 8, and conduct a sensitivity analysis based on MMAT scores.

CONCLUSION

This review of 53 studies shows that research on mathematical representation translation concentrates on symbolic-target translation, especially between graphs and algebra (RQ1). The translation process is rarely modeled stage by stage, and failure most often occurs when deconstructing the meaning of the source representation (RQ2). Interpretation errors dominate, and translation direction determines the level of difficulty (RQ3). Dynamic technology, scaffolded task design, and meaningful language are the most studied supports, but causal evidence remains limited (RQ4). Structural translation, rather than surface translation, is associated with explaining, generalizing, and justifying (RQ5).

The main contribution of this review is a map of translation directions and a representation–translation–reasoning model, which affirms that students need to be helped to connect, not merely translate, representations. This contribution is conceptual and synthesis-based; causal claims cannot yet be drawn because most of the evidence is qualitative.

For teaching practice, teachers are advised to vary translation directions and ask students to explain equivalence of meaning across representations. For future research, it is recommended to study neglected translation directions, conduct experiments that test instructional supports, model the translation stages, and develop instruments that measure translation quality and reasoning together.

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APPENDIX

Table A1. Characteristics of all reviewed studies with MMAT scores (codes S01–S53 are used throughout the manuscript)

Code	Author (year)	Country	Level	Topic	Design	MMAT
S01	Saiman et al. (2026)	Indonesia	Upper secondary	Algebra (reasoning tasks)	Qualitative	5/5
S02	Gregersen (2024)	Denmark	Lower secondary	Algebra/geometry (variables), GeoGebra	Qualitative	5/5
S03	Nuñez-Gutierrez et al. (2025)	Colombia	Lower secondary	Linear functions (variational thinking)	Qualitative	3/5
S04	Uribe & Prediger (2021)	Germany (Colombian and Turkish contexts)	Mixed	Fractions/basic concepts (multilingual)	Qualitative	5/5
S05	Rodríguez-Nieto et al. (2021)	Mexico	University	Derivatives	Qualitative	4/5
S06	Xonkulov et al. (2026)	Uzbekistan	Upper secondary	Trigonometry	Mixed methods	3/5
S07	Muhandaz et al. (2026)	Indonesia	Lower secondary	Geometry (digital learning)	Mixed methods	3/5
S08	Zentgraf & Prediger (2024)	Germany	Lower secondary	Functions (bottle-filling graph)	Qualitative	5/5
S09	Bakar et al. (2020)	Malaysia	Mixed	Addition (preschool)	Qualitative	3/5
S10	Korntreff et al. (2026)	Germany	Pre-service teachers	Conditional probability (part–whole structure)	Randomized trial	3/5
S11	Rodríguez-Nieto et al. (2023a)	Colombia & Mexico	University	Derivatives (projectile motion)	Qualitative	5/5
S12	Tytler et al. (2023)	Australia	Primary	Ecology/data (science–mathematics)	Qualitative	5/5
S13	Waisman et al. (2023)	Israel	Upper secondary	Functions	Quantitative descriptive	3/5
S14	Bach et al. (2024)	Denmark/Sweden	Lower secondary	Geometry and functions in DGE	Qualitative	4/5
S15	Post & Prediger (2024)	Germany	Upper secondary	Conditional probability information	Qualitative	5/5
S16	Morais et al. (2018)	Portugal	Primary	Rational numbers	Qualitative	5/5
S17	Tondorf & Prediger (2022)	Germany	Primary	Equivalence of expressions (early algebra)	Qualitative	5/5
S18	Li et al. (2022)	China	Lower secondary	Linear functions	Mixed methods	3/5
S19	Parr & Lippe (2024)	USA	University	Calculus (distance on graphs)	Qualitative	5/5
S20	De Bock et al. (2015)	Belgium	University	Proportional, inverse, and affine functions	Quantitative descriptive	3/5
S21	González-Polo & Castaneda (2024)	Mexico	Upper secondary	Functions (free fall)	Qualitative	4/5
S22	Hurst & Linsell (2020)	New Zealand	Primary	Multiplication and division	Qualitative	4/5
S23	Kim (2025)	South Korea	Primary	Multiplication (distributive property)	Qualitative	5/5
S24	Faria et al. (2021)	Portugal	Primary	Sequences and patterns	Qualitative	4/5

Code	Author (year)	Country	Level	Topic	Design	MMAT
S25	Kynigos & Karavakou (2023)	Greece	Lower secondary	3D turtle geometry (MaLT2)	Qualitative	5/5
S26	Peters & Prediger (2026)	Germany	Lower secondary	Mass unit conversion	Qualitative	5/5
S27	Nurrahmawati et al. (2021)	Indonesia	Lower secondary	Systems of linear equations in two variables	Qualitative	4/5
S28	Yao (2022)	USA	Pre-service teachers	Generalization of figural patterns	Qualitative	5/5
S29	Nasrullah et al. (2026)	Indonesia	Upper secondary	Systems of linear inequalities in two variables	Qualitative	5/5
S30	Urhan et al. (2026)	Turkey	University	Calculus (concavity and tangent lines)	Qualitative	5/5
S31	Swidan & Fried (2021)	Israel	Upper secondary	Fundamental Theorem of Calculus	Qualitative	5/5
S32	Fonger (2019)	USA	Upper secondary	Linear equations (CAS)	Qualitative	5/5
S33	Malambo (2022)	Zambia	Pre-service teachers	Functions	Qualitative	4/5
S34	Abraham & Prediger (2025)	Germany	Primary	Multiplicative structures (dot arrays)	Qualitative	5/5
S35	Kara & İncikabı (2018)	Turkey	Lower secondary	Addition and subtraction of fractions	Quantitative descriptive	2/5
S36	Van den Eynde et al. (2019)	Belgium/Ireland	University	Functions (graph–equation)	Quantitative descriptive	3/5
S37	Rodríguez-Nieto et al. (2023b)	Mexico	University	Derivatives (graphs of f and f')	Qualitative	5/5
S38	Siregar et al. (2026)	Indonesia	Upper secondary	Quadratic functions	Qualitative	5/5
S39	Barana (2021)	Italy	Lower secondary	Algebraic formulas in geometry	Mixed methods	2/5
S40	Cuevas-Vallejo et al. (2023)	Mexico	University	Vectors	Qualitative	4/5
S41	Wahab et al. (2026)	Indonesia	University	Cyclic covariation	Mixed methods	3/5
S42	Yerushalmy et al. (2023)	Israel	Upper secondary	Functions (exemplary tasks)	Qualitative	5/5
S43	Rahmawati et al. (2025)	Indonesia	Upper secondary	Quadratic functions	Qualitative	3/5
S44	Ceuppens et al. (2018)	Belgium	Lower secondary	Linear functions (mathematics and physics)	Quantitative descriptive	4/5
S45	Ariawan et al. (2026)	Indonesia	University	Plane geometry proofs	Quantitative non-randomized	3/5
S46	Mosese & Ogbonnaya (2021)	South Africa	Upper secondary	Trigonometric function graphs	Quantitative non-randomized	3/5
S47	Ovodenko & Kouropatov (2025)	Israel	University	Derivatives (optimization)	Qualitative	4/5
S48	Rodríguez-Nieto et al. (2025)	Colombia	University	Derivatives (graphs of f and f')	Qualitative	5/5
S49	Swastika et al. (2023)	Indonesia	University	Calculus (graphs)	Qualitative	4/5

Code	Author (year)	Country	Level	Topic	Design	MMAT
S50	Xu et al. (2022)	China	Primary	Fraction notation	Quantitative descriptive	4/5
S51	Popović et al. (2022)	Serbia	Lower secondary	Proportions (problems of varying levels)	Quantitative descriptive	2/5
S52	Hankeln & Prediger (2025)	Germany	Primary	Multiplication	Quantitative descriptive	3/5
S53	Testa & Catena (2022)	Italy	Upper secondary	Linear functions (mathematics and kinematics)	Quantitative descriptive	3/5