CRIME IMPACT MODEL OF NARCOTIC DRUG ABUSE IN NIGERIA

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Abstract
This study developed a 6-dimensional nonlinear ordinary differential equation model to investigate the criminal activity associated with narcotic drug abuse in Nigeria. The model was structured around the dynamics of disease transmission, with the goal of limiting and potentially eliminating the threat posed by drug abuse crimes. The analysis of the basic reproduction number, R0, revealed that narcotic drug abuse will die out when R0<1, and become endemic when R0>1. The model's stability analysis showed that the narcotic drug abuse-free equilibrium is asymptotically stable when the Routh-Hurwitz criteria are satisfied. The study emphasizes the importance of consistent law enforcement, effective rehabilitation programs, and a coordinated, multi-sectoral approach to addressing the escalating narcotic drug abuse crisis in Nigeria. The findings of this research are recommended for consideration by policymakers, public health professionals, and community stakeholders to mitigate the far-reaching consequences of this pressing issue. The use and trafficking of narcotic drugs have emerged as a growing public health and social concern in Nigeria in recent years. The misuse of narcotics, including substances such as opioids, cannabis, and cocaine which is alarming in the prevalence of narcotic drug abuse across the country, with profound
implications for individual well-being, public safety, and socioeconomic development

**Keywords**: Lyapunov Function, Narcotic Drug, Stability Analysis

**Introduction**

The use and trafficking of narcotic drugs have emerged as a growing public health and social concern in Nigeria in recent years Akpienbi et al. (2018). The misuse of narcotics, including substances such as opioids, cannabis, and cocaine which researchers have documented the alarming rise in the prevalence of narcotic drug abuse across the country, with profound implications for individual well-being, public safety, and socioeconomic development.

In a comprehensive nationwide survey, Adebisi and Ogunleye (2020) examined the patterns and determinants of narcotic drug abuse among Nigerian youth, finding that the most commonly abused substances included heroin, cocaine, and synthetic opioids. The study, published in the African Journal of Drug and Alcohol Studies, identified risk factors such as peer pressure, economic instability, and limited access to mental health services as key contributors to the initiation and escalation of narcotic drug use.

Corroborating these findings, Chukwuma et al. (2018), in their report published in the International Journal of Drug Policy, conducted in-depth interviews with law enforcement officials and healthcare providers in Lagos, Nigeria. The authors emphasized the significant strain that narcotic drug abuse places on the country's already overburdened criminal justice and public health systems, underscoring the urgent need for a multifaceted, evidence-based approach to prevention, treatment, and rehabilitation.

Building on these insights, Okafor and Egбуonu (2021), in their review article in the Nigerian Medical Journal, highlighted the unique sociocultural and economic factors that contribute to narcotic drug abuse in the Nigerian context. The researchers called for the development of culturally-sensitive intervention programs that address the root causes of substance use disorders, such as social marginalization, economic instability, and limited access to mental health services.
Complementing these studies, Duru et al. (2019), in their work published in the International Journal of Environmental Research and Public Health, examined the impact of narcotic drug abuse on family dynamics and interpersonal relationships in Nigeria. Their findings revealed the profound disruptions and breakdowns in family structures, further exacerbating the cycle of substance use and its associated harms.

Addressing the issue from a public policy perspective, Odejide (2015), in an article in the Journal of Substance Abuse Treatment, advocated for the development of comprehensive national drug control policies in Nigeria, emphasizing the need for a balanced approach that combines demand reduction, harm reduction, and supply control strategies.

Highlighting the role of the media, Akande and Akande (2017), in their study published in the Journal of Substance Use, explored the framing of narcotic drug abuse narratives in Nigerian news outlets, underscoring the potential for the media to shape public perceptions and influence policy decisions.

Furthermore, Akinola and Akinsola (2018), in their work in the Journal of Psychoactive Drugs, investigated the unique challenges faced by women struggling with narcotic drug use disorders in Nigeria, advocating for gender-responsive treatment and support services.

Building on these insights, Nwanna and Adesokan (2020), in their article in the International Journal of Mental Health and Addiction, examined the comorbidity of narcotic drug abuse and mental health disorders in Nigeria, emphasizing the need for integrated, holistic approaches to addressing these intersecting issues.

Nevertheless, Oshodi et al. (2011), in their study published in Substance Abuse Treatment, Prevention, and Policy, explored the role of traditional and faith-based approaches in the prevention and management of narcotic drug abuse in Nigeria, highlighting the potential for these culturally-rooted interventions to complement conventional treatment modalities.

**Formulation of the narcotic drug abuse model**

The narcotic drug abuse model was developed using the following assumptions:
- Populations that are vulnerable to drug addicts who have either received drug misuse education or have not received drug-addicted education can be mild level drug addicts.
- Population of heavy drug addicts cannot be mild level drug addicts.
- Population of heavy narcotics addicts will undergo criminal law if caught.
- Population of drug addicts who have
undergone criminal law can return to severe drug addicts. natural death rate and death rate due to drug abuse are considered separately. The population of narcotics addicts who can be exposed is considered as 14 years or older and all parameters are non-positive.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Descriptions of variables</th>
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<tbody>
<tr>
<td>$S(t)$</td>
<td>Number of individuals which are susceptible to narcotic drug abuse at time</td>
</tr>
<tr>
<td>$A(t)$</td>
<td>The number of individuals vulnerable to being a narcotic addict but has received drugs misuse education at time</td>
</tr>
<tr>
<td>$T(t)$</td>
<td>The number of mild drugs addicts at time</td>
</tr>
<tr>
<td>$H(t)$</td>
<td>The number of heavy level drugs addicts at time</td>
</tr>
<tr>
<td>$M(t)$</td>
<td>The number of drugs addicts who undergo criminal law at time</td>
</tr>
<tr>
<td>$R(t)$</td>
<td>The number of recovered narcotic drug user</td>
</tr>
</tbody>
</table>

Schematic diagram of narcotic drug abuse model

![Figure 1. Model of Narcotic drug abuse](image)

Then, the system of ordinary differential equations is obtained from the figure 1 above as:
\[
\begin{align*}
\frac{dS}{dt} &= \Lambda - \beta TS - dS \\
\frac{dA}{dt} &= \beta TS - (\gamma + d) A \\
\frac{dT}{dt} &= \gamma A - (\theta + \delta_1 + d) T \\
\frac{dH}{dt} &= \theta A + \eta T + \sigma R - (\delta_2 + d + \omega) H \\
\frac{dM}{dt} &= \omega H - (\eta + d + e) M \\
\frac{dR}{dt} &= eT - (\sigma + d) R
\end{align*}
\]

\[\text{(01)}\]

<table>
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<th>Parameters</th>
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</tr>
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<tbody>
<tr>
<td>(d)</td>
<td>Natural human death rate</td>
</tr>
<tr>
<td>(\Lambda)</td>
<td>Recruitment rate of drug abuse</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>The rate which human individual that receiving drugs misuse education progress to mild user</td>
</tr>
<tr>
<td>(\beta)</td>
<td>The direct contact rate of S with T individuals</td>
</tr>
<tr>
<td>(\omega)</td>
<td>The rate at which heavy drug user being caught and guilty of drug abuse crime</td>
</tr>
<tr>
<td>(\theta)</td>
<td>The transition rate of mild user to heavy user</td>
</tr>
<tr>
<td>(\delta_1)</td>
<td>The death rate of mild drugs addicts</td>
</tr>
<tr>
<td>(\delta_2)</td>
<td>The death rate of heavy drug addicts</td>
</tr>
<tr>
<td>(\eta)</td>
<td>The transition rate of drug abuse crime individual after their rehabilitation (correctional) period return to abuse habit</td>
</tr>
<tr>
<td>(e)</td>
<td>Recovered rate of drug abuse</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>The rate at which the recovered becomes a heavy user</td>
</tr>
</tbody>
</table>

**Existence of equilibrium state of the model**

The equilibrium state for the system was obtained by setting the model equations (01) to zero, and solving the resultant algebraic equations simultaneously to obtain the narcotic drug abuse free and endemic narcotic drug abuse in Nigeria.
Case I: Narcotic drug abuse free

Here let \( (A = 0, T = 0, H = 0, M = 0, R = 0) \) then solve to obtain

\[
E_0 = \left( S = \frac{\Lambda}{d}, A = 0, T = 0, H = 0, M = 0, R = 0 \right)
\]

The equilibrium point as

Case II: Endemic narcotic drug abuse

Solve equation (01) simultaneously to obtain the following equilibrium point

\[
E_1 = \left( S = \frac{(d + \gamma)(d + \theta + \delta)}{\beta \gamma}, A = \frac{\Lambda \beta y - (\gamma + d)(\theta + d + \delta)}{\beta y (d + \gamma)}, T = \frac{\Lambda \beta y - (\gamma + d)(\theta + d + \delta)}{(\gamma + d)(\theta + d + \delta) \beta}, R = \left( \frac{\Lambda \beta y - c(y + d)(\theta + d + \delta)}{\beta (d + \gamma)^2 (d + \theta + \delta)} \right) \right)
\]

Invariant region of the narcotic drug abuse model in Nigeria

Let consider this theory and it proof to establish the narcotic drug abuse model in Nigeria

**Theorem 1:** Supposed there exists a domain \( D \) in which the closed set \( \{S, A, T, H, M, R\} \) is contained and bounded.

**Proof:**

The model population \( N = S + A + T + H + M + R \)

Then, let differentiate \( N \) both sides with respect to \( t \) leads to

\[
\frac{dN}{dt} = -dS + dA - (\delta_1 + d)T - (\delta_2 + d)H - dM - dR
\]

\[
\frac{dN}{dt} \leq -dN
\]

Using method of separation of variable and integrate (02), we have \( -dN \geq Be^{-dt} \)

where \( B \) is constant.
We need to apply the initial condition $N_0 \geq N(0)$ in equation (03), to get
\[ B = \wedge - dN_0 \]  
(04)

Then, substitute (04) into (03), to have
\[ \wedge - dN \geq (\wedge - dN_0) e^{-dt} \]
\[ N \leq \frac{\wedge}{d} \left( \frac{\wedge - dN_0}{d} \right) e^{-dt} \]  
(05)

At $t \to 0$, in equation (4.0), the population size $N \to \frac{\wedge}{d}$

Thus, the feasible solution set of the system equation of human population model enter and remain in the region
\[ D = \{ S, A, T, H, M, R \} \in [0, \infty] : N = \frac{\wedge}{d} \]

**Positivity of solutions of narcotic drug abuse system**

Likewise, there is a need to know if the model positive initial data will remain positive for all time $t > 0$.

**Theorem 2.** The narcotic drug abuse model governed in equation (01) is positive given the initial condition then, the solutions $S(t), A(t), T(t), H(t), M(t), R(t)$ are positive for $t > 0$

**Proof.** The first equation of the narcotic drug abuse model (01) gives;
\[ \frac{dS}{dt} = \wedge - (\lambda + d)S \geq - (\lambda + d)S \]

Thus, \[ \frac{dS}{dt} \geq - (\lambda + d)S \]

Using separation of variable approach, Let $\beta T = \lambda$
\[ \frac{dS}{S} \geq -(\lambda + d) dt \]

Integrate both sides of the inequality yields
\[ \int \frac{dS}{S} \geq -\int (\lambda + d) \, dt \]

\[ S(t) \geq -(\lambda + d) t + T \]

\[ S(t) \geq T \exp(-(\lambda + d) t) \]

Where \( T = e^T \) \hspace{1cm} (06)

Applying the initial condition, at \( t = 0 \) which gives; \( S(t) \geq S(0) e^{-(\lambda + d) t} \) that is \( S \geq S(0) \geq 0 \)

Similarly, same procedure would verified that \( A(t), L(t), H(t), C(t) \) and \( R(t) > 0 \) for all positive at \( t \geq 0 \)

**The Basic Reproduction Number, \( R_0 \)**

The basic reproduction number \( R_0 \) measures the average number of new infections generated by a single infected person during his or her infectious period in a population that is fully susceptible as cited in Akpienbi *et al* (2021). One can easily predict if an infection will spread in exponential progression, die off after some time or remain constant with no further spread judging from the value of the reproduction number. When \( R_0 < 1 \), the narcotic drug abuse will die off because every infected person will transmit the disease to less than one person in the transmittable period. When \( R_0 = 1 \), the narcotic drug abuse will become endemic and will stay with each infected person transmitting to one new person. When \( R_0 > 1 \), narcotic drug abuse will spread and the infected people will grow exponentially which will in the end lead to a pandemic. Using next generation method, the basic reproduction number \( \left( R_0 \right) \) is given as \( R_0 = \rho(FV^{-1}) \). Where \( \rho \) is the spectral radius the spectral radius \( FV^{-1} \) which is the maximum eigenvalue

Given the matrices \( F \) and \( V \), then take the partial derivative of \( F \) and \( V \) then, substitute the equilibrium of the narcotic drug abuse free

\[ E_0 = \left( S = \frac{A}{d}, A = 0, T = 0, H = 0, M = 0, R = 0 \right) \]

\[ R_0 = \frac{\beta \Lambda \gamma}{d (\gamma + d) (\theta + d + \delta_1)} \]
Stability analysis of free narcotic drug abuse equilibrium point of the model

The Routh-Hurwitz conditions are used in order to determine local asymptotical stability of the equilibria. The Routh-Hurwitz criterion states that a necessary and sufficient condition that the equation \( p(\lambda) = \lambda^n + a_1\lambda^{n-1} + \cdots + a_n = 0 \), (with real coefficients) has only roots of negative real part if the values of the determinants of the matrices are all positive, if and only if Routh Hurwitz criteria satisfy

\[
\begin{align*}
R_1 &> 0, R_2 > 0, \\
A &> 0
\end{align*}
\]

For polynomials of degree \( p = 2, 3, \) and 4 the Routh-Hurwitz criteria are summarized as follows:

\( p = 2 \): \( a_1 > 0 \) and \( a_2 > 0 \)
\( p = 3 \): \( a_1 > 0, a_3 > 0 \) and \( a_1a_2 > a_3 \)
\( p = 4 \): \( a_1 > 0, a_3 > 0, a_4 > 0, \) and \( a_1a_2a_3 > a_3^2 \)

Then the equilibrium point \( E_0 \) is locally stable if \( R_0 < 1 \) and unstable if \( R_0 > 1 \)

**Proof:** To determine the local stability of \( E_0 \), the Jacobian matrix below is computed corresponding to narcotic drug abuse free equilibrium

\[
J(E_0) = \begin{bmatrix}
-d & 0 & -\frac{\beta \lambda}{d} & 0 & 0 & 0 \\
0 & -(\gamma + d) & 0 & 0 & 0 \\
0 & \gamma & -(\theta + \delta_1 + d) & 0 & 0 & 0 \\
0 & \theta & 0 & -(\omega + \delta_2 + d) & \eta & \sigma \\
0 & 0 & 0 & \omega & -(e + \eta + d) & 0 \\
0 & 0 & e & 0 & 0 & -d \\
\end{bmatrix}
\]

Then,

\[
\det[J_{E_1}] = d \neq 0
\]

\[
D = (\gamma + d), C = \frac{\beta \lambda}{d}, F = (\theta + \delta_1 + d), X = (\omega + \delta_2 + d), B = (e + \eta + d), Y = (\sigma + d)
\]
Using scientific workplace 5.5. We compute characteristic polynomial as

\[(\lambda + Y) \left( \lambda^2 + (B + X) \lambda + BX + \omega \eta \right) \left( \lambda^2 + (D + E) \lambda - \gamma C + FD \right) \left( \lambda + d \right)\]

Now after expansion it becomes

\[
\begin{align*}
\lambda^6 + (Y + B + X + F + D + d) \lambda^5 + (BD + BF + BX + BY + Bd - C \gamma + DF + DX \\
+ D Y + D d + FX + FY + F d + XY + Xd + Yd + \eta \omega) \lambda^4 + (-B C \gamma + BDF \\
+ BDX + BDY + BD d + BF X + B F Y + BF d + BXY + BX d + BY d - C X \gamma \\
- C Y \gamma - C d \gamma + DX Y + D F Y + DX Y + D X Y + DX d + D Y d + D \eta \omega + F X Y \\
+ F X d + F Y d + F \eta \omega + X Y d + Y \eta \omega + d \eta \omega) \lambda^3 + (-B C X \gamma - B C Y \gamma - B C d \gamma \\
+ BDF X + BDF Y + BD d + BDX Y + BDX d + BD Y d + B F X Y + BF X d \\
+ B F Y d + B X Y d - C X Y \gamma - C X d \gamma - C Y d \gamma - C \eta \gamma \omega + DF Y X + D F X d \\
+ D F Y d + D F \eta \omega + D X Y d + D Y \eta \omega + D d \eta \omega + F X Y d + F Y \eta \omega + F d \eta \omega \\
+ Y d \eta \omega) \lambda^2 + (-B C X Y \gamma - B C X d \gamma - B C Y d \gamma + B D F X Y + B D F X d \\
+ B D F Y d + B D X Y d + B F X Y d - C X Y d \gamma - C Y \eta \gamma \omega - C d \eta \gamma \omega + D F Y X d \\
+ D F \eta \omega + D F d \eta \omega + D Y d \eta \omega + F Y d \eta \omega) \lambda - Y d (B C X \gamma - B D F X + C \eta \gamma \omega \\
- D F \eta \omega)
\end{align*}

Can be written as \(A_6 \lambda^6 + A_5 \lambda^5 + A_4 \lambda^4 + A_3 \lambda^3 + A_2 \lambda^2 + A_1 \lambda + A_0 = 0\) to obtain these eigenvalues

\[
\begin{align*}
\lambda_1 &= -d \\
\lambda_2 &= -Y \\
\lambda_3 &= -\frac{1}{2} B - \frac{1}{2} X + \frac{1}{2} \sqrt{B^2 - 2BX + X^2 - 4 \eta \omega} \\
\lambda_4 &= -\frac{1}{2} B - \frac{1}{2} X - \frac{1}{2} \sqrt{B^2 - 2BX + X^2 - 4 \eta \omega} \\
\lambda_5 &= -\frac{1}{2} D - \frac{1}{2} F + \frac{1}{2} \sqrt{4C \gamma + D^2 - 2DF + F^2} \\
\lambda_6 &= -\frac{1}{2} D - \frac{1}{2} F - \frac{1}{2} \sqrt{4C \gamma + D^2 - 2DF + F^2}
\end{align*}
\]

And the polynomial root has negative real parts where \(A_4 > 0, A_3 > 0, A_2 > 0, A_1 > 0\) and \(A_6 > 0\) if \(R_0 < 1\). Therefore, the reproduction number is less than one which signifies that the narcotic drug abuse free equilibrium is locally asymptotically stable.
Global stability of endemic narcotic drug abuse equilibrium point of the model

The global asymptotic stability of \( E_1 \) is obtained using Lyapunov’s direct method. The ideal of (De Leon) were employed in constructing a common quadratic Lyapunov function of the model equations (01)

**Theorem 4.** If \( R_0 < 1 \), the equilibrium point \( E_1 \) of model Equations (01) is globally asymptotically stable for \( R_0 \leq 1 \) and unstable if \( R_0 > 1 \) in the interior of region \( \square \).

**Proof.** Let define \( L \) on \( D_1 \) in the interior of \( \square \), where \( E_4 \) denotes the global minimum of \( L \) on \( \square \) and

\[
L(S, A, T, H, M, R) = \frac{1}{2} \left[ (S - S^*) + (A - A^*) + (T - T^*) + (H - H^*) + (M - M^*) + (R - R^*) \right]^2
\]

(07)

Now, we differentiate \( L \) along the solution of the model system equations (01), we obtain

\[
\frac{dL}{dt} = \left[ (S - S^*) + (A - A^*) + (T - T^*) + (H - H^*) + (M - M^*) + (R - R^*) \right] \frac{d}{dt} \frac{\partial L}{\partial S}
\]

(08)

where

\[
\frac{d}{dt} \frac{\partial L}{\partial S} = -d \left( S + A + T + H + M + R \right) - \delta_1 T - \delta_2 H
\]

Then, equations (01) becomes

\[
\frac{dL}{dt} = \left[ (S - S^*) + (A - A^*) + (T - T^*) + (H - H^*) + (M - M^*) + (R - R^*) \right] \times \wedge - d \left( S + A + T + H + M + R \right) - \delta_1 T - \delta_2 H
\]

(09)

At the equilibrium point \( E_1 \), \( \wedge \) in equation (09) becomes

\[
\wedge = d \left( S + A + T + H + M + R \right) + \delta_1 T + \delta_2 H
\]
Therefore, equations (09) can be rewritten as

\[
\frac{dL}{dt} = \left[\left( S - S^* \right) + \left( A - A^* \right) + \left( T - T^* \right) + \left( H - H^* \right) + \left( M - M^* \right) + \left( R - R^* \right) \right] \\
\times \left[ d\left( S^* + A^* + T^* + H^* + M^* + R^* \right) - d\left( S + A + T + H + M + R \right) \right] \\
+ \delta T^* + \delta_2 H^* + \delta_1 T + \delta_2 H
\]

(10)

From equations (10) we have

\[
\frac{dL}{dt} = \left[\left( S - S^* \right) + \left( A - A^* \right) + \left( T - T^* \right) + \left( H - H^* \right) + \left( M - M^* \right) + \left( R - R^* \right) \right] \\
\times \left[ d\left( S^* + A^* + T^* + H^* + M^* + R^* \right) - d\left( S + A + T + H + M + R \right) \right] \\
+ \delta_1 \left( T - T^* \right) + \delta_2 \left( H - H^* \right)
\]

(11)

Let

\[ N_1 = (S - S^*), N_2 = (A - A^*), N_3 = (T - T^*), N_4 = (H - H^*), N_5 = (M - M^*), N_6 = (R - R^*), \]

\[ N_7 = N_1 + N_2 + N_3 + N_4 + N_5 + N_6 \]

Thus, equation \((E)\) becomes

\[
\frac{dL}{dt} = N, \left[ -dN_7 - \delta_1 N_3 - \delta_2 N_4 \right]
\]

It can also be rewritten as

\[
\frac{dL}{dt} = -\left[ \mu N_7^2 + \delta_1 N_2 N_7 + \delta_2 N_4 N_7 \right]
\]

Therefore,

\[
\frac{dL}{dt} = -\left[ dN_7^2 + \delta_1 N_5 N_7 + \delta_2 N_4 N_7 \right] \leq 0
\]

Also, \(\frac{dL}{dt} = 0\) if \(S = S^*, A = A^*, T = T^*, H = H^*, M = M^*, R = R^*\) in equation (11). So, the largest compact invariant set in \((S, A, T, H, M, R)\) is singleton, where the endemic equilibrium point. By Lassalle’s invariance principle \(E_i\) is globally asymptotically stable in the interior of \(\Delta\).
Conclusion

The narcotic drug abuse system of 6-dimension of nonlinear ordinary differential equation was developed to study criminal activity associated to narcotic drug abuse in Nigeria in order to limit it abuse and if possible eliminate it crime threat to mankind. The models were conceptualized and structured along with the dynamics of disease transmission. The result of the basic reproduction number $R_0$ which determines whether the narcotic drug abuse crime will die off, become endemic and pandemic was carefully analyse using the method of next generation matrix and it revealed that narcotic will die off whenever $R_0 < 1$ and become endemic if $R_0 > 1$. The model stability analysis was carried out and it asymptotically stable for narcotic drug abuse free equilibrium after satisfying the Routh-Hurwiz criteria condition which state that a necessary and sufficient condition that the equation polynomial with real coefficients has only roots of negative real part if the values of the determinants of the matrices are all positive likewise, the global stability of the narcotic drug. This model is very useful and helpful to the Nigeria Government and policy maker to put an end to the effect of narcotic drug abuse crime. If the government and policy maker would constantly and consistently check drug abuse crime and allowing those guilty to face the wrath of law without any bias will go a long way to reduce the rate of drugs abuse in the country. When correctional center is equipped with all the necessary rehabilitation resources with good managerial execution the basic reproduction number would be less than one. To this extend, these studies have helped to shed light on the escalating narcotics drug abuse crisis in Nigeria.

After this finding this research is recommended to policymakers, public health professionals, and community stakeholders to address this pressing issue. As the country continues to grapple with the far-reaching consequences of narcotic drug abuse, a coordinated, multi-sectoral response will be crucial to mitigate the impact and support those affected.
References


