UNVEILING THE POWER OF EXPONENTIAL FUNCTIONS:
APPLICATIONS IN OUR DAILY LIVES

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Abstract

In this research paper, we have obtained some new applications of exponential and logarithmic functions. This report deals all the validity of previous works. This work is motivated by the work of [1-12].

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Introduction

Input-output analysis delves into various aspects of economic dynamics. It explores the relationships between producer and consumer surplus even in unconventional market conditions where demand may be positive, but supply is negative (Sahani, 2023). Sahani’s work indicates that even in such scenarios, there exists untapped value for both producers and consumers, underscoring the resilience and complexity of economic systems. This insight is not only intriguing but also carries significant implications for economic policies.
Another study was done by the help of input-output analysis to study the international pricing of risk within the global capital market (Sahani et al., 2023). Sahani and his colleagues underscore the role of input-output analysis in unravelling the complex relationships between different sectors of the world economy and illuminating the channels through which risk is transmitted across borders. These findings bear relevance for international financial regulation, as understanding the dynamics of risk transmission can inform policies aimed at curbing financial contagion. Furthermore, input-output analysis is applied to the realm of non-linear science, where it uncovers the intricate interactions within non-linear systems, providing predictive insights into their behaviour (Sahani & Prasad, 2023). Sahani and Prasad's work highlights the adaptability of input-output analysis as a versatile tool for understanding complex systems beyond traditional economic domains. Turning our attention to Africa, input-output analysis is instrumental in dissecting the employment and job creation challenges faced by the youth in African countries (Sahani, 2023). Sahani's research identifies critical sectors like agriculture, construction, and trade as significant contributors to youth employment. Addressing the challenges in these sectors becomes imperative for fostering economic growth and harnessing the demographic dividend of Africa's youthful population (see [1-12]).

A particularly important exponential function is where the base is ‘e’ (approximately 2.71828). This specific function, denoted by $e^x$, is ubiquitous in mathematics, science, and engineering due to its unique properties and prevalence in natural phenomena. In the 1600s, mathematicians like Jacob Bernoulli started looking closer at exponential growth because of its importance in things like interest rates.

Around the same time, another mathematician named Christian Huygens explored similar ideas while studying how populations grow. Later in the 1600s, Johann Bernoulli (Jacob’s brother) even figured out how to use calculus with exponential functions. By the 1700s, Leonhard Euler, a super brainy mathematician, really dug into exponential functions. He’s the one who introduced the number “e” (Euler’s number), which is super important in this area of math.

People like the Babylonians and Egyptians dealt with exponential growth for ages, even though they didn’t have a fancy mathematical term for it. A good example is how they calculated interest, which grows more and more over time. The 18th century marked a breakthrough with Leonhard Euler’s extensive work on exponential functions. Euler not
only studied them in depth but also introduced a fundamental constant, ‘e’ (Euler’s number). This number, approximately equal to 2.71828, is the base of the natural logarithm and plays a crucial role in exponential functions. He essentially discovered a specific exponential function with unique properties that became the standard for studying exponential growth and decay.

According to Sahani (2023), input-output analysis delves into various aspects of economic dynamics. It explores the relationships between producer and consumer surplus even in unconventional market conditions where demand may be positive, but supply is negative. His work indicates that even in such scenarios, there exists untapped value for both producers and consumers, underscoring the resilience and complexity of economic systems. This insight is not only intriguing but also carries significant implications for economic policies.

Economic sectors and revealing the pathways by which risk is transnationally dispersed is emphasized by Sahani and his associates. These results are relevant to international financial regulation because actions intended to prevent financial contagion can be informed by knowledge of the dynamics of risk transmission. Additionally, input-output analysis is utilized in the field of non-linear research, revealing the complex relationships among non-linear systems and offering prognostications regarding their conduct (Sahani & Prasad, 2023). The work of Sahani and Prasad demonstrates how flexible input-output analysis is as a tool for comprehending complex systems outside of conventional economic fields. Now that we are focusing on Africa, input-output analysis is crucial for analysing the issues related to employment and job development that young people in African nations confront (Sahani, 2023). According to Sahani’s research, important industries that significantly contribute to young employment include trade, construction, and agriculture. In order to promote economic growth and capitalize on the demographic dividend of Africa’s youthful population, it becomes necessary to address the difficulties in various sectors.

**Impact of exponential function in today’s world**

Today, the exponential function is a cornerstone of mathematics. Its applications span a vast array of fields, including:
Population modeling: Exponential functions can be used to model how populations grow or decline over time.

Economics: Compound interest, a cornerstone of finance, relies heavily on exponential functions.

Physics: Radioactive decay, a process where unstable atomic nuclei lose energy and transform into different nuclei, follows an exponential pattern.

Engineering: Exponential functions find applications in various engineering disciplines, from analyzing signal behavior in circuits to modeling heat transfer processes.

Application and their examples of exponential function in daily life:

Exponential functions are all around us, even though we might not realize it! They are useful for understanding situations where things are rapidly growing or decaying. Here are some everyday examples of exponential functions at work:

Financial growth

Ever wondered how your savings account grows over time? When you earn interest on your money, that interest is often compounded. This means you earn interest on both the original amount you deposited and the interest you’ve already earned. Exponential functions can be used to model this compound interest, showing how your savings can grow significantly over time.

Disease spread

Unfortunately, exponential functions can also be used to model the spread of diseases. As a disease infects more people, those people can then infect even more people, and so on. This rapid growth can be exponential, especially in the early stages of an outbreak.

Radioactive decay

On the other hand, some things decay exponentially. Radioactive materials, for instance, lose their radioactivity over time at a predictable rate. This decay can be modeled with an exponential function, which helps scientists understand how long it takes for radioactive materials to become safe.
Bacterial growth

Bacteria can reproduce very quickly, often doubling in number every few minutes. This explosive growth can be modeled by an exponential function, which is helpful for understanding how quickly bacteria can populate an environment.

Social media reach

When you share a post on social media, it has the potential to reach a widening audience. If each person who sees your post shares it with a few of their followers, the number of people who see it can grow exponentially. This can be especially impactful for viral content that spreads quickly.

Signal strength

Our mobile phones rely on radio waves to communicate with cell towers. The strength of these signals weakens exponentially as the distance from the tower increases. Exponential functions help engineers design cellular networks to ensure sufficient coverage and signal strength for users.

Computer performance

Moore’s Law, an observation in the tech industry, states that the number of transistors on a microchip doubles roughly every two years. This exponential growth has led to the dramatic increase in processing power and miniaturization of computers we’ve seen over the decades.

Light dimmer switch

The way a light dimmer works is often based on an exponential function. When you turn the knob, the resistance changes exponentially, not linearly. This creates a smoother dimming effect as opposed to an abrupt on/off transition.

Chemical reaction

Many chemical reactions proceed at a rate that can be modeled by exponential functions. The concentration of reactants can decrease exponentially over time as products are formed. Understanding these reaction rates is crucial in various fields like medicine and materials science.
Some applications of exponential function in real life:

Problem 1:
We start with 100% stain intensity (P=100) and the stain halves each day (b=1/2), how much stain intensity (A) will be left after 3 days (t=3)?

Answer

\[ A = 100 \left( \left( \frac{1}{2} \right)^t \right) \approx 4 \] (This represents about 4% of the original stain intensity remaining).

Problem -2

A town initially has a population of 1000 people (f(0) = 1000). The population grows at a rate of 10% annually (represented by the factor 1.1).

Answer

To find the population after 5 years (t = 5), we substitute t = 5 into the function: \( f(5) = 1000 \times (1.1)^5 \approx 1610.51 \). Therefore, the estimated population after 5 years is around 1610.

You can use this function to predict future population based on the initial value and growth rate.

Problem -3

You invest $1000 (P = 1000) at an annual interest rate of 5% (r = 0.05).

Answer

To find the total amount after 3 years (t = 3), we substitute the values: \( f(3) = 1000 \times (1 + 0.05)^3 \approx $1157.63 \). So, the total amount after 3 years with compounded interest is approximately $1157.63.

This function allows you to calculate future investment values considering interest.

Problem-4

We have a sample with 10 grams (A = 10) of radioactive material that decays at a rate where half the amount remains every year (represented by the base 0.5).
Answer

To find the amount of material remaining after 2 years (t = 2), we substitute t = 2 into the function: \( f(2) = 10 \times (0.5)^2 = 2.5 \). Therefore, there are approximately 2.5 grams of material remaining after 2 years.

This function helps model the decay of radioactive elements over time.

Conclusion

It is concluded that exponential function has many application in real life. It helps us to easily determine the compounded growth and decay of any possible objects in given period of time with at a proportional rate. There appears to be much to study and understand in the world of exponential function and its applications to our world. I can now look and recognize that the Math in the courses have always had topics related to exponential function. It seems interesting that such a concept appears so frequently in different topics within mathematics.

References


