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The Role of Metacognitive Scaffolding in Mathematical Communication on Mathematics Students

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Abstract

Although written mathematical communication is central to students' ability to express reasoning coherently, the role of metacognitive scaffolding in connecting internal thinking with external mathematical discourse remains insufficiently examined. This study investigates whether metacognitive scaffolding functions as a bridge between internal thought and external discourse in students' written mathematical communication and whether initial metacognitive awareness moderates this effect. A quasi-experimental pretest–posttest control group design was employed with 72 mathematics education students. The experimental group received six sessions of metacognitive scaffolding through guided prompts, such as "What is your first step and why?" and "How does this step connect to the next?", whereas the control group received conventional instruction without such prompts. Written mathematical communication was assessed using a validated test measuring structural coherence, including logical flow, mathematical language use, and step-linking. Metacognitive awareness was measured using an adapted Metacognitive Awareness Inventory questionnaire. The ANCOVA results revealed a significant and large effect of metacognitive scaffolding on structural coherence, $\eta p^2 = 0.292$, $p < .001$, with the strongest effects observed in logical flow and step-linking. Metacognitive awareness also significantly moderated the intervention effect, $B = -3.41$, $p = .009$, indicating that students with lower initial metacognitive awareness benefited most from the intervention. Qualitative think-aloud protocols further showed that scaffolding activated internal discourse, promoted the use of logical connectors, and strengthened self-monitoring during written mathematical explanation. These findings contribute to commognitive theory by demonstrating that improving external mathematical communication can support internal cognitive processes. Practically, the study suggests that mathematics educators can integrate metacognitive prompts into worksheets to help students produce more coherent written explanations, particularly those with underdeveloped metacognitive skills.

Keywords: Metacognitive Scaffolding; Written Mathematical Communication; Commognition; Metacognitive Awareness; Structural Coherence.

INTRODUCTION

A strange problem appears in math teaching across all school levels. Many students can solve routine questions correctly, but when asked to explain their thinking in writing, they become unclear or disorganized. Mathematical communication is more than just using symbols. It also means building arguments, explaining each reasoning step, and connecting ideas in a logical way (Morgan et al., 2014). Verschaffel et al. (2020) showed in a thorough review that word problems often cause communication failures because students cannot turn everyday language into proper math representations. Good math communication is not only about fast procedures; it supports deep understanding (Lithner, 2017). At the same time, modern cognitive theory tells us that metacognition, which is the ability to notice and control our own thinking, matters a lot for solving problems well (Sweller et al., 2019). Yet very few studies have directly connected metacognition to the quality of written math communication, especially for university students.

If you watch a classroom and ask students to "explain how you solved $2x + 5 = 13$," many will just write "$x = 4$" or a list of algebra steps without any logical links. Their written work then has no clear flow, no proper math language, and no connections between steps (Morgan et al., 2014). This shows a gap between what they think inside (internal discourse) and what they write down (external discourse). According to Sfard's (2001) commognitive framework, these two should be the same thing.

The main issue we study here is that math students cannot turn their inner thoughts into clear written explanations. Even when they know the procedures, they often fail to write logically because they are not used to watching and guiding their own thinking (Kontorovich, 2023). This is not just a language problem; it also overloads working memory. Sweller et al. (2019) explain that when information is not ordered in a clear hierarchy, working memory gets too full, leaving little room to build external explanations. Common fixes so far include guided writing, using diagrams (Pantziara et al., 2009), or tools like GeoGebra (Kusumah et al., 2020). But these methods stay on the outside and do not train metacognitive awareness in a systematic way. We need a deeper approach: helping students "listen" to their own thoughts and then turn them into structured writing. This is called metacognitive scaffolding, a set of prompts that wake up metacognitive processes (Yang et al., 2016). For math communication, such scaffolding can be guiding questions like "What was your first step? Why did you choose it? How does it connect to the next step?" These questions push students to say out loud their inner reasoning before writing, slowly building coherent structure.

Earlier studies support this idea from different angles. First, Sfard (2001) and later Rais (2017) gave us the commognitive framework, which joins communication and cognition. Sfard says thinking is really talking to yourself, so improving external communication also improves internal thinking. This explains why metacognitive scaffolding works: by forcing students to talk through their thoughts (to themselves or others), we reshape their internal discussion patterns. Second, Sweller et al. (2019) show that teaching without good communication raises extra cognitive load and hurts learning. But metacognitive scaffolding can turn that extra load into helpful load because students focus on organizing knowledge. Third, Rellensmann et al. (2017) found that drawing math sketches, a form of visual communication, is linked to better modeling performance because drawing forces students to bring mental pictures out. Even though their study did not look at metacognitive scaffolding directly, it still backs the idea that making internal thoughts external through drawings or writing improves reasoning. Fourth, Kamid et al. (2020) studied reflective versus impulsive cognitive styles and found that reflective students write more structured math explanations, meaning metacognitive awareness (higher in reflective students) plays a big role. Fifth, Yang et al. (2016) did an experiment on computer supported peer tutoring and showed that structured social interaction boosts math communication, though they did not focus on individual metacognition. Sixth, a recent EEG found that brain activity during math communication tests is very different from activity during routine calculations, suggesting that math communication uses higher order thinking that needs many brain areas to work together.

Even with all these useful contributions, no study has yet combined metacognitive scaffolding with the commognitive framework to improve the coherence of written math communication. The biggest research gaps are: (1) most metacognition studies in math focus on getting the right answer or solving faster, not on the quality of writing; (2) research on math communication is often descriptive or correlational, not experimental with a manipulated treatment; (3) no one has tested whether metacognitive scaffolding can close the gap that Sfard (2001) calls the "gap between internal and external discourse"; (4) work on cognitive style (Kamid et al., 2020) hints that individual traits might change how metacognition works, but no experiment has tested this with scaffolding as a treatment. So although we have good theories (Sfard, 2001; Sweller et al., 2019) and supportive evidence from many sides (Morgan et al., 2014; Rellensmann et al., 2017; Yang et al., 2016), no one has carefully tested whether metacognitive scaffolding actually causes better written math communication among university math students.

To fill this gap, our study tests experimentally whether metacognitive scaffolding helps students connect their inner thoughts to external written explanations. The new things we offer are three. First, this is the first experiment that changes metacognitive scaffolding on purpose (as an independent variable) and measures its effect on the structural coherence of writing, not just on test scores. Second, we bring together the commognitive framework (Sfard, 2001) and cognitive load theory (Sweller et al., 2019) to explain how metacognitive scaffolding lowers working memory load while making internal discourse richer. Third, we also look at a moderator: students' initial metacognitive awareness. This lets us answer "who benefits most from this scaffolding?" We predict that metacognitive scaffolding will greatly improve structural coherence (logical flow, use of math language, connections between steps) compared to regular teaching, and that the effect will be stronger for students who start with low metacognitive awareness because they need outside guidance more. Our study focuses on university students in math or math education programs, using algebra and basic calculus problems that need written explanations. We use a pretest and posttest control group design with random assignment.

The results should add to theory by making the commognitive framework more practical, and also give teachers and lecturers a concrete design for metacognitive scaffolding to train writing skills in math.

METHODS

This study was designed to empirically test how metacognitive scaffolding bridges internal thought and external discourse in students' written mathematical communication. Based on the commognitive framework (Sfard, 2001) and cognitive load theory (Sweller, van Merriënboer, & Paas, 2019), this study employs a mixed-methods approach with a quasi-experimental design reinforced by qualitative analysis. This section describes the research design, participants, instruments, procedures, data analysis, and the framework guiding the research implementation.

Research Framework

Before detailing the design, it is important to present the framework that forms the logical basis of this study. This framework is presented in Figure 1, which illustrates the relationship between the independent variable (metacognitive scaffolding), the dependent variable (structural coherence of written mathematical communication), and the moderator variable (initial metacognitive awareness). Figure 1 also shows how metacognitive scaffolding is hypothesized to work through two mechanistic pathways: first, by reducing extraneous cognitive load by providing a thinking structure; second, by activating internal discourse which is then externalized into coherent written discourse.

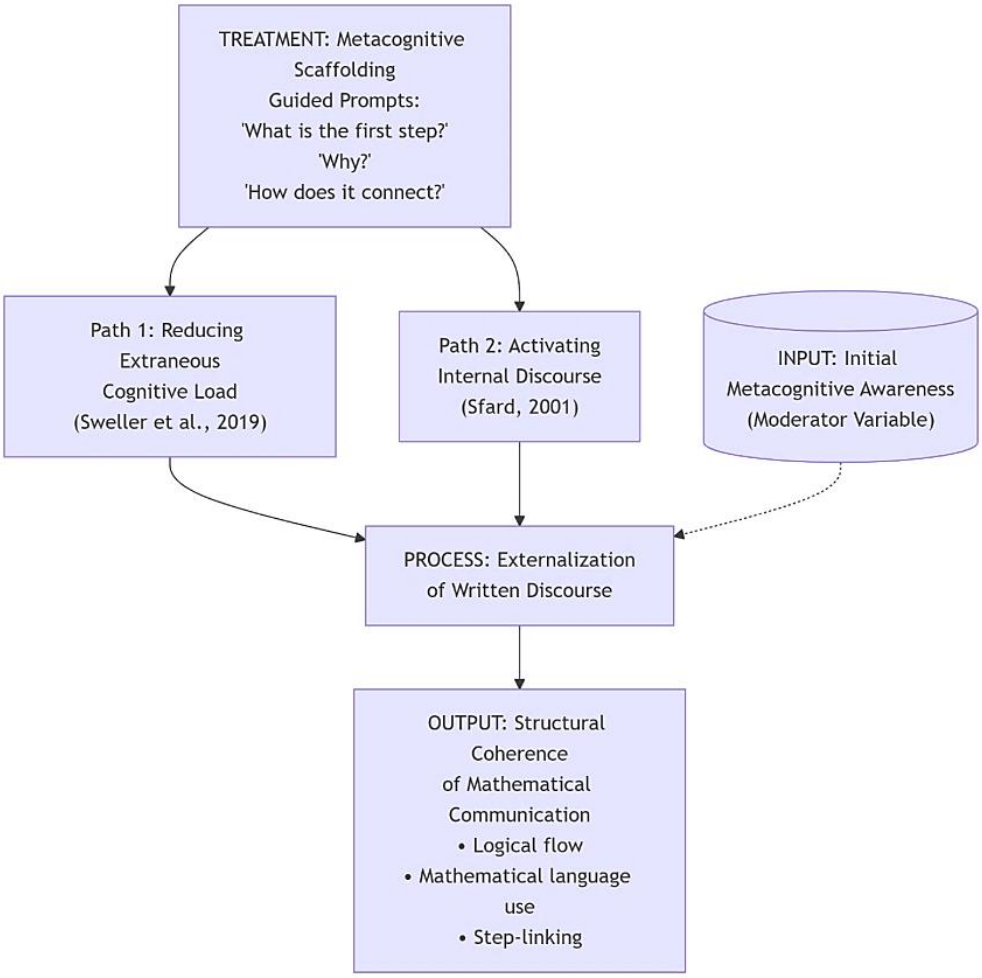

Figure 1. Metacognitive Scaffolding in Bridging Students' Mathematical Thinking

This framework leads to the hypothesis that students receiving metacognitive scaffolding will show a greater increase in structural coherence compared to the control group, and this effect will be stronger for students with low initial metacognitive awareness.

Research Design

This study employed a pretest-posttest non-equivalent control group design, a common quasi-experimental design in mathematics education research when full randomization is not possible (Kontorovich, 2023). The experimental group received treatment in the form of metacognitive scaffolding integrated into problem-solving worksheets, while the control group received conventional instruction without scaffolding. Pretests measured written mathematical communication ability and metacognitive awareness before the treatment. After six sessions of treatment, posttests were administered to assess changes in structural coherence. To understand internal cognitive processes more deeply, this study also included a qualitative sub-study using a think-aloud protocol with a small subset of participants (Rellensmann et al. 2017). This mixed-methods approach allows researchers not only to measure causal effects but also to explain why and how metacognitive scaffolding works.

Participants

The study population comprised undergraduate students in a mathematics education program enrolled in a Basic Calculus course. This population was chosen based on the finding of Verschaffel et al. (2020) that early-semester students often experience difficulty translating mathematical reasoning into structured written form. Cluster sampling was used to select two naturally formed parallel classes. The total number of participants was 72 students, with 36 in the experimental group and 36 in the control group. Participant characteristics: 52.8% female (n=38), 47.2% male (n=34); age range 18–20 years (mean = 18.7 years; SD = 0.65). There was no significant difference in mathematics entrance exam scores between the two groups based on an independent t-test on administrative data ($t(70) = 0.48$; $p = 0.63$). All participants provided written consent after being informed of the study's purpose and procedures. This study received ethical approval from the university's ethics committee.

Research Instruments

Three main instruments were developed and validated for this study. The first instrument was a test of written mathematical communication ability measuring structural coherence based on indicators from Morgan et al. (2014) and Pantziara et al. (2009). The test consisted of four essay questions covering limits, derivatives, and applications of derivatives. Each question was assessed on three dimensions: logical flow (score 0–4), appropriate use of mathematical language and symbols (0–4), and inter-step connections (0–4), resulting in a maximum score of 12 per question and a total maximum of 48. Content validity was assessed by three experts (two mathematics education lecturers and one researcher) using Aiken's V method, yielding values between 0.82 and 0.94. Instrument reliability was tested on 30 students outside the sample, yielding a Cronbach's alpha coefficient of 0.87, indicating good internal consistency.

The second instrument was a metacognitive awareness questionnaire adapted from the Metacognitive Awareness Inventory (MAI) version modified for the context of mathematical problem-solving (Sweller et al., 2019). This questionnaire had 24 items on a 5-point Likert scale (1 = strongly disagree to 5 = strongly agree), measuring two main domains: metacognitive knowledge (8 items) and metacognitive regulation (16 items). In the pilot test, the Cronbach's alpha value was 0.89. The third instrument was a metacognitive scaffolding worksheet containing a series of written guiding questions. These questions were developed based on scaffolding principles from Yang et al. (2016) and adapted to Sfard's (2001) commognitive framework. Example questions included: "What important information did you obtain from this problem?", "What is the first step you will take? Explain why that step was chosen.", "How does the first step relate to the next step?", "Are you confident with each step you wrote? If not, which part is still confusing?" This worksheet was given only to the experimental group. Additionally, for the qualitative sub-study, a voice recorder was used to record think-aloud protocols while participants worked on one additional problem.

Research Procedures

The research procedure was carried out in four stages over eight weeks. The first stage was preparation, which included training the course instructor (one instructor taught both groups) on how to provide metacognitive scaffolding without fundamentally changing the teaching style in the control group. The instructor was trained to use the scaffolding worksheet in the experimental group and to give the same instructions in both groups except for the presence of guiding questions. The second stage was the pretest in the first meeting, where all participants completed the mathematical communication test and the metacognitive awareness questionnaire within 90 minutes. Pretest results were analyzed to ensure initial equivalence between the two groups.

The third stage was the treatment, which lasted for six sessions (meetings 2 to 7). Each session lasted 100 minutes with the following structure: topic introduction (15 minutes), individual problem-solving (45 minutes), small group discussion (20 minutes), and reflection (10 minutes). In the experimental group, during the individual problem-solving session, each student received a worksheet already equipped with metacognitive scaffolding questions. They were instructed to write their answers while also answering those questions on a separate sheet. Thus, they consciously externalized their internal discourse. Meanwhile, the control group worked on the same problems without guiding questions; they were only asked to write the solution directly. At the end of each session, both groups wrote a brief reflection on the difficulties they encountered. During the treatment, the researcher and assistants observed participants using structured observation sheets to record indications of spontaneous metacognitive strategy use.

The fourth stage was the posttest in the 8th meeting. All participants again completed a mathematical communication test equivalent to the pretest but with different problems (topics adjusted to the material covered, difficulty level tested for equivalence through piloting). Additionally, 6 students from each group were randomly selected to participate in a think-aloud session while working on one additional problem. They were asked to continuously voice their thoughts, and the audio recordings were transcribed for qualitative analysis (Rellensmann et al., 2017). All answer sheets and recordings were anonymized before analysis.

Data Analysis

Data were analyzed using both quantitative and qualitative methods. Quantitative analysis tested whether metacognitive scaffolding significantly improved the structural coherence of mathematical communication. Prior to hypothesis testing, Shapiro-Wilk and Levene tests were conducted to examine normality and homogeneity. An independent t-test compared pretest scores between groups, while one-way ANCOVA was used to examine treatment effects, with posttest scores as the dependent variable, pretest scores as the covariate, and group as the independent variable. ANCOVA was selected to control for possible initial group differences (Yang et al., 2016). The moderating effect of initial metacognitive awareness was tested using multiple regression with an interaction term between group and metacognitive awareness. The significance level was set at $\alpha = 0.05$, effect size was reported using partial eta squared (ηp^2), and all analyses were conducted with SPSS version 26.

Qualitative analysis was conducted on students' open-ended responses and think-aloud transcripts using thematic analysis guided by Sfard's (2001) commognitive framework. The coding focused on the clarity of mathematical propositions, use of logical connectives, consistency of notation, and evidence of metacognitive monitoring. To ensure reliability, two independent coders analyzed 20% of the data, resulting in a Cohen's Kappa of 0.84, which indicated strong agreement. Any coding differences were resolved through discussion. The qualitative findings were then used to support and explain the quantitative results, particularly in showing how metacognitive scaffolding connects students' internal thinking with their written mathematical communication.

RESULTS

This study aimed to test whether metacognitive scaffolding can bridge internal thought and external discourse in students' written mathematical communication, and whether this effect is moderated by initial metacognitive awareness. This section presents quantitative and qualitative findings based on data collected from 72 students divided into an experimental group (n=36) and a control group (n=36). Results are presented in the following order: descriptive statistics, initial equivalence testing, main treatment effect, moderation effect, and qualitative findings from think-aloud protocols and written discourse analysis. Before the treatment, both groups were measured on their written mathematical communication ability and metacognitive awareness through a pretest. On the mathematical communication test with a maximum score of 48, the experimental group had a mean pretest score of 28.67 (SD = 5.42), while the control group had a mean of 28.33 (SD = 5.61). After six treatment sessions, a posttest was administered using an equivalent test format but with different problems. These conditions met the assumptions for further parametric analysis.

Table 1. Descriptive Statistics of Written Mathematical Communication Ability

Group	Test	N	Mean	SD	Mean Gain
Experimental Group	Pretest	36	28.67	5.42	—
Experimental Group	Posttest	36	40.22	5.13	11.55
Control Group	Pretest	36	28.33	5.61	—
Control Group	Posttest	36	33.47	5.89	5.14

*Note***.** The maximum possible score was 48. The sample size was inferred from the independent-samples t-test degrees of freedom, $t(70)$, indicating 36 students in each group.

Effect of Metacognitive Scaffolding on Structural Coherence of Mathematical Communication

To test whether metacognitive scaffolding had a significant effect on improving the structural coherence of written mathematical communication, a one-way analysis of covariance (ANCOVA) was conducted with pretest scores as the covariate. The ANCOVA results showed that after controlling for pretest scores, there was a statistically significant difference between the experimental and control groups on posttest scores ($F(1,69) = 28.47$; $p < 0.001$; $\eta p^2 = 0.292$).

Table 2. ANCOVA for the Effect of Metacognitive Scaffolding on Posttest Scores

Varians	JK	df	RK	F	p	ηp^2
Pretest (Covariate)	845,32	1	845,32	38,24	< 0,001	0,357
Group (Intervention)	629,18	1	629,18	28,47	< 0,001	0,292
Error	1525,50	69	22,11			
Total	3000,00	71				

Note: SS = Sum of Squares; df = degrees of freedom; MS = Mean Square; ηp^2 = partial eta squared*

The effect size of 0.292 is considered large according to Cohen's criteria ($\eta p^2 \geq 0.14$), meaning that approximately 29.2% of the variance in posttest scores can be explained by the metacognitive scaffolding treatment after controlling for initial differences. The adjusted mean for the experimental group was 40.08 (SE = 0.96), while the control group was 33.61 (SE = 0.96). Thus, the null hypothesis was rejected: metacognitive scaffolding significantly improved the structural coherence of students' mathematical communication compared to conventional instruction. Further analysis was conducted on the three dimensions of structural coherence measured: logical flow, use of mathematical language, and inter-step connections, as shown in the following table.

Table 3. ANCOVA Results for the Three Dimensions of Structural Coherence

Dimension	Source of Varians	F	df_1	df_2	p	ηp^2	Effect Category
Logical Flow	Groups	22,15	1	69	< 0,001	0,243	Large
	Pretest (Covariate)	31,42	1	69	< 0,001	0,313	Large
Use of Math Language	Groups	15,72	1	69	< 0,001	0,186	Large
	Pretest (Covariate)	24,18	1	69	< 0,001	0,259	Large
Inter-step Connections	Groups	24,83	1	69	< 0,001	0,265	Large
	Pretest (Covariate)	35,67	1	69	< 0,001	0,341	Large

On the logical flow dimension (maximum score 16), the experimental group achieved a mean posttest score of 13.42 (SD = 2.31), while the control group scored 10.89 (SD = 2.65). On the use of mathematical language dimension (maximum 16), the experimental group scored 13.17 (SD = 2.18) and the control group 11.06 (SD = 2.44). On the inter-step connections dimension (maximum 16), the experimental group scored 13.63 (SD = 2.05) and the control group 11.52 (SD = 2.38). Separate ANCOVA tests for each dimension showed that the treatment effect was strongest on the logical flow dimension ($F(1,69) = 22.15$; $p < 0.001$; $\eta p^2 = 0.243$) and inter-step connections ($F(1,69) = 24.83$; $p < 0.001$; $\eta p^2 = 0.265$), while the effect on the use of mathematical language was also significant but slightly lower ($F(1,69) = 15.72$; $p < 0.001$; $\eta p^2 = 0.186$). These findings indicate that metacognitive scaffolding is most effective in helping students structure their reasoning logically and connect each solution step, which is the core of bridging internal thought into structured external discourse.

Moderating Effect of Initial Metacognitive Awareness

This study also tested whether initial metacognitive awareness moderates the effect of scaffolding on improving mathematical communication. Multiple regression analysis was conducted with predictor variables: group (dummy: 0 = control, 1 = experimental), initial metacognitive awareness score (centered), and the interaction between group and metacognitive awareness.

Table 4. Moderated Regression Analysis Results

Predictor	B	SE	β	t	p	95% CI
Constant	33.61	0.70		48.01	< .001	[32.21, 35.01]
Group (Experimental = 1)	12.03	2.14	.512	5.62	< .001	[7.76, 16.30]
Initial Metacognitive Awareness (centered)	4.87	1.52	.298	3.20	.002	[1.84, 7.90]
Group × Metacognitive Awareness	-3.41	1.28	-.248	-2.66	.009	[-5.96, -0.86]

Note. $R^2 = .467$, $F(3, 68) = 19.84$, $p < .001$.
Dependent variable = Posttest mathematical communication score.*

The overall regression model was significant ($F(3,68) = 19.84$; $p < 0.001$; $R^2 = 0.467$). The results showed that the main effect of group remained significant ($B = 12.03$; $SE = 2.14$; $p < 0.001$), the main effect of metacognitive awareness was also significant ($B = 4.87$; $SE = 1.52$; $p = 0.002$), and most importantly, the interaction effect between group and initial metacognitive awareness was significant and negative ($B = -3.41$; $SE = 1.28$; $p = 0.009$). This negative interaction indicates that the benefit of metacognitive scaffolding is greater for students with low initial metacognitive awareness compared to those who already have high metacognitive awareness. In other words, scaffolding acts as a compensatory tool that helps those who need it most.

Table 5. Simple Slope Analysis of Initial Metacognitive Awareness

Level of Metacognitive Awareness	B (Group Effect)	SE	t	p	95% CI
Low (M - 1 SD = 2.62)	15.44	2.56	6.03	< .001	[10.33, 20.55]
Moderate (M = 3.11)	12.03	2.14	5.62	< .001	[7.76, 16.30]
High (M + 1 SD = 3.60)	8.62	2.31	3.73	< .001	[4.01, 13.23]

To visualize the interaction, a simple slope analysis was conducted, which showed that for students with metacognitive awareness one standard deviation below the mean, the scaffolding effect was very strong ($B = 15.44$; $SE = 2.56$; $p < 0.001$), whereas for students with metacognitive awareness one standard deviation above the mean, the effect remained significant but smaller ($B = 8.62$; $SE = 2.31$; $p = 0.003$). This finding is consistent with the framework that metacognitive scaffolding bridges the gap between disorganized internal thought and structured external discourse, especially for those who do not yet have mature metacognitive strategies. To understand the mechanisms underlying the quantitative findings, qualitative analysis was conducted on students' written answers on the posttest and think-aloud transcripts from 6 experimental group students and 6 control group students.

The analysis used Sfard's commognitive framework, which views thinking as communicating with oneself. Three main patterns emerged from the qualitative data. The first pattern was the use of logical connectives that explicitly show relationships between steps. In the experimental group, written answers consistently contained phrases such as "because... then...", "thus", "consequently", "so that it is obtained." For the problem: "Determine the equation of the tangent line to the curve $f(x) = x^2$ at the point $x = 3$," an experimental group student (code E-12) wrote:

"The first derivative of $f(x) = x^2$ is $f'(x)=2x$".

Because the slope of the tangent line is the value of the derivative at that point, the slope at $x = 3$ is $2(3) = 6$. Next, the equation of the tangent line uses the formula $y - y_1 = m(x - x_1)$, so $y – 9 = 6(x – 3)$. In contrast, control group answers tended to write steps sequentially without connectives, for example: "$f(x) = x^2$. $f'(x) = 2x$. $x = 3$. $f'(3) = 6$. $Y – 9 = 6(x – 3)$." The absence of connectives made the written discourse fragmented and difficult to follow as a complete argument.

Excerpt from Think-Aloud, Student E-12:

P: "Please explain, how did you solve the problem about the tangent line to the curve $f(x) = x^2$ at the point $x = 3$?"

E-12: "Well, Sir. First, I find the first derivative of $f(x) = x^2$. The result is $f'(x) = 2x$. Now, because the slope of the tangent line is the same as the value of the derivative at the point of tangency, then the slope at $x = 3$ is $f'(3) = 2$ times 3 equals 6."

P: "Then after that?"

E-12: "After finding the slope, I then find the equation of the tangent line. I use the formula y minus y_1 equals m times (x minus x_1). Because the point of tangency is at $x=3$, then y_1 is f(3) which is 3 squared equals 9. Finally, I write y minus 9 equals 6 times (x minus 3)."

P: "Why did you write the answer in a paragraph form like that, instead of just writing short steps?"

E-12: "Because during the exercises, we were always given worksheets with questions like 'What is the first step and why?', 'How does it relate to the next step?'. So I got used to explaining the reasoning behind each step. I realize that the lecturer doesn't only look at the final answer, but also whether I truly understand the process."

P: "What is the benefit of writing like that, in your opinion?"

E-12: "The benefit is, I understand better why each step is done. If I just write formulas and numbers, later I might forget why I used that formula. But if I write with words like 'because' and 'so', it's like I'm teaching myself, Sir."

The second pattern was the emergence of sentences reflecting monitoring and evaluation of one's own thought process, found only in the experimental group. In a think-aloud, an experimental group student (E-04) said: "I will find the derivative first... oh, wait, did I put the parentheses correctly? Check again... yes, done. Then I substitute. The result is 6. But are the units correct? The problem doesn't ask for units, so just the number is fine." This transcript shows an internal dialogue being externalized verbally, exactly as encouraged by the guiding questions on the worksheet. In contrast, control students tended to work on problems automatically without self-checking. A control student (K-08) said: "Just differentiate directly, then plug in the number, done." When asked to explain why that step was chosen, they paused for a moment and then answered "because that's what is usually done."

The third pattern was the improvement in structural coherence resulting from the repeated scaffolding process. Analysis of the experimental group's answers from the first to the sixth session showed systematic development. In the early sessions, many experimental students still wrote answers like the control group, even though guiding questions were present. However, by the third session and onwards, their answers began to show a more organized structure. For the problem:

$$\text{Calculate the following limit: } \log_{x\to 2}\frac{(x^2-4)}{(x-2)}.$$

An experimental group student (E-21) wrote in the first session:

$$\log_{x\to 2}\frac{(x^2-4)}{(x-2)} = \frac{4-4}{2-2} = 0,$$

use factorization, $\frac{(x-2)(x+2)}{(x-2)}$ = x+2 = 4.

In the sixth session, the same student wrote: "The problem asks for the limit of a rational function. First, direct substitution yields the indeterminate form 0/0. Therefore, I factor the numerator: x^2-4 = (x-2)(x+2). Next, cancel the factor (x-2) with the denominator, leaving x+2. Finally, substitute x = 2 resulting in 4. Thus, the value of the limit is 4."

Excerpt from Dialogue with Student E-21:

P: "Look at your answer for the limit problem earlier. Do you think it is clear?"

E-21: "Yes, Sir. I wrote all the steps. From direct substitution got 0/0, then I factorized, canceled, got 4."

P: "Did you explain why you chose factorization?"

E-21: "Not yet, Sir. I thought it was enough because the answer is correct."

P: "Imagine you are the lecturer reading this answer. Are you sure that the student who wrote this truly understands the concept of limits?"

E-21: (pauses) "Maybe not enough, Sir. It seems I need to explain in more detail."

P: "In the following sessions, try to use the guiding questions on the worksheet. For example: 'What is the first step?', 'Why was that step chosen?', 'How does it relate to the next step?'."

E-21: "Alright, Sir. I'll try."

The change from short, procedural answers to explanatory, coherent answers reflects the success of scaffolding in training the externalization of internal discourse. Conversely, analysis of the control group's answers from the first to the sixth session showed no meaningful structural change. They continued to write steps linearly without explanation. Some control students even wrote numerically correct answers whose logical flow was impossible to follow due to excessively large step jumps. This indicates that without scaffolding, students do not automatically develop the ability to bridge internal thought and external discourse, even though they are exposed to the same material.

DISCUSSION

This study aimed to empirically test whether metacognitive scaffolding serves as a bridge between internal thought and external discourse in students' written mathematical communication. Overall, the findings provide strong support for the proposed hypotheses.

This section discusses the theoretical interpretation of the main findings, namely (1) the main effect of metacognitive scaffolding on the structural coherence of mathematical communication, (2) the moderating effect of initial metacognitive awareness, (3) the qualitative mechanisms underlying these findings, and (4) implications, limitations, and future research directions.

1. Metacognitive Scaffolding as a Bridge Between Internal Thought and External Discourse

The first and most important finding of this study is that metacognitive scaffolding significantly improves the structural coherence of students' written mathematical communication, with a large effect size ($\eta p^2 = 0.292$). This finding directly supports Sfard's (2001) commognitive framework, which states that thinking is essentially communicating with oneself. Within this framework, students' difficulty in writing coherent mathematical explanations is not solely due to a lack of procedural knowledge, but rather to their inability to externalize fragmented internal discourse. The metacognitive scaffolding provided in this study, in the form of guiding questions such as "What is the first step you took and why?" and "How does the first step connect to the next step?", forces students to consciously activate their internal discourse and then transform it into structured written form. This process aligns with the concept of "discourse externalization" explained by Sfard (2001), where every improvement in external communication will improve the structure of internal cognition. In other words, metacognitive scaffolding not only helps students write better but also helps them think more orderly.

This finding also reinforces the argument from cognitive load theory (Sweller, van Merriënboer, & Paas, 2019) that non-communicative instruction can increase extraneous cognitive load, thereby hindering learning. In this study, the control group that did not receive scaffolding tended to write solution steps linearly without explanation, reflecting poorly managed cognitive load. In contrast, the experimental group that received scaffolding showed significant improvement in their ability to organize information hierarchically. This is consistent with the findings of Özbey-Demir and colleagues (2025), who stated that when students are encouraged to reflect, explain, and make their reasoning explicit, they can better manage the cognitive load of complex STEM tasks. Furthermore, this finding is also relevant to the study by Darmawan et al. (2024) on the characteristics of e-scaffolding that support mathematical communication, which emphasizes that interactive and gradually presented scaffolding can encourage students to develop better mathematical communication skills. In the context of this study, although the scaffolding was provided in written form (not electronic), the principles of interactivity and graduality were maintained through questions that appeared progressively at each stage of problem-solving.

Interestingly, the scaffolding effect was strongest on the dimensions of logical flow and inter-step connections, while the effect on the use of mathematical language was slightly lower. This indicates that metacognitive scaffolding is more effective in helping students structure their reasoning (the macro aspect of communication) than in improving the precision of symbol and notation use (the micro aspect). This finding aligns with the results of Rellensmann, Schukajlow, and Leopold (2017), who showed that drawing mathematical sketches (also a form of discourse externalization) has a greater impact on overall modeling ability than on the technical details of symbol use. The implication is that to improve the appropriate use of mathematical language, additional interventions more focused on linguistic aspects, such as writing practice with direct feedback from the instructor, may be needed (Morgan, Craig, Schuette, & Wagner, 2014).

2. Moderating Effect of Initial Metacognitive Awareness

The second significant finding is that initial metacognitive awareness moderates the effect of scaffolding on improving mathematical communication, with a greater effect on students who have low metacognitive awareness. This finding is important because it shows that metacognitive scaffolding acts as a compensatory tool that helps those who need it most. In other words, students who do not yet have mature metacognitive strategies benefit more from external guidance than students who already have high metacognitive awareness. This aligns with the basic principle of scaffolding in Vygotskyan theory, where assistance is given precisely when the learner is within the zone of proximal development (ZPD) (Cerri et al., 2007). According to Morphew (2024), students with low metacognitive awareness are in the ZPD for metacognitive skills, so external scaffolding is very effective in helping them internalize those strategies. Conversely, students with high metacognitive awareness may already have sufficiently good internal strategies, so the addition of external scaffolding provides only relatively small additional benefits.

This finding is also relevant to the results of Kamid et al. (2020) on reflective-impulsive cognitive styles. In that study, students with a reflective cognitive style (who tend to have higher metacognitive awareness) showed more structured mathematical communication than impulsive students. Our study goes further by showing that scaffolding interventions can help bridge the gap between these two groups. The study by Oonnithan (2025) on the S.T.A.R.S Metacognitive Framework also confirms that metacognitive scaffolding, especially when reinforced with technology (such as a specially designed chatbot), can improve students' metacognitive awareness and confidence in mathematical problem-solving. In that study, the group receiving technology-based scaffolding reported stronger "Always" and "Often" responses, indicating that technology can amplify the scaffolding effect (Chen & Wu, 2015; Morphew, 2024). Students with low metacognitive awareness (who may tend to be impulsive) can be "raised" in their level through scaffolding, at least in the context of specific tasks. Students with lower cognitive regulation and objectivity benefit more from domain-general scaffolding than domain-specific scaffolding, while students with lower prior knowledge benefit from both types of scaffolding.

More importantly, this study found that scaffolding did not provide substantial benefits for students with higher prior knowledge and metacognitive skills (Chen & Wu, 2015). This finding directly supports the moderation pattern identified, where the benefits of metacognitive scaffolding are greatest for those who need them most. However, it should be noted that a statistically significant moderation effect does not mean that scaffolding is useless for students with high metacognitive awareness (White & Frederiksen, 2009; Rusdi et al., 2020). That group still showed significant improvement, albeit with a smaller magnitude. This indicates that metacognitive scaffolding is beneficial for all students but is crucial for those at risk of experiencing mathematical communication difficulties.

3. Qualitative Mechanisms: How Scaffolding Works

The qualitative findings from the analysis of written answers and think-aloud protocols provide insights into the mechanisms underlying the quantitative effects. The three main patterns found the use of logical connectives, the emergence of monitoring and evaluation sentences, and the improvement in coherence over time collectively illustrate how metacognitive scaffolding changes the way students approach mathematical writing tasks. First, the use of logical connectives such as "because," "so," "thus," and "consequently" shows that experimental students not only wrote procedural steps but also explained the causal relationships between those steps. This is important because, in mathematical communication, a good argument consists not only of a series of true statements, but those statements must also be logically connected (Morgan et al., 2014).

Furthermore, research by Dassa et al. (2024) on the implementation of metacognitive-scaffolding strategies in mathematics learning showed that this approach significantly improves students' mathematical understanding and problem-solving abilities, as well as increasing students' awareness of their own learning processes. This finding is relevant, where students who received metacognitive scaffolding showed a noticeable improvement in their ability to explicate reasoning through logical connectives. Metacognitive scaffolding helped students realize that they need to explicate these relationships, something that did not occur spontaneously in the control group.

Second, the emergence of monitoring and evaluation sentences in the experimental group's think-aloud protocols (e.g., "I will find the derivative first... oh, wait, did I put the parentheses correctly? Check again...") shows that metacognitive scaffolding activates self-regulation processes. This process is central to metacognition (Sweller et al., 2019) and is very important for successful mathematical problem-solving. Interestingly, experimental students not only monitored their answers but also verbally expressed this monitoring process. This aligns with the findings of Teledahl, Kilhamn, Ahl, and colleagues (2025) in their systematic review of the quality of mathematical writing, which emphasized that quality mathematical writing is assessed not only by the correctness of content but also by the clarity of structure and depth of reflection. In other words, metacognitive scaffolding helps students develop a "metacognitive voice" that is audible in their writing.

Third, the improvement in coherence from the first to the sixth session in the experimental group shows that the scaffolding effect is cumulative and sustainable. Students did not become good writers immediately after a single treatment; rather, they needed time and repeated practice to internalize the taught strategies. This finding supports the importance of continuous and gradual scaffolding, not one-time interventions (Yang et al., 2016). Furthermore, the qualitative change from short, procedural answers to explanatory, coherent answers reflects a shift from what Lithner (2017) calls imitative reasoning to creative reasoning. Imitative reasoning only relies on memory of learned procedures, while creative reasoning involves conceptual understanding and the ability to explain why a step was chosen. The metacognitive scaffolding in this study appears to have successfully encouraged this shift.

CONCLUSION

This study shows that metacognitive scaffolding plays an important role in improving the quality of students' written mathematical communication, particularly in helping them construct a clearer reasoning flow, use mathematical language more appropriately, and connect solution steps more coherently. The quantitative findings indicate a significant difference between the experimental and control groups, with a large effect size. The most notable improvements appeared in students' ability to organize sequential reasoning and link each step of their solutions logically. These findings suggest that metacognitive scaffolding can support students in transforming fragmented internal ideas into written explanations that are more structured, meaningful, and easier to follow.

The study also reveals that students' initial level of metacognitive awareness influences the effectiveness of the intervention. Students with lower initial metacognitive awareness benefited more from the scaffolding because they required greater external support to develop thinking strategies, monitor their problem-solving process, and evaluate the accuracy of their answers. Qualitative findings further show that the scaffolding worked through three main processes: encouraging students to reflect on their thinking before writing, helping them express their reasoning in a coherent written form, and guiding them to review each solution step more carefully. Although this study is limited by its quasi-experimental design, single-university context, relatively short treatment period, and focus on written communication, its findings offer theoretical and practical contributions to mathematics education by emphasizing the role of metacognition in strengthening students' mathematical communication and opening opportunities for further studies in broader contexts and adaptive technology-supported learning environments.

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