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Misconceptions to Mathematical Understanding: Synthesizing the Roles of Cognitive Conflict, Reflection, and Conceptual Reconstruction

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Abstract

Mathematical misconceptions can hinder the development of deep and lasting conceptual understanding. This study aimed to analyze and synthesize the roles of cognitive conflict, reflection, and conceptual reconstruction in transforming misconceptions into mathematical understanding. A systematic literature review employing a descriptive qualitative approach examined relevant scholarly articles published between 2020 and 2026. The analysis focused on the characteristics of misconceptions, mechanisms of cognitive conflict, functions of reflection, and processes of conceptual reconstruction. The synthesis indicates that cognitive conflict promotes learners' awareness of discrepancies between their existing conceptions and mathematical evidence; reflection supports the evaluation and refinement of their thinking processes; and conceptual reconstruction enables the development of more accurate, coherent, and meaningful knowledge structures. Together, these components provide an integrated account of how misconceptions can be transformed into mathematical understanding. The study contributes an integrative synthesis framework that connects these mechanisms through the theoretical perspectives of conceptual change, constructivism, and metacognition. This framework highlights the complementary roles of the three components and provides a theoretical and pedagogical basis for developing mathematics instruction that addresses misconceptions and strengthens conceptual understanding.

Keywords: Mathematical Misconceptions; Cognitive Conflict; Reflection; Conceptual Reconstruction; Mathematical Understanding

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INTRODUCTION

The fundamental aim of mathematics education is not merely to develop learners' ability to perform calculations and solve mathematical problems, but also to foster deep, logical, and sustainable conceptual understanding. Mathematical understanding enables learners to connect different concepts, explain the reasoning underlying solution procedures, and apply their knowledge in varied situations. However, achieving these goals remains challenging because of the occurrence of misconceptions, that is, understandings or interpretations of concepts that are inconsistent with established mathematical principles. Misconceptions may develop when learners rely on prior experiences, intuition, memorized procedures, or faulty generalizations to understand a topic. The problem becomes more complex because inaccurate prior knowledge can affect the processing of new information and create obstacles to learning subsequent mathematical concepts. Thus, misconceptions are not merely errors in obtaining an answer; rather, they are fundamental problems associated with learners' knowledge structures and the process through which mathematical understanding is constructed.

The relevance of this issue can be seen in Indonesian students' mathematics literacy performance in the Programme for International Student Assessment (PISA). According to the OECD report on PISA 2022, Indonesia obtained an average mathematics score of 366 points, substantially below the OECD average of 472 points (OECD, 2023). These data indicate that Indonesian students continue to face challenges in mathematical proficiency, particularly in using mathematical knowledge to understand, reason about, and solve contextual problems.

Although PISA performance does not directly measure the prevalence of misconceptions, the results may indicate broader challenges in mathematical competence, including the ability to develop in-depth conceptual understanding. This situation suggests that mathematics instruction should not focus solely on content delivery, problem-solving practice, and final outcomes; it should also address how learners construct, test, evaluate, and revise their understanding. Accordingly, efforts to address misconceptions should be directed toward conceptual change, enabling learners to abandon inaccurate understandings and develop more accurate knowledge structures.

More specifically, mathematical misconceptions can occur across various topics, including fractions, negative numbers, algebraic operations, equations, functions, geometry,

and other abstract concepts. For example, learners may assume that multiplication always produces a larger number, struggle to understand a variable as a representation of an unknown number, or apply operational rules mechanically without understanding their mathematical meaning. These problems may arise from several factors, including insufficient prerequisite knowledge, instruction that places excessive emphasis on procedures, inappropriate use of representations, and limited opportunities to articulate and evaluate mathematical reasoning. Another challenge is that misconceptions are often difficult to identify through conventional achievement tests because learners may obtain correct answers by memorizing procedures without understanding the underlying concepts. Conversely, an error in an answer does not necessarily indicate a misconception, as it may result from carelessness or a procedural mistake. Therefore, instructional approaches are needed that not only identify errors but also reveal learners' patterns of thinking and systematically facilitate changes in understanding.

One relevant approach to addressing this issue is cognitive conflict, which occurs when learners encounter a discrepancy between their existing understanding and information, evidence, or mathematical situations that contradict their beliefs. Such conflict may encourage learners to question the validity of their prior knowledge and seek more appropriate explanations. In mathematics instruction, cognitive conflict can be elicited through contradictory problems, counterexamples, comparisons of different solution strategies, or explorations of mathematical representations. However, the occurrence of cognitive conflict does not automatically lead to conceptual change. Learners need opportunities for reflection to reconsider the reasoning, strategies, and assumptions they have used and to compare their initial understanding with the mathematical evidence they encounter. Conceptual reconstruction is then required so that learners do not merely recognize their errors but also rebuild their understanding based on accurate relationships among concepts, principles, and representations. These three processes are complementary: cognitive conflict creates the need for change, reflection helps learners evaluate and organize their understanding, and conceptual reconstruction supports the development of more meaningful mathematical knowledge.

Previous studies have provided empirical and theoretical foundations for the importance of conceptual change in learning. Through a systematic review of eight articles on the application of cognitive conflict in mathematics instruction, Ganesha et al. (2026) found that cognitive-conflict strategies have the potential to reduce misconceptions and

improve mathematical problem-solving skills. Their study also indicated that the success of cognitive conflict depends on teachers' ability to facilitate interaction and guide conflict resolution in ways that address learners' needs. Lestari et al. (2026) conducted a systematic literature review of 31 articles published between 2016 and 2026 and identified fractions, algebraic variables, and functions as topics in which misconceptions are frequently found. Their study also revealed that interventions involving cognitive conflict, inquiry-based learning, digital technology, and bilingual approaches may help reduce misconceptions. Meanwhile, in a meta-analysis of 218 primary studies involving 18,051 learners in science education, Pacaci et al. (2024) found that conceptual-change strategies had a large positive effect on learning. This finding strengthens the theoretical basis for the potential of conceptual-change strategies, although the review did not specifically focus on mathematics education.

Although various studies have examined misconceptions, cognitive conflict, and conceptual change, several limitations remain and create opportunities for further research. First, some studies tend to investigate cognitive conflict as a stand-alone instructional strategy, leaving the systematic relationship among cognitive conflict, reflection, and conceptual reconstruction insufficiently explained as a unified process of change in mathematical understanding. Second, research on misconceptions often emphasizes the identification of error types, causal factors, or the effectiveness of particular interventions rather than providing an in-depth explanation of the mechanisms through which learners' knowledge structures change. Third, existing literature reviews could be further developed by integrating findings according to mathematical content, educational level, types of misconceptions, and pedagogical mechanisms that support conceptual change. For example, Ganesha et al. (2026) specifically focused on cognitive conflict and covered eight articles, whereas Lestari et al. (2026) placed greater emphasis on the prevalence of misconceptions and the effectiveness of various interventions across three groups of mathematical topics. The limited integration of these three components therefore indicates the need for a more comprehensive review to explain how learners move from conceptual inconsistency toward accurate, coherent, and applicable mathematical knowledge.

In response to this research gap, the study entitled "Misconceptions to Mathematical Understanding: Synthesizing the Roles of Cognitive Conflict, Reflection, and Conceptual Reconstruction" proposes a novel conceptual synthesis that integrates these three components as mechanisms of change in mathematical understanding. The novelty lies in

explaining the relationships among the emergence of cognitive conflict, learners' reflective engagement, and the process of conceptual reconstruction in developing more comprehensive mathematical understanding. Unlike previous studies that tend to examine instructional strategies or interventions separately, this study analyzes how the three components are interconnected, the conditions that support their effectiveness, and their implications for reducing misconceptions and strengthening conceptual understanding. Through a literature review, this study seeks to develop a synthesis framework that explains the stages of conceptual change, identifies supporting and inhibiting factors, and formulates pedagogical implications for mathematics instruction. The urgency of this study lies in the need to shift the focus of instruction from merely correcting incorrect answers to understanding and improving learners' ways of thinking. Accordingly, this review is important for enriching the theoretical foundations of mathematics education and providing direction for the development of more reflective, constructive, and sustainable approaches to mathematical understanding.

METHODS

This study employed a Systematic Literature Review (SLR) with a descriptive qualitative approach to examine and synthesize previous research on the roles of cognitive conflict, reflection, and conceptual reconstruction in addressing misconceptions and developing mathematical understanding. The literature was collected by searching scholarly articles in the Google Scholar database using the keywords mathematical misconceptions, cognitive conflict, reflective thinking, conceptual reconstruction, and mathematical conceptual understanding. Priority was given to publications from 2020 to 2026, taking into account the relevance and quality of the scholarly sources. Article selection involved several stages: literature searching, screening based on the relevance of titles and abstracts, examination of the full article content, and selection of literature aligned with the focus of the study. The selected articles were then analyzed to identify information on the characteristics of misconceptions, the implementation of cognitive conflict, the role of reflection, and the process of conceptual reconstruction. The data were subsequently grouped according to thematic similarities, compared, and synthesized to identify relational patterns, research trends, and research gaps. The synthesis was used to formulate a conceptual framework explaining the relationships among cognitive conflict, reflection, and

conceptual reconstruction as processes through which misconceptions are transformed into deeper and more meaningful mathematical understanding.

RESULTS

The findings are based on a synthesis of ten scholarly articles relevant to the topic "Misconceptions to Mathematical Understanding: Synthesizing the Roles of Cognitive Conflict, Reflection, and Conceptual Reconstruction," published between 2020 and 2026. The synthesis focuses on four main aspects: (1) the characteristics and causes of mathematical misconceptions; (2) the role of cognitive conflict in promoting changes in understanding; (3) the contribution of reflection to the evaluation and improvement of mathematical thinking; and (4) conceptual reconstruction as a process of developing more meaningful mathematical understanding. Article selection takes into account topical relevance, methodological clarity, and the contribution of findings to the focus of the study.

Table 1. Synthesis of Previous Research

No.	Author(s) and year	Research focus	Key findings and relevance
1	Van Hoof et al. (2021)	Learners' misconceptions about fraction magnitude addressed through a refutation-text intervention.	Misconceptions about fraction magnitude tend to persist even after learners receive correct explanations. The intervention had long-term learning effects, although outcomes varied across individuals. This highlights the need for approaches that take learners' prior understanding into account.
2	Setiawan et al. (2023)	Cognitive conflict arising from thinking errors in the construction of mathematical concepts.	The study found that cognitive conflict emerges when learners encounter thinking errors while constructing mathematical concepts. This suggests that cognitive conflict can be used to reveal thinking processes and inconsistencies in conceptual understanding.
3	Lestari et al. (2026)	Misconceptions involving fractions, algebraic variables, and functions.	The review of 31 articles showed that misconceptions are frequently found in fractions, algebraic variables, and functions. Interventions involving cognitive conflict, inquiry-based learning, and digital technology have the potential to reduce misconceptions.
4	Ganesha et al. (2026)	The application of cognitive conflict in mathematics instruction.	The review of eight articles showed that cognitive conflict has the potential to reduce misconceptions and improve problem-solving skills. Its effectiveness depends on teacher facilitation and conflict resolution that addresses learners' needs.

5	Aulia et al. (2026)	Representation-based remedial instruction to address algebraic misconceptions.	The study examined changes in types of misconceptions and improvements in conceptual understanding through remedial instruction based on mathematical representations. The findings are relevant to conceptual reconstruction through connections among different mathematical representations.
6	Rahayu et al. (2026)	Identification of cognitive conflict in solving problems involving the Pythagorean theorem.	A study involving six ninth-grade students identified three types of cognitive conflict. Errors in identifying the hypotenuse and modeling geometric problems triggered conflict. The findings highlight the importance of diagnosing learners' initial knowledge structures.
7	Toikka et al. (2026)	Activation of metacognitive skills through problem solving and reflective prompts.	A study involving 225 learners showed that planning, monitoring, and evaluation skills do not always emerge spontaneously. Structured reflective prompts are needed to activate metacognitive processes in mathematics learning.
8	Kholid et al. (2026)	Problem-solving patterns in arithmetic sequences from the perspective of reflective thinking.	The study identified three solution patterns: the use of variables, visual representations, and direct solutions. Learners who used variables and visual representations demonstrated more complete elements of reflection than those who used direct solutions.
9	Nindiasari et al. (2026)	Development of a local-wisdom-based numeracy website using a metacognitive approach and targeting mathematical reflective-thinking ability.	The study connected digital numeracy media, a metacognitive approach, and the development of reflective ability. The findings suggest that learning environments can be designed to support evaluation and the development of mathematical understanding.
10	Hasanah et al. (2024)	Learners' reflective-thinking skills in mathematics learning: a systematic review.	The systematic review of studies published between 2017 and 2023 mapped publication trends, educational levels, methods, conceptual frameworks, and indicators and levels of reflective thinking. It is relevant for positioning reflection as a construct with diverse indicators and measurement approaches.

Active Misconceptions as a Barrier to Mathematical Understanding

Based on the studies by Van Hoof et al. (2021), Setiawan et al. (2023), and Lestari et al. (2026), misconceptions are conceptual problems that cannot always be resolved by providing explanations or correcting answers. Learners may retain inaccurate initial understandings even after receiving correct information. In the topic of fractions, for example, learners may continue to assume that the larger the denominator, the greater the

value of the fraction. In algebra and functions, misconceptions are associated with the interpretation of symbols, variables, and relationships among concepts. These findings indicate that misconceptions should be understood as part of learners' knowledge structures; addressing them therefore requires conceptual change rather than simply repeating material or providing additional exercises.

Cognitive Conflict as a Trigger for Conceptual Change

The studies by Setiawan et al. (2023), Ganesha et al. (2026), Rahayu et al. (2026), and Wijayanti & Awantagusnik (2026) show that cognitive conflict plays a role in helping learners recognize discrepancies between their prior knowledge and the mathematical evidence they encounter. Conflict may arise when learners obtain a solution that contradicts their beliefs, make errors in identifying geometric elements, or encounter problems that cannot be solved using procedures they have previously considered valid. Ganesha et al. (2026) showed that cognitive conflict has the potential to reduce misconceptions and improve problem-solving skills. Meanwhile, Wijayanti & Awantagusnik (2026) found changes in learners' responses related to conceptual understanding after cognitive-conflict-based interviews. However, these changes were not consistent across all indicators of understanding. Thus, cognitive conflict can be regarded as a trigger for conceptual change, but its effectiveness requires appropriate guidance and conflict-resolution support.

Reflection as a Mechanism for Evaluating and Strengthening Understanding

The studies by Toikka et al. (2026), Kholid et al. (2026), Hasanah et al. (2024), Kuncoro et al. (2025), and Nindiasari et al. (2026) indicate that reflection plays an important role in helping learners evaluate their thinking processes, monitor solution strategies, and connect prior knowledge with the concepts being learned. Toikka et al. (2026) found that learners' metacognitive skills do not always emerge spontaneously when solving mathematical problems. Structured reflective prompts are needed to encourage planning, monitoring, and evaluation. Furthermore, Kholid et al. (2026) showed that learners who used visual representations and variables tended to demonstrate more complete reflective processes than those who used direct solution methods. Through a systematic review of studies on reflective thinking in mathematics education published between 2017 and 2023, Hasanah et al. (2024) identified various frameworks, indicators, and levels of reflective-thinking ability used in research. Kuncoro et al. (2025) also discussed reflective thinking in mathematics learning as a basis for understanding the thinking processes underlying problem

solving. Nindiasari et al. (2026) developed a local-wisdom-based numeracy website using a metacognitive approach to support mathematical reflective-thinking ability. These findings emphasize that reflection is not merely the act of reviewing an answer but an important process for evaluating and improving mathematical understanding.

Conceptual Reconstruction as a Process of Developing Mathematical Understanding

The studies by Aulia et al. (2026), Wijayanti & Awantagusnik (2026), and Lestari et al. (2026) indicate that changes in mathematical understanding require learners to rebuild relationships among concepts through appropriate learning experiences. Aulia et al. (2026) emphasized the use of various mathematical representations in remedial instruction to address algebraic misconceptions. Wijayanti & Awantagusnik (2026), meanwhile, showed that cognitive conflict can lead to changes in learners' responses concerning representation, understanding of necessary and sufficient conditions, and concept application. The findings of Lestari et al. (2026) also indicate the potential of digital technology, inquiry-based learning, and cognitive conflict in addressing misconceptions. Overall, these findings suggest that conceptual reconstruction requires the reorganization of knowledge, the use of multiple representations, and the strengthening of relationships among concepts so that the resulting understanding is not merely procedural but also conceptual and applicable to different problems.

The ten studies reviewed indicate that cognitive conflict, reflection, and conceptual reconstruction make complementary contributions to the process of changing mathematical understanding. Cognitive conflict acts as a trigger that makes learners aware of inconsistencies in their initial understanding; reflection enables them to evaluate and reconsider their thinking; and conceptual reconstruction allows them to develop a more accurate knowledge structure. Nevertheless, these three components have not yet been fully synthesized as a unified mechanism of conceptual change in mathematics education. Some studies focus more on reducing misconceptions through cognitive conflict, whereas others emphasize the development of reflective abilities or the use of representations to strengthen conceptual understanding. Accordingly, this study proposes an integrative synthesis that places the three components within a single framework for transforming misconceptions into mathematical understanding.

DISCUSSION

The literature synthesis indicates that the transition from misconceptions to mathematical understanding cannot be regarded as a simple process of replacing an incorrect answer with a correct one. Rather, it is a process of knowledge restructuring involving cognitive, constructive, reflective, and metacognitive dimensions. This finding can be explained through the integration of three major theoretical perspectives: conceptual change theory, constructivism, and metacognition. From the perspective of conceptual change, Vosniadou (2021) explains that learners develop knowledge on the basis of initial conceptual frameworks shaped by everyday experiences, intuition, and previous learning. These frameworks may lead to interpretations that differ from formal mathematical concepts, making it difficult for learners to accept new information that contradicts their existing beliefs. For example, learners who are accustomed to understanding numbers through natural numbers may experience difficulties when learning negative fractions or rational numbers because these new concepts require adjustments to their prior knowledge structures. This view is reinforced by diSessa, (2022) through the Knowledge in Pieces perspective, which regards learners' knowledge as a collection of intuitive reasoning elements that may be activated differently depending on the problem context. From this perspective, misconceptions are not necessarily single, unified conceptual errors; rather, they may arise from partially developed pieces of knowledge that have not yet been connected consistently. Mathematics instruction should therefore provide experiences that enable learners to test, connect, and reorganize the knowledge elements they possess. Furthermore, Chi & Roscoe, (2002), in their discussion of the processes and challenges of conceptual change, emphasize that addressing misconceptions requires changes in representations and knowledge categories rather than merely adding new information. Thus, the synthesis indicates that cognitive conflict has an important role as a trigger for conceptual change because it can reveal discrepancies between intuitive knowledge and more defensible mathematical evidence.

With regard to cognitive conflict, conceptual change theory explains that learners need to become dissatisfied with their initial explanations before they are willing to consider more appropriate alternative conceptions. This perspective is consistent with the conceptual change framework developed by Posner et al. (1982), which identifies dissatisfaction with an initial conception, the intelligibility of a new conception, the plausibility of the new explanation, and its potential to generate more fruitful understanding as important

conditions for conceptual change. Nevertheless, cognitive conflict should not be treated as a condition that automatically produces understanding. Vosniadou (2021) shows that conceptual change can be complex because learners must adjust various assumptions and relationships within their existing knowledge. Chi & Roscoe, (2002) also explain that different types of misconceptions require different forms of intervention. In mathematics instruction, cognitive conflict can be designed through counterexamples, the presentation of two solutions that lead to different conclusions, predictive questions, visual demonstrations, or problems that cannot be solved using procedures learners have previously considered valid. The discrepancy between their prediction and the calculated result may generate cognitive conflict and encourage them to question their previous generalization. The meta-analysis by Pacaci et al. (2024), involving 218 studies and 18,051 learners in science education, found that conceptual-change strategies had a large positive effect on learning outcomes. Cognitive-conflict strategies also demonstrated a large positive effect. Although these findings come from science education and cannot be directly generalized as an estimate of effectiveness in mathematics education, they provide cross-disciplinary empirical support for the potential of cognitive conflict in addressing misconceptions. Thus, in mathematics education, cognitive conflict should be understood as an initial stage that creates opportunities for learners to question their prior conceptions and recognize the need for more accurate mathematical explanations.

Furthermore, the constructivist perspective provides an epistemological foundation for understanding how learners construct knowledge through active engagement with their experiences and learning environments. Through cognitive constructivism, Piaget, (1952) explains that knowledge develops through the processes of assimilation and accommodation. Assimilation occurs when new information is integrated into an existing knowledge structure, whereas accommodation occurs when that structure must be adjusted because it can no longer explain new experiences or information. In the context of this study, misconceptions may be understood as a condition in which learners continue to assimilate information on the basis of an inaccurate initial understanding. Cognitive conflict then creates cognitive disequilibrium, which encourages accommodation and the reorganization of knowledge. This view can be complemented by Vygotsky's, (1978) social constructivism, which emphasizes that the development of understanding occurs not only through individual activity but also through social interaction, language, discussion, and assistance from more capable individuals. The concept of the Zone of Proximal Development (ZPD) explains the

importance of support or scaffolding in enabling learners to understand concepts that they cannot yet master independently. Therefore, cognitive conflict in mathematics instruction should be accompanied by dialogue, guiding questions, group discussions, and explanations that help learners identify relationships between their initial conceptions and formal concepts. The constructivist perspective is further reinforced by von Glasersfeld, (1995), whose radical constructivism views knowledge not as a passive copy of reality but as an individual construction based on experience and conceptual viability. In mathematics instruction, this view suggests that teachers should not merely present formulas or solution procedures; they should provide opportunities for learners to construct meaning through exploration, representation, argumentation, and problem solving. The study by Rochmah & Alifiani, (2026) on deep learning pedagogy from a constructivist perspective further supports the relevance of active learner engagement in constructing conceptual understanding, while Putra et al. (2026), through the DECADE model, demonstrate the importance of integrating different learning theories in designing coherent mathematics instruction. Accordingly, conceptual reconstruction in this study can be positioned as the outcome of knowledge construction through interactions among individual experience, mathematical evidence, and support from the learning environment.

Within this process of conceptual change, reflection serves as a mechanism linking cognitive conflict and conceptual reconstruction. The metacognitive perspective introduced by Flavell, (1979) explains that metacognition concerns individuals' knowledge and awareness of their own thinking processes, including their ability to plan, monitor, and evaluate cognitive activities. In mathematics learning, these abilities enable learners to recognize the reasons for using a particular procedure, identify errors in reasoning, and assess whether the strategy employed is consistent with mathematical principles. This idea was further developed by Efklides & Schwartz, (2024) through the Metacognitive and Affective Model of Self-Regulated Learning (MASRL), which emphasizes that metacognitive processes are intertwined with affective experiences, motivation, and learners' subjective judgments about task difficulty. The model explains that feelings of doubt, difficulty, uncertainty, and confidence in an answer can provide important information that encourages individuals to monitor and regulate their thinking. In the context of mathematical misconceptions, learners who experience a conflict between a predicted answer and mathematical evidence should be encouraged not merely to accept correction but also to reflect on the reasons underlying their errors. Smortchkova & Shea, (2020) also emphasize the relationship between metacognitive

development and conceptual change in children, particularly as learners become increasingly able to evaluate and reconsider their beliefs. The findings of Toikka et al. (2026) reinforce this view by showing that metacognitive skills in mathematics learning, such as planning, monitoring, and evaluation, do not always emerge spontaneously and therefore require reflective prompts and structured instructional support. Thus, reflection in this study is understood not simply as repeating or checking an answer but as a metacognitive process that enables learners to identify the sources of misconceptions, test the consistency of their thinking, and determine strategies for improving their understanding.

Furthermore, the relationship between metacognition and mathematical understanding can be explained through the theoretical framework of Cai et al. (2026), which identifies five dimensions of mathematical understanding: factual, procedural, conceptual, metacognitive, and affective understanding. This framework offers a broader perspective, suggesting that mathematical understanding should not be assessed solely in terms of the ability to recall facts and apply procedures; it also includes the ability to understand relationships among concepts, regulate thinking processes, and develop positive attitudes and beliefs toward mathematical activity. This framework is relevant to the findings of the present synthesis because the transition from misconceptions to mathematical understanding involves simultaneous development across several dimensions. Learners who successfully address a misconception need not only to know the correct concept but also to understand why their previous conception was inaccurate, how the new concept is constructed, and under what conditions it can be applied. Therefore, conceptual reconstruction should aim to develop coherent knowledge, in which logical relationships are established among definitions, principles, procedures, symbols, representations, and applications. In this regard, visual, symbolic, numerical, and verbal representations can help learners connect different forms of mathematical knowledge. Studies on the use of representations in algebraic remedial instruction and on reflective and metacognitive approaches also support the importance of activities that reorganize knowledge. Thus, conceptual reconstruction is not merely a matter of correcting one answer or replacing one formula; it involves rebuilding a network of conceptual relationships so that learners can explain, connect, and transfer their understanding to different situations.

By integrating these ten scholarly perspectives and theoretical developments, this study proposes that the transition from misconceptions to mathematical understanding occurs through relationships that are both sequential and dynamic. Vosniadou's perspective

explains the existence of initial knowledge frameworks; diSessa explains the nature of knowledge as consisting of intuitive elements; Chi & Roscoe (2002) explain the complexity of conceptual change; Posner et al. (1982) provide a basis for understanding the conditions under which new conceptions are accepted; Piaget (1952) explains the mechanisms of assimilation and accommodation; Vygotsky (1978) emphasizes the role of social interaction and scaffolding; von Glasersfeld (1995) highlights the active construction of knowledge; Flavell, (1979) provides the foundation for understanding metacognition; Efklides and Schwartz explain the relationship between metacognition, affect, and self-regulated learning; and Cai et al. (2026) broaden the concept of mathematical understanding to encompass factual, procedural, conceptual, metacognitive, and affective dimensions. These ten perspectives demonstrate that conceptual change cannot be explained through a single theory or instructional strategy. Cognitive conflict raises awareness of inconsistencies in understanding, constructivism explains how knowledge is constructed and reorganized, and metacognition explains how learners monitor, evaluate, and regulate this process of change. This relationship also indicates that the three main components of the study should not be understood as entirely linear and separate stages. In practice, reflection may generate new cognitive conflict, conceptual reconstruction may reveal previously unrecognized inconsistencies, and learners may re-evaluate their conceptions when confronted with more complex mathematical problems. Accordingly, conceptual change is more appropriately viewed as a recursive process involving continuous interaction among prior knowledge, learning experiences, metacognitive evaluation, and the formation of new knowledge structures.

Based on the overall discussion, the main theoretical contribution of this study lies in developing an integrative perspective that connects conceptual change theory, constructivism, and metacognition to explain the mechanisms through which misconceptions change in mathematics learning. Previous studies have tended to position cognitive conflict as a strategy for creating cognitive disequilibrium, constructivism as a foundation for knowledge construction, and metacognition as the ability to regulate thinking processes. This study integrates the three perspectives into a single synthesis framework by positioning cognitive conflict as the trigger for change, reflection as the mechanism for evaluating and regulating understanding, and conceptual reconstruction as the process of reorganizing knowledge to produce deeper mathematical understanding. Another novel contribution is the emphasis on reciprocal relationships among these components. Thus,

conceptual change is viewed not merely as a transition from incorrect to correct understanding but as a process that may recur according to the complexity of concepts and learners' learning experiences. This framework also broadens the meaning of mathematical understanding by considering conceptual, procedural, representational, metacognitive, and applied dimensions. Theoretically, this synthesis is expected to enrich mathematics education research, particularly by explaining how learners reconstruct knowledge after encountering conceptual inconsistencies. Pedagogically, the framework provides a basis for teachers to design instruction that goes beyond identifying errors by creating productive conflict situations, providing opportunities for reflection, and facilitating conceptual reconstruction through discussion, guiding questions, exploration of representations, and problem solving. Therefore, this study offers a conceptual foundation for developing mathematics instruction that is more strongly oriented toward thinking processes, metacognitive awareness, and the development of sustainable conceptual understanding.

CONCLUSION

The literature synthesis indicates that the transition from misconceptions to mathematical understanding involves three interrelated components: cognitive conflict, reflection, and conceptual reconstruction. Cognitive conflict raises awareness of inconsistencies in prior understanding; reflection helps learners evaluate and improve their thinking processes; and conceptual reconstruction enables the development of a more accurate and meaningful knowledge structure. The integration of these three components strengthens the perspectives of conceptual change theory, constructivism, and metacognition in explaining the process of conceptual change. The main contribution of this study is an integrative synthesis framework that explains the mechanisms through which misconceptions can be transformed into deep, coherent, and applicable mathematical understanding.

Future research should develop and test instructional models that integrate cognitive conflict, reflection, and conceptual reconstruction across different mathematical topics and educational levels. Empirical testing is needed to determine the effectiveness of integrating these three components in reducing misconceptions, improving conceptual understanding, and strengthening learners' metacognitive abilities. In addition, longitudinal studies could

examine the sustainability of changes in mathematical understanding and the potential use of digital technology to support the process of conceptual change.

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