

A STUDY AND EXAMINED OF EXPONENTIAL FUNCTION: A JOURNEY OF ITS APPLICATIONS IN REAL LIFE

Kishan Kumar Sah¹, Suresh Kumar Sahani², Kameshwar Sahani³, Binod Kumar Sah⁴

¹MIT Campus, T.U., Janakpurdham, Nepal; ²Rajarshi Janak University, Janakpurdham,
Nepal; ³Kathmandu University, Nepal;

⁴R.R.M. Campus, T.U., Janakpurdham, Nepal
sahkishan940@gmail.com; sureshkumarsahani35@gmail.com

Article Info:

Submitted:	Revised:	Accepted:	Published:
Jan 10, 2024	Feb 27, 2024	Mar 5, 2024	Mar 9, 2024

Abstract

In this report, we have examined the real life applications of exponential function on the following topics: depreciation, population growth, bacterial growth, compound interest, population of one-horned rhino.

Keywords: Exponential function, Population, Depreciation, Bacterial growth, and Compound interest.

Introduction

Exponential function was developed by Leonard Euler. It is considered as the most important function in applied mathematics due to its ubiquitous occurrence. It is also called natural exponential function. It analyses the growth or decay of a quantity at a rate proportion to its current value. The intensity of exponential function's study began in late

17th and early 18th centuries, when great mathematicians, such as Jacob Bernoulli, Leonard Euler, Isaac Newton began to delve into the depth of exponential functions and logarithmic functions. It brought the creation of new calculus. The number “e” is named in honor of Leonard Euler, who spent amounts of time exploring its properties, although Jhon Napier used it to develop his conceptual understanding and development of logarithmic functions almost a century before Euler. Some believe that “e” received its notation simply because it stood for “exponential”, while others believe that the letter a-d had already been given mathematical meanings so, “e” was the next obvious choice Aleff, H.P (2005).1718 (18th Century) Thomas Malthus was a British economist and demographer who studied about unlimited growth. He published a book “An Essay on the principle of population”.1838, Pierre – Francois Verhulst is a Belgian mathematician who studied about biologist who studied about unlimited growth in mid 20th century. In 2004, Donella was the mathematician who studied the limits to growth. He published a book named, “The limits to Growth”(see [7-12]).

According to of Sahani (2023), input-output analysis delves into various aspects of economic dynamics. It explores the relationships between producer and consumer surplus even in unconventional market conditions where demand may be positive, but supply is negative. His work indicates that even in such scenarios, there exists untapped value for both producers and consumers, underscoring the resilience and complexity of economic systems. This insight is not only intriguing but also carries significant implications for economic policies.

Another study was done by the help of input-output analysis to study the international pricing of risk within the global capital market (Sahani et al., 2023). Sahani and his colleagues underscore the role of input-output analysis in unraveling the complex relationships between different sectors of the world economy and illuminating the channels through which risk is transmitted across borders. These findings bear relevance for international financial regulation, as understanding the dynamics of risk transmission can inform policies aimed at curbing financial contagion.

Furthermore, input-output analysis is applied to the realm of non-linear science, where it uncovers the intricate interactions within non-linear systems, providing predictive insights into their behaviour (Sahani & Prasad, 2023). Sahani and Prasad's work highlights the

adaptability of input-output analysis as a versatile tool for understanding complex systems beyond traditional economic domains.

Turning our attention to Africa, input-output analysis is instrumental in dissecting the employment and job creation challenges faced by the youth in African countries (Sahani, 2023). Sahani's research identifies critical sectors like agriculture, construction, and trade as significant contributors to youth employment. Addressing the challenges in these sectors becomes imperative for fostering economic growth and harnessing the demographic dividend of Africa's youthful population (see [1-6]).

The works that I have presented here are matching with the above given works of Suresh Kumar Sahani. I have examined the applications of exponential functions through the following functions.

Definition:

1. **Unlimited Growth Function:** The function modeled by the equation $f(t) = (ae^{rt})$, where 'a' and 'e' are constants, is called unlimited growth function. It was given by Thomas Malthus in 18th Century.
2. **Limited Growth Function:** The function modeled by the equation $f(t) = M(1 - e^{-rt})$, where 'M' and 'e' are constants, is called limited growth function. It was given by Thomas Malthus in 18th Century.
3. **Logistic Growth Function:** The function which is modeled by $f(t) = (M / (1 + ae^{-rt}))$, where 'M', 'r' and 't' are constants, is called logistic growth function. It was given by Pierre François Verhulst in between 1838 and 1847 under the guidance of Adolphe Quetelet.
4. **Unlimited Decay Function:** The Function modeled by the equation $f(t) = ae^{-rt}$, where 'a' and 'r' are constant, is called unlimited decay function. It was given by Rutherford and Soddy.

Discussion

Problem-1: The price valued at \$400,000,000 depreciated continuously at the rate of 10% per for 2 years.

Problem-2: The population of population of Nepal is 31,240,315 with a PGR of 0.92.

Problem-3: The number of bacteria in the flask is 20,000,000,000 and it grows by 100% every 20 minutes.

Problem-4: A sum of \$60,000 is deposited in bank with an interest rate of 6% per annum for 10 years.

Problem-5: The population of one-horned rhino in 2024 in Nepal is 752 and is increasing with a rate of 16%.

Solution-1:

Initial value of airplane (a) = \$400,000,000

And, rate of depreciation (r) = 10% per annum.

We know, unlimited decay function $f(t) = ae^{-rt}$.

So, let's examine the scrape value of airplane in next 2 years.

We have,

$$i = r/100$$

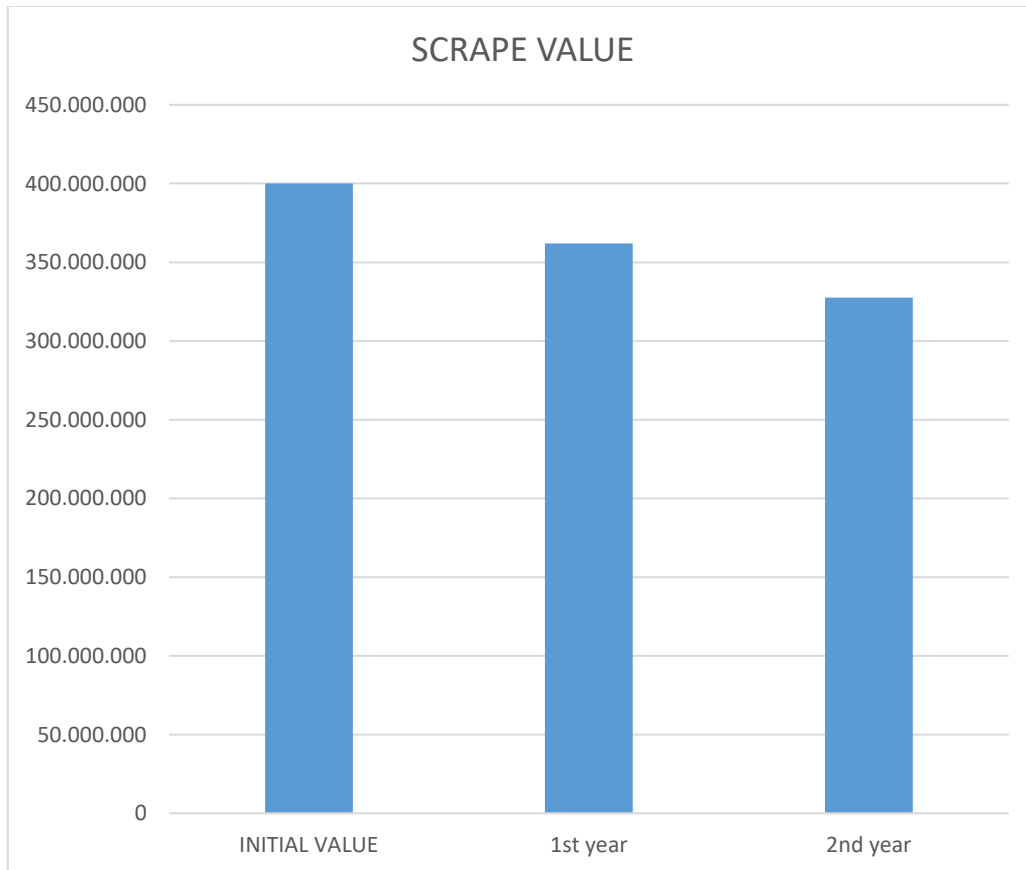
$$= 10/100$$

$$= 0.1$$

$$\text{Scrape value}(s) = ae^{-ri}$$

$$= 400,000,000 * e^{-0.1 * 2}$$

$$= \$327492301.2$$



Solution-2:

Present population of Nepal in 2024 (p)=31,240,315

And, population growth rate (r)=0.92%

We know, unlimited growth function $f(t) = ae^{it}$.

Let's examine the population of Nepal in next 10 years.

We have,

$$i = r/100$$

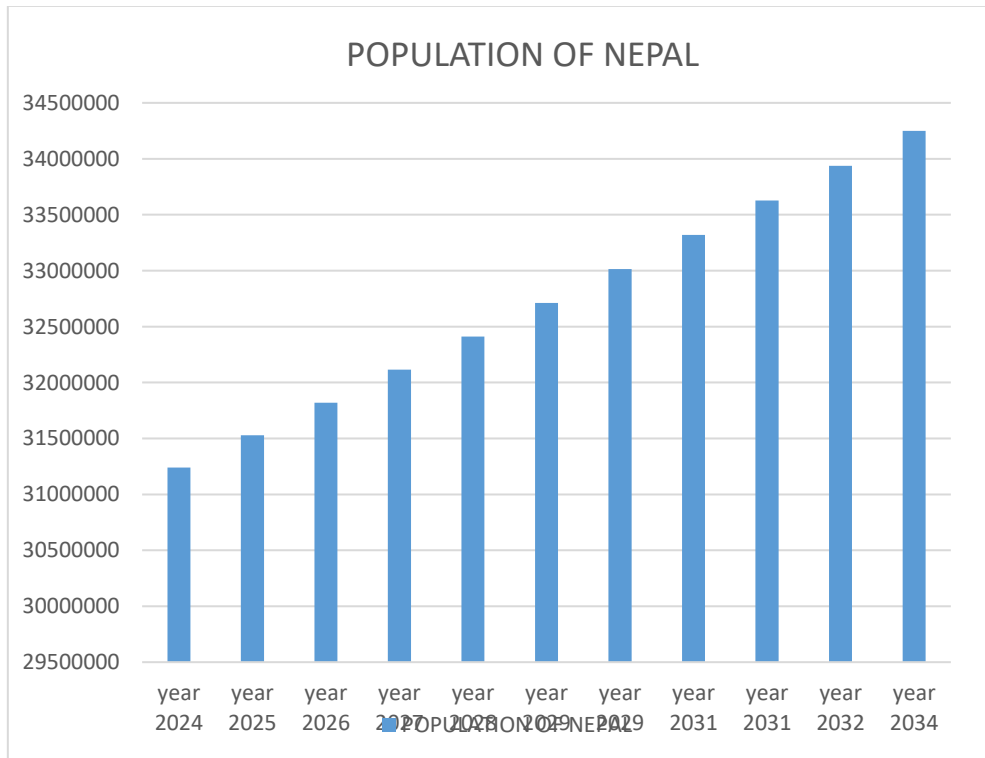
$$= 0.92/100$$

$$= 0.0092$$

So, population of Nepal in next 10 years (P)= ae^{it}

$$= 31,240,315 * e^{0.0092 * 10}$$

$$= 34,250,782$$



Solution-3:

Number of bacteria in a flask (p)=20,000,000,000

And, rate of growth(r)=100% per 20 minutes.

So, it will grow by (100/20) % per minute. (i.e 5% per minute).

We know unlimited growth function $f(t)=ae^{it}$.

Let's examine its growth in 6 minutes.

We have,

$$i=r/100$$

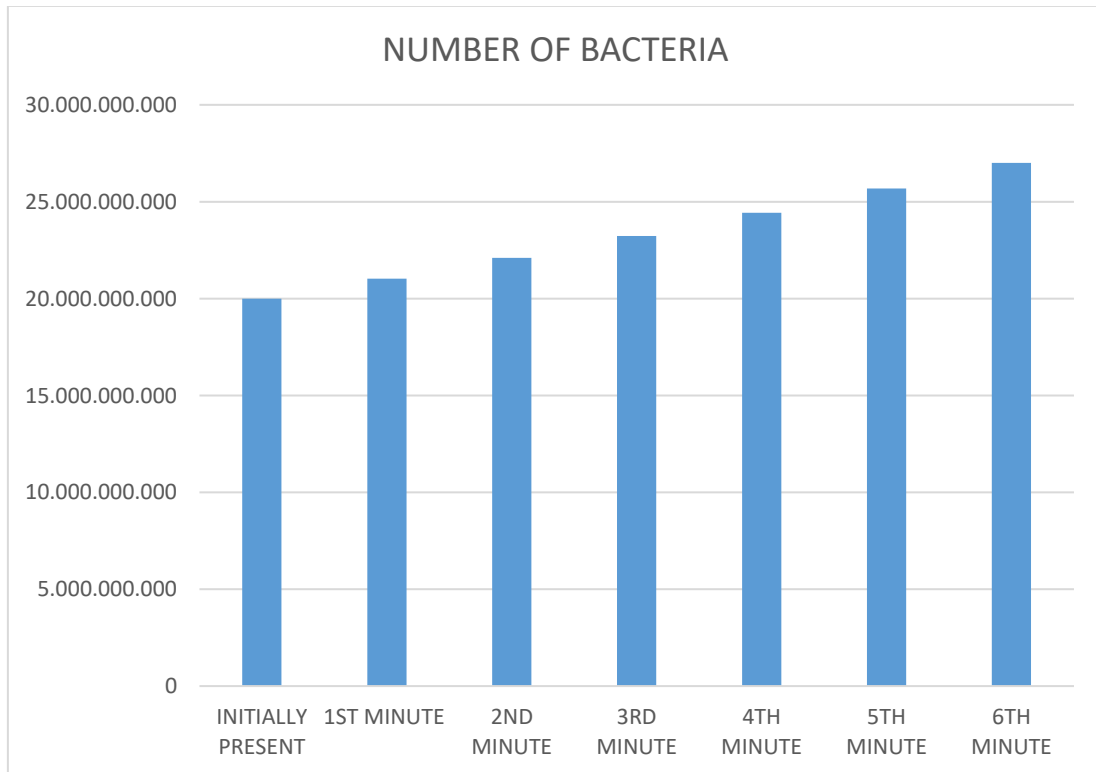
$$=5/100$$

$$=0.05$$

So, number of bacteria in the flask in 1day $=ae^{it}$

$$=20,000,000,000 * e^{0.05 * 6}$$

$$=26,997,176,150$$



Solution-4:

We have sum (a)=\$60,000 invested for 10 years

And, rate of interest (r)=6% per annum.

We have,

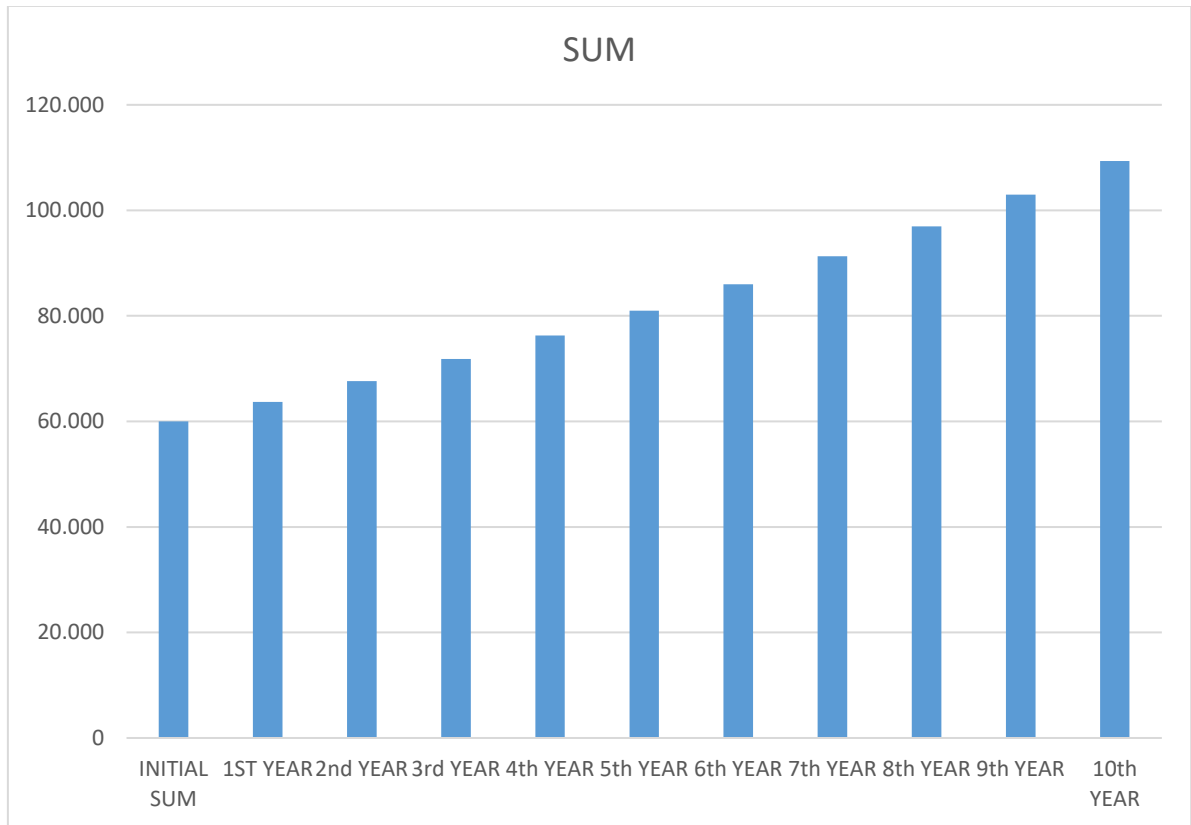
$$i=6/100$$

$$=0.06$$

So, the sum in 10 years = ae^{it}

$$=60,000 * e^{0.06 * 10}$$

$$=\$109327.128$$



Solution-5:

We have, population of one-horned rhino in 2024 (p)=752

And, rate of increase in its population (r)=16% per annum.

We know, unlimited growth function $f(t)=ae^{rt}$.

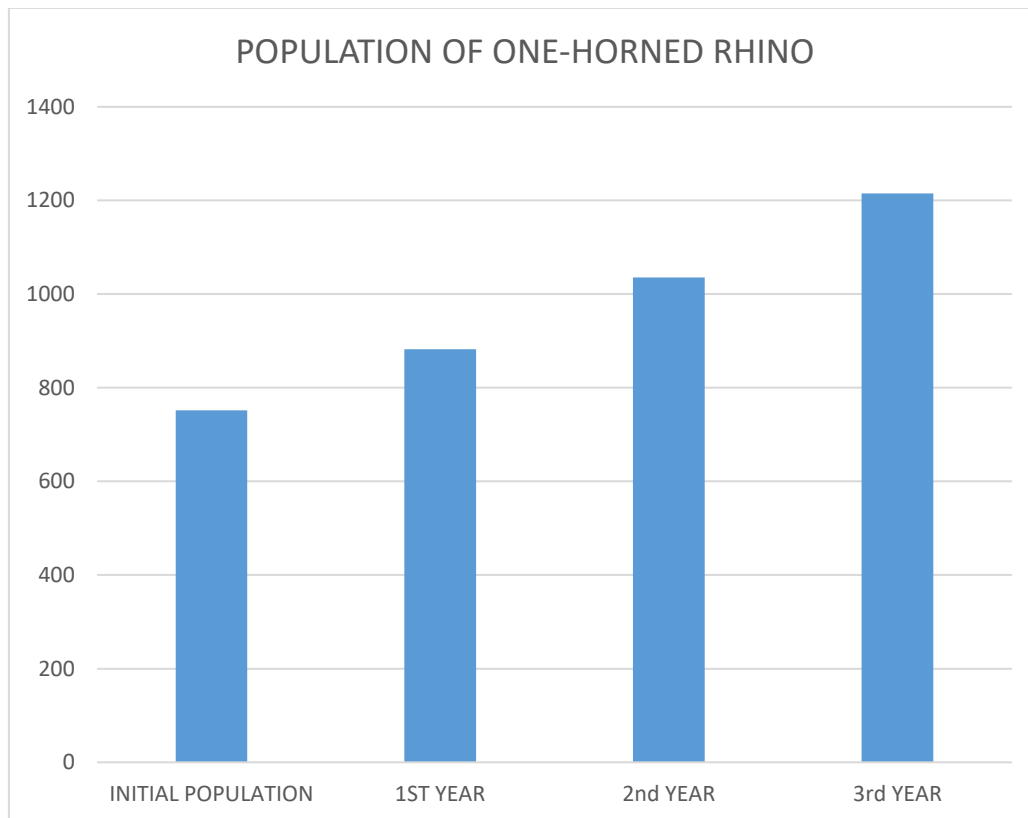
Let's examine its population in next 3 years.

So, the population of one-horned rhino in next 3 years = ae^{rt}

$$= 752 * e^{0.16 * 3}$$

$$= 1215.28795$$

$$= 1215$$



Conclusion

In problem (1), the final scrape value of airplane reaches to \$327492301.2 in 2 years with a depreciation rate of 10% per annum. In problem (2), the population of Nepal reaches 34250782 with a PGR of 0.92 in the of 10 years from 2024. In problem (3), the number of bacteria increases from 20,000,000,000 to 26,997,176,150 in 6 minutes with a growth rate of 100% per 20 minutes. In problem (4), the sum increased from \$60,000 to \$109,327.128 in 10 years with an interest rate of 6% per annum. In problem (5), the population of one-horned rhino increased from 752 to 1215 in 3 years at 16% population growth rate per annum.

It is concluded that exponential function has many application in real life. It helps us to easily determine the compounded growth and decay of any possible objects in given period of time with at a proportional rate. There appears to be much to study and understand in the world of exponential function and its applications to our world. I can now look and recognize that the Math in the courses have always had topics related to exponential function. It seems interesting that such a concept appears so frequently in different topics within mathematics.

References

- [1] Sahani, S.K.2023.The Effective Theory of Producer and Consumer Surplus; Positive Demand, Negative Supply, Central Asian Journal of Mathematics Theory and Computer Sciences,Vol.4,Issue 8,70-76.
- [2] Sahani, S.K. et al.2023.The International Pricing of Risk; An Empirical Investigation of the World Capital Market Structure, American Journal of Science and Learning Development,Vol,2,No.7,49-65.
- [3] Sahani, S.K. and Prasad, K.S.2023.Study and Analysis of Input| Output Analysis to Non-Linear Science ,ACCST Research Journal, Vol. XXL,No.1,22-30.
- [4] Sahani, S.K.2023.Youth on Employment and Job Creation in African Countries; Opportunities, Challenges, Characteristics Determinants and Performance of Self-Employment Among Them the Youth in African Countries, Central Asian Journal of Mathematics Theory and Computer Sciences, Vol.04, No.7,27-41.
- [5] Sahani, S.K. et al.2023.Economic Insights Unveiled: A Journey Through Input-Output Analysis in Non-Linear Mathematics, MIKAILALSYS Journal of Multidisciplinary Science, Vol.1,Issue 1,240-259.
- [6] Sahani, S.K. et al.2023.Unraveling the Interdependence of Input and Output Analysis in Business Sector; A Case Study, International Journal of Education, Management, and Technology, Vol.1,Issue 1, 27-45.
- [7] Allendoerfer, C., Oakely, C., & Kerr, D. Jr. (1977). *Elementary functions*. McGrawHill, Inc.
- [8] Bartlett, A., Fuller, R., Plano, V., & Rogers, J. (2004). *The Essential Exponential! For the future of our planet*. Lincoln, NE.: Center for Science, Mathematics & Computer Education.
- [9] Bogley, W. & Robson, Robby. (1999)
- [10] Larson, R. & Hostetler, R. (1979). *Calculus with analytic geometry*. Lexington, MA.: D.C Health and Company.
- [11] Lial, M & Miller, C. (1975). *Essential calculus with applications in business, biology and behavioral sciences*. Glenview, II.: Scott, Foresman and Company.
- [12] Sharma, S., Sahani, S., Chaudhary, B., Sahani, K., & Sah, N. (2023). Asan, The Traditional Market of Kathmandu Valley: Analyzing Market Trend and Consumer Buying Behavior of Such Market. EDUMALSYS Journal of Research in Education Management, 1(2), 215-226. <https://doi.org/10.58578/edumalsys.v1i2.2048>