MIKAILALSYS
Journal of Multidisciplinary Sciences
e-ISSN : 2987-2286
p-ISSN : 2987-3924
Index : Dimensions, Scilit, Crossref, Garuda, Semantic, Lens,Google Scholar,Base, etc
https://doi.org/10.58578/mikailalsys.v2i1.2286

A CERTAIN STUDIES ON MATHEMATICAL MODELING: AN OPTIMIZATION ORIENTED THE REPLACEMENT OF ITEMS WHOSE EFFICIENCY DETERIORATES WITH TIME

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Article Info:

Submitted:	Revised:	Accepted:	Published:
Nov 26, 2023	Dec 15, 2023	Dec 18, 2023	Jan 1, 2024

Abstract

According to the definitions of operation research, it may be regarded as a methodology, an approach, a set of techniques, a team activity, a mixing of various disciplines, an extension of a particular discipline, a new discipline, a vocation, even a religion. There are many problems in today's world that involve the replacement of items, and in this paper we solve a few of these problems and also discuss the day-to-day replacement of items with maintenance costs that increase over time as well as their value as time goes on.

Keywords: Replacement, Items, Maintenance, Money

Introduction

In operation research, replacement theory is used when determining whether it is necessary to replace a used piece of equipment due to its deteriorating characteristics or failure. In the case of replacing existing items, the theory is applied when the items are no longer economically viable or have reached the end of their lifetime, or the items may have been destroyed either by accident or by intentional destruction. The above discussed situations can be mathematically analyzed and categorized on some basis like

i) Items that deteriorate with time

ii) Items becoming out of date due to new developments like

Ordinary weaving looms by automatic, manual accounting by tally etc.

iii) Items which do not deteriorate but fall completely after

Certain amount of use like electronic parts, street lights.

iv) The existing working staff in an organization gradually diminishing due to death, retirement, retrenchment and otherwise (see [1-9].

Replacement of items whose efficiency deteriorates with time:

The first and foremost problem here is the criteria for measuring efficiency. The discounted value of the failure costs associated with each policy is generally the discounted value of those failure costs. In the case of machine manufactured systems, the maintenance costs usually increase over time, and at a certain point, the maintenance costs become so high that it becomes economically and practically beneficial to replace the item. Sometimes in a system there are alternative choices. We may like to compare various alternatives by considering the costs involved in each choice. There are many factors that can affect the cost of this service. In making such comparisons we may not consider those costs which are different for different alternative choices at various stages of the analysis. In terms of how these situations are analyzed, there are a number of different approaches that can be classified into the following categories.

Replacement of items whose Maintenance cost increases with time and value of money remains same during the period.

Criteria of present value for comparing replacement alternatives.

Replacement of items whose maintenance cost increases with time and value of the money also changes with time

Model: Replacement of items whose maintenance cost increases with time and the value of the money remains same during the period.

Theorem: The cost of maintenance of a machine is given as a function increasing with time t, whose scrap value is constant.

i) If time is measured continuously then the average annual cost will be minimized by replacing the machine when he average cost to data becomes equal to the current maintenance cost.

ii) If time is measured in discrete units, then the average annual cost will be minimized by replacing the machine when the next period's maintenance cost becomes greater than the current average cost.

Proof: Let C= the capital or purchasing cost of item,

S= Scrap value of the item

Case i) When t is a continuous variable

Let u(t) be the maintenance or running cost at time t.

If the item is used in the system for a period y then the total maintenance cost incurred during period y will be

$$M(y)= \int_0^y u(t)dt ------------ (1)$$

The total cost incurred on the item during period y=

C+M (y)-S

Thus average cost per unit of times incurred during period y on the item is given by

$$G(y)=\frac{C+M(y)-S}{y}--------(2)$$

We want to find the value of y for which G(y) is minimum.

$$\therefore \frac{dG}{dy}=-\frac{c-s}{y^2}+\frac{u\,(y)}{y}-\frac{1}{y^2}\int_0^y u(t)dt=$$

By using equation (i), we obtain

$$-\frac{c-s}{y^2}+\frac{u\,(y)}{y}-\frac{1}{y^2}M(y)=0$$

$$u(y)=\frac{C-s+M(y)}{y}$$

= G(y) (by using equation$------(3)$

We conclude that the replace the item when the average annual cost reaches at the minimum which will to the current maintenance cost i.e. If time is measured continuously then the average annual cost will be minimized by replacing the machinelike items when the average cost to date becomes equal to the current maintenance cost.

Case (i) when time t is a discrete variable. In this case, the period of time is taken as one year and t can take the values 1, 2, 3,… etc.

Case (ii) When time t is a discrete variable. In this case the period of time is taken as one year and t can take the values 1,2,3,….etc.

Then M(y)=$\sum_{t=1}^{y} u(t) = total\ running\ cost\ of\ years ---(4)$

$\therefore$ $total\ cost$ incurred on the item during y years

T(y)=C+M(y)-S

= C-S+$\sum_{t=1}^{y} u(t)$

$\therefore$ $average\ annual$ cost incurred during y years

$$G(y)=\frac{T(y)}{y}=\frac{[C+M(y)-S]}{y}----------(5)$$

Now, G(y) will be minimum for that valu of y, for which

G(y+1)>G(y)

and G(y-1)>G(y)

thus,

$G(y+1)=\frac{c-s+\sum_{t=1}^{y+1}u(t)}{(y+1)}$

$\therefore G(y+1)-G(y)=$

$$\frac{(c-s)+\sum_{t=1}^{y+1}u(t)}{y+1}-\frac{(c-s)+\sum_{t=1}^{y}u(t)}{y}$$

$=\frac{u(y+1)}{y+1)}+\frac{(c-s)+\sum_{t=1}^{y}u(t)}{y+1}-\frac{(c-s)+\sum_{t=1}^{y}u(t)}{y}$

$=\frac{u(y+1)}{y+1)}+\frac{(c-s)+\sum_{t=1}^{y}u(t)}{y+1}-\frac{(c-s)+\sum_{t=1}^{y}u(t)}{y}$

$=\frac{u(y+1)-G(y)}{y+1}$

Thus G(y+1)-G(y)>0, if u(y+1)> G(y)

Similarly,

G(y-1)-G(y)>0 if u(y)<G(y-1)----------------------------(5)

Equation(5) shows that do not replace if the next year running cost is less than the previous years total unit but replace at the end of years if the next year's [i..e. $(y+1)^{th}$ year's] running cost is more than the average cost of y^{th} year i.e. when time is measured discrete units, then the average annual cost will be minimized by replacing the machine when the next periods mainetance cost becomes greater than the current average cost.

Example:i) The cost of a machine us Rs. 6100 and its scrap value is Rs. 100. The maintenance costs fund from experience are as follows:

Year	1	2	3	4	5	6	7	8
Cost in Rs:	100	250	400	600	900	1200	1600	2000

When should the machine be replaced?

Solution: The problem can be solved by minimizing G(y) of equation (3).In this problem t is discrete.

Here C= 6100, S=100. The value of G(y) can be calculated for different years from the following table

years t, y	u(t)	M(y)=$\sum_{t=1}^{y} u(t)$	T(y)=C-S+M(y)	$\frac{T(y)}{y} = G(y)$
1	100	100	6100	6100/1=6100
2	250	350	6350	6350/2=3175
3	400	750	6750	6750/3=2250
4	600	1350	7350	7350/4=1837.50
5	900	2250	8250	8250/5=1650
6	1200	3450	9450	9450/6=1575
7	1600	5050	11050	11050/4=1578.52
8	2000	7050	13050	13050/8=1631.25

From above table, we find that G(y) is minimum in the 6^{th} year being equal to 1575.00. Also the maintenance cost in the 7^{th} year is more then 1575.00. Hence the machine should be replaced at the end of 6^{th} year otherwise the average annual cost would increase.

Example2: A fleet owner finds from his past records that the costs per year of running a truck and resale values whose particular price is Rs 6000 are given as under. At what stage the replacement is due?

years	1	2	3	4	5	6	7
Running cost	1000	1200	1400	1800	2300	2800	3400
Rescale value	3000	1500	750	375	200	200	200

The problem can be solved easily by minimizing G(y) given in equation.

Here the scrap value decreases with time. Let S(y) denote

the scrap value of truck at the end of y^{th} year.

Here C=6000. The value of G(y) can be calculate from the following table.

Years	Running cost G(y)	Cumulative running cost M(y)=in Rs	Resale value S(y) in Rs	C-S(y) in Rs.	T(y)=C-S(y)+M(y)in Rs.	$\frac{T(y)}{y} = G(y)$
1	1000	1000	3000	3000	4000	4000
2	1200	2200	1500	4500	6700	3350
3	1400	3600	750	5250	8850	2950

4	1800	5400	375	5625	11025	2756
5	2200	7700	200	5800	13500	2700*
6	2600	10,500	200	5800	16300	2717
7	3400	13,900	200	5800	19700	2814
8	4000	17,900	200	5800	23700	2962

It is observed from above table, that the value of G(y) in the fifth year is minimum. Here the truck should be replaced at the end of fifth year, otherwise the average annual cost would increase.

Replacement of items whose maintenance cost increases with time and the value of money also changes with time: -

This is more complicated problem and it can be solved under two different ways: -

i) The maintenance cost varies with time and we want to find optimum value of time at which the item should be replaced. The value of money decreases with a constant rate which is known as depreciation ratio and is denoted by d.

Here the value of money changes can be seen from the following example: -

Suppose, we borrow Rs. 100 at an interest of 10% per year. After one year we have to return Rs. 110.00. Therefore, Rs. 110.00 after one year from now are equivalent to Rs. 100.00 at present. Thus Re 1., after one year from now is equal to $(1.1)^{-1}$ at present. This is known as present value.

ii) The manufacture takes load for a certain period at a given interest and agrees to pay it in a number of installments. Then we are interested in finding the most suitable period during which the load should be repaid.

Situation 1: Let the equipment cost be Rs. A and the maintenance cost be Rs. $C_1, C_2, \ldots\ldots, C_n, C_{n+1} \ldots\ldots\ldots (C_{n+1} > C_n)$ during the first year, second year, etc. respectively. If d is depreciation value per unit of money during a year then to find the optimum replacement policy which minimizes the total of all figure discounted costs.

Solution: Let us assume that the expenditure is incurred at the begin of the year and the resale value of the item is Zero. The problem can be solved by finding the total expenditure incurred on the equipment and its maintenance during the desired period and its present value.

Let us assume that it is desirable to replace the item after X years of service. Then we calculate the expenditure made in different years and their present values in the following tabular form:-

year	Capital cost	Maintence cost	Total cost in i^{th} year	Present value of total expenditure
1	A	C_1	A+C_1	A+C_1
2	-	C_2	C_2	dC_2
3	-	C_3	C_3	d^2C_3
⋮	⋮	⋮	⋮	⋮
X	-	C_x	C_x	$d^{x-1}C_x$
X+1	A^*	C_1^*	A+ C_1^*	$d^*(C_1+A)$
⋮	⋮	⋮	⋮	⋮
2x	-	C_x	C_x	$d^{2\,x-1}\,C_x$
And sen	------	---------	------------	----------

The present value of total future expenditure incurred with a x-yearly replacement policy will be

$$G(x)=[A + C_1 + dC_2 + --- + d^{x-1}C_x] + (A + C_1)^{d^x} + C_2 d^{x+1} + \quad -- + C_x d^{2x+1}$$

$$= \frac{A+\sum_{i=1}^{x} Ci\propto^{i-1}}{1-d^x} --------- (6)$$

Above equation (6) gives the total amount of money required at present to pay all future costs of purchasing and maintaining the equipment with a replacement policy after x-year. The value of x preferred for which

G(x+1)> G(x)

-----------------(7)

G(x-1)> G(x)

For G(x+1)> (x)

= AG(x)>0

thus

$$AG(x)=\frac{A+\sum_{i=1}^{x} c_i\, d^{i-1}}{1-d^{x+1}} - \frac{A+\sum_{i=1}^{x} c_i\, d^{i-1}}{1-d^{x}}$$

$= \frac{C_{x+1} - G(x)(1-d)d^x}{1-d^{x+1}}$

We are given that d is the depreciation value of money and hence $\frac{d^x}{1-d^{x+1}} > 0.$

Thus,

$\Delta G(x) > 0$

$= C_{x+1} > G(x)(1-d)$

G(x)<$\frac{C_{\propto+1}}{1-d}$

now for (1-d) $G(x)_0$=$\frac{A+\sum_{i=1}^{x} Ci\, d^{i-1}}{1+d+d^2+----+d^{x+1}}$--------(8)

equation (8) gives the weighted average cost of all previous X years with weights 1, d, d^2, ……, respectively.

Similarly for G(x-1)>G(x)

$=\Delta G(x-1) < 0$

and proceeding in the same way as in equation (8), we get

$= \frac{d^{x-1}}{1-d^{x-1}}$ [C_x=(2^{nd})

now, $\frac{d^{x-1}}{1-d^{x-1}} > 0$

= Q(x)-Q(x-1)<0

We should have

$C_x < (1-d)G(x)$

$= C_x < \frac{A+\sum_{i=1}^{x} Ci\, d^{i-1}}{1+d+d^2+----+d^{x-1}} < C_{x+1}$-------------- (8)

We combine that do not replace it the operating cost of next period is less than the weighted average of previous cost, where weights are 1, d, d^2, ….. respectively and replace if the operating cost of next period is greater than the weighted average of the previous costs.

Remarks: if i→ $0, d = \frac{1}{1+i} \to 1 - - - - (9)$

$$\therefore \textit{the limiting value of } \frac{A+\sum_{i=1}^{x} Ci\, d^{i-1}}{1+d+d^2+----+d^{x+1}} \textit{as } d \to 1$$

= G(x)

i.e.$C_x < G(x) < C_{x+1}$ *is* same as previous result.

Example: A Machine costs Rs. 1000. Operating costs are Rs. 500 per year for the first five years. In the 6th and succeeding years operating costs increases by Rs. 100 per year. Assuming a 10% discount rate of money per year, find the optimum length if time to hold the machine before

Solution: The discount rate per rupee will be 0.1.

d= $\frac{1}{1+0.1} = 0.9091$ *and* A=10000.

We know that

G(x)= $\frac{A+\sum_{i=1}^{x} c_i\, d^{i-1}}{\sum_{i=1}^{x} \propto^{i-1}}$ *can be* calculated in the tabular form of following table: -

(1) X	(2) c_i	(3) d^{i-1}	(4) $c_i d^{i-1}$	(5) $A + \sum_{i=1}^{x} Ci\ d^{i-1}$ (2)/(3)	(6) $A + \sum_{i=1}^{x} d^{ix-1}$	(7) G(x) (5)/(6)
1	500	1.0000	500	10500	1.0000	10500
2	500	0.9091	456	10956	1.9091	5738.8
3	500	0.8264	413	11369	2.7355	4156.3
4	500	0.7513	376	11745	3.4868	3368.4
5	500	0.6830	342	12087	4.1698	2898.7
6	600	0.6209	373	12460	4.7907	2600.8
7	700	0.5645	395	12855	5.3552	2400.8
8	800	0.5132	411	13266	5.8684	2260.5
9	900	0.4665	420	13686	6.3349	2160.4
10	1000	0.4241	424	14110	6.7590	2087.5
11	1100	0.3856	424	14534	7.1446	2034.2
12	1200	0.3506	421	14955	7.4952	1995.2
13	1300	0.3187	414	15369	7.8139	1996.8
14	1400	0.2897	406	15775	8.1036	1946.6
15	1500	0.2637	396	16171	8.3673	1932.6
16	1600	0.2397	384	16555	8.6070	1923.4
17	1700	0.2179	370	16925	8.8249	1971.8

18	1800	0.1983	357	17282	9.0230	1915.3
19	1900	0.1801	342	17624	9.2031	1915.0
20	2000	0.1637	327	17951	9.3668	1916.4

Columns (2) and (7), we find that $C_{20} > C_{19}$ and$C_{19} < C_{18}$. Hence from above both equations, the optimum length of time to hold the machine before we replace it, is 19 rears. Alsothe minimum weighted average cost occurs at the end of 19 years.

Situation (ii), the manufacturer borrows the total sum at the fixed interest rate, which is equal to discounted value of the money. Then if be agrees to pay equal installments of Rs. y per year then to find the optimum value of the repayment period x.

Solution: For this, we know that

$$d=\frac{1}{1+i},$$

The present value of the installments over a period of time can be calculates as in the following table:-

year	Installment paid	Present value of the installments
1	y	y
2	Y	dy
3	Y	d^2y
----	--------------	------------------
X	Y	d^{x-1}
X+1	Y	d^x
⋮	⋮	⋮
2x	y	d^{2x-1}
----	--------------	------------------

Thus the present value of total money paid of him

$$T(x)= y+dy+\text{--------}+d^{2x-1}+\text{-----}=\frac{y}{1-d} ----(10)$$

now the best period is that for which

$$T(x)=G(x) \text{ -------------------- } (11)$$

$$\text{Thus } T(x)=\frac{y}{1-d}=\frac{A+\sum_{i=1}^{x} Ci\, d^{i-1}}{1-d^x}=G(x) ------(12)$$

$$y=\frac{A+\sum_{i=1}^{x} Ci\, d^{i-1}}{1+d+d^2+--d^{x-1}} -------(13)$$

∴ annual installment= weighted average for x years.

∴ from equation(13), we conclude that in situation (ii)the

optimuk value of y should be equal to the weighted average of previous costs, the weights being $1, d, d^2, \dots \dots$ respectively.

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