

MATHEMATICAL ANALYSIS OF FERTILITY IN NEPAL USING ARRIAGA'S APPROACH

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Abstract

This article has judged how well the P/F changing ratio technique and Arriaga's approach forecast fertility levels in Nepal. The study's goal was accomplished and the logistic curve function was legalized by estimating national fertility rates. Using information from the 2011 till 2021 censuses together with Arriaga's methodology, the following factual investigates the calculation of fertility rates in Nepal. By using cutting edge indirect fertility estimation techniques on a national level, this study seeks to evaluate the fertility change in Nepal. Utilizing information from the 2011 and 2021 Censuses, fertility estimates were produced with an emphasis on last-year births. The findings show that Nepal's fertility rates have recently dropped.

Keywords: Fertility, National data, Arriga method, Changing P/F ratio, Logistic, Estimation, Projection and logistic curve

Introduction

The US Census Bureau demographer Eduardo Arriaga [1] has suggested an alternate method for calculating fertility when data from two surveys about live births, categorized by mother's age, are available. Mendes et.al. explained Brass and Arriaga's indirect fertility estimation methods are explained and used with data from the 1940–1980 Brazilian censuses. It is also taken into consideration if Arriaga's method can be used to survey data [2,13].

Numerous writers have made contributions to the field of fertilization, including Albuquerque et al. [3], Leite et al. [4], Arretx et al. [5], Juarez et al. [6], Frias et al. [7], and many others.

Objective of the study

To evaluate the accuracy of Arriaga's technique and the P/F changing ratio method in estimating Nepal's fertility.

Arriaga estimation

Prior fertility estimation methods relied on the premise of constant fertility, which is a shortcoming. Arriaga's method [9] on the other hand, does not necessitate this assumption. It uses a simulation model to study how, when fertility decreases, the number of children born to mothers under 35 varies linearly. By linearly interpolating data from two or more censuses, it forecasts the number of children born in the year before or after the census date. This method is more dependable when fertility is declining.

It is assumed that birth records are uniformly complete, that women under the ages of thirty or thirty-five accurately report all of their births, and that there is no age misreporting among women who are of reproductive age. It is anticipated that the average number of children born per woman between two reporting dates will be impacted linearly by changes in fertility.

The method uses factors (Z_i factors) to assess the consistency and quality of data; close-to-one factors indicate consistent information. This method requires no change when comparing children born to date with the cumulative fertility trend. It may also be applied, supposing continuous fertility in such cases, where data on births and fertility trends are limited to a single date [10,11,12].

Methodological Procedure

Step1. Determine the stated average parities for each survey. $P(i, j)$
 The average parities obtained from the first survey are written as $P(i, 1)$, while the average parities obtained from the second survey are written as $P(i, 2)$. The reported number of children ever born to women in age group (i) is divided by the total number of women in age group (i) in order to get these averages in both scenarios.

Mathematically, it can be expressed as:

$$P(i, j) = \frac{CEB(i, j)}{W(i, j)} \quad (1)$$

$CEB(i, j)$ = Number of children ever born in age group 'i' in the survey or census 'j'.

$W(i, j)$ = Total number of women in age group 'i' in the survey or census 'j'.

Here, 'i' represents the age group of women from 1 to 7, including the group 15-19, and 'j' corresponds to the survey or census number, with 1 denoting the first survey or census and 2 representing the second survey or census.

For the first survey:

$$P(i, 1) = \frac{CEB(i, 1)}{W(i, 1)} \quad (2)$$

And for the second survey:

$$P(i, 2) = \frac{CEB(i, 2)}{W(i, 2)} \quad (3)$$

$CEB(i, 1)$ is the reported number of children ever born to women in age group (i) in the first survey.

$W(i, 1)$ is the total number of women in age group (i) in the first survey.

Similarly, for the second survey:

$CEB(i, 2)$ is the reported number of children ever born to women in age group (i) in the second survey.

$W(i, 2)$ is the total number of women in age group (i) in the second survey.

Step2. The average parities are depending upon the length of inter-survey interval

The parity increment between survey for corresponding cohort is equal to $P(i + 1, 2) - P(i, 1)$ in Hypothetical cohort parities.

$$\text{€P}(i + 1) = P(i + 1, 2) - P(i, 1) \text{ for } i = 1, \dots, 6$$

$$\text{And } P(i, S) = \text{Error! } \text{€P}(j)$$

The parity increment $\text{€P}(i + 1)$ is for the youngest age group ($i = 0$), is directly set equal to $P(1, 2)$. When fertility is undergoing fast changes, this value of $\text{€P}(1)$ line up more closely with the period rates observed in the second survey rather than the midpoint of the interval.

$$\text{€P}(i + 2) = P(i + 2, 2) - P(i, 1) \text{ for } i = 1, \dots, 5.$$

Similarly, for age groups of odd numbers, the theoretical cohort parities are determined by adding the parity increments for other age groups with odd numbers. Thus,

$$P(2, S) = \text{€P}(2) = P(2, 2)$$

$$P(4, S) = \text{€P}(2) + \text{€P}(4)$$

$$\text{and } P(6, S) = \text{€P}(2) + \text{€P}(4) + \text{€P}(6)$$

where,

$$P(1, S) = \text{€P}(1) = P(1, 2)$$

$$P(3, S) = \text{€P}(1) + \text{€P}(3)$$

$$P(5, S) = \text{€P}(1) + \text{€P}(3) + \text{€P}(5)$$

$$\text{And } P(7, S) = \text{€P}(1) + \text{€P}(3) + \text{€P}(5) + \text{€P}(7).$$

Step 3. Calculation of age - precise fertility rates for inter - survey period $f(i)$

We first compute age-specific fertility rates for each survey year in order to get age-specific fertility rates for the inter-survey interval. To calculate this, divide the total number of women in age group (i) for each survey by the number of births that occurred in the year before to the survey.

$$f(i, 1) = \text{Error!}$$

$$f(i, 2) = \text{Error!}$$

The age specific fertility rates for inter-survey period are calculated as the average of observed of age-specific rates for two surveys.

$$f(i) = \text{Error!}$$

Where, $f(i, 1) = \text{ASFRRS for First survey}$

$f(i, 2) = \text{ASFRS for Second survey}$

Step 4. Calculation of cumulated fertility for hypothetical inter-survey cohort $\zeta(i)$.

The calculation of cumulated fertility is denoted by $\zeta(i)$.

$$\zeta(i) = 5 \text{Error! } f(j)$$

$f(j) = \text{inter-survey ASFR from younger age group up to the upper limit of the age group considered.}$

$\zeta(i) = \text{Cumulated ASFRs for a hypothetical inter-survey cohort.}$

Step 5. Estimation of average parity equivalent for the hypothetical inter-survey cohort $F(i)$

The average parity equivalent for hypothetical inter-survey cohort is $F(i)$. The average parity equivalent for hypothetical inter-survey cohort of age group (i) is estimated by inter-polarity of cumulated inter-survey age-specific fertility rates.

$$F(i) = \zeta(i-1) + a(i) f(i) + b(i) f(i+1) + c(i) \zeta(7)$$

$a(i), b(i), \& c(i)$ are coefficients of constant

Step 6. Calculation of inter-survey age-specific fertility rates for conventional five-year age group $f+(i)$

It is calculated by following relationship is

$$f^+(i) = [1 - w(i-1)] f(i) + w(i) f(i+1)$$

where $w(i) = x(i) + y(i) \text{Error!} + z(i) \text{Error!}$

$X(i), y(i)$ and $z(i)$ are constants for weighting factors.

Step 7. Calculation of adjusted inter-survey ASFRs for conventional five years age group

$$f^*(i)$$

It is calculated by multiplying the inter-survey ASFRs Adjusted for congenital five year age group by adjusted factor 'k'.

$$f^*(i) = k \times f^+(i)$$

where k is selected from consistent **Error!**

Then, $\text{TFR} = 5 \times \text{Error!} f^*(i)$

$$f(x) = \frac{1}{1 + e^{-x}}$$

In this equation, "e" represents Euler's number.

The logistic function was called by Pierre François Verhulst in 1844 or 1845[2] when he investigated its application to population expansion. Because of its S-shaped form, it is also known as the logistic curve. The study of these demographic concerns has benefited greatly from the use of mathematical and statistical models, with an emphasis on predicting fertility and mortality in relation to nuptiality. Logistic functions are frequently used to model diffusion processes. The sigmoid curve with equation: is the logistic function.

$$\frac{d}{dx} f(x) = f(x) \cdot (1 - f(x)). \quad (4)$$

The Total Fertility Rate (TFR) for each of the two country populations is predicted using this variance. The complement of the TFR—that is, the difference between the TFR for a particular year and the lower bound of the logistic curve—is the basis for defining the logistic model. This special specification ensures that corresponding modifications to the national TFR are likewise made to the regional TFR. This technique essentially guarantees that the predicted national TFR and the regional TFR are in line.

Estimation of fertility Arriaga method at national level, 2011

Table 1 displays the outcomes of applying Arriaga's approach to lifetime and current fertility in the censuses of 2011 and 2021. The number of children born and reported Age-Specific Fertility Rates (ASFR) are two measures of lifetime fertility indices that in Nepal generally start to show significant disparities after age 20. This is due to the fact that most Nepali women in the 20–24 and 25–34 age groups have higher fertility rates. Younger women in developing nations also show a similar tendency of increased average parity[8].

The statistics from the 2011 and 2021 censuses are used to classify women into reproductive age groups and create the Children Ever Born (CEB) data, which is the main focus of this study. The data for the 2011 census is organized in Table 2.1 based on age categories of women who are fertile at the national level. Age-Specific Fertility Rates (ASFR) and the cumulative ASFR pattern were used to calculate the corrected values. When the age group grows, these adjusted values exhibit a decreasing tendency.

Table 1: Estimation of ASFR based on 2011

Age	ASFR from CEB	Cum. ASFR	ASFR Pattern	Cum. ASFR pattern	Adj factors	Adj. Fertility f*
5-19	0.060	0.060	0.028	0.028	2.177	0.1085
20-24	0.179	0.240	0.096	0.124	1.932	0.2149
25-29	0.142	0.382	0.079	0.203	1.884	0.1253
30-34	0.082	0.464	0.043	0.246	1.888	0.0752
35-39	0.040	0.504	0.023	0.269	1.873	0.0417
40-44	0.017	0.521	0.010	0.279	1.864	0.0129
45-49	0.006	0.526	0.003	0.283	1.862	0.0014
TFR						2.631

Source: Census, 2011 and 2021.

The adjusting factor of P_2/F_2 values has been calculated to be (1.932) resulted adjusted ASFR for all age group and the adjusted TFR value was 2.631 in February 2011. The mean age of childbearing was 27.2 years.

Estimation of fertility from Arriaga method at national level 2021

Although women of reproductive age, or those between the ages of 15 and 49, are the focus of most fertility investigations, there are benefits to going beyond this traditional age range when it comes to fertility and childhood mortality problems. This method was used in the censuses of 2011 and 2021, when women who were twelve years of age or older were questioned about the number of live births (i.e., children born) they had ever had. In addition, women who were 50 years of age or younger than 12 or 14 at the time of the census were questioned on the number of children they had given birth to in the previous year (current births).

Analyzing the number of births within the previous 12 months (current births) and children ever born (CEB) that women of reproductive age report during a survey or census is the goal. This approach assumes that fertility has changed recently and that inaccuracies in the data on current births do not correspond to the mother's age. When used, age is a good way to gauge how accurate and consistent the two sets of data are. Arriaga (1983) has

provided an early analysis of CEB and current births reported at the 2021 census, illustrating how the P/F ratio approach may be used to assess fertility when it has been changing in a community.

Table 2: Estimation of ASFR based on 2021 census at national level

Age	ASFR from CEB	Cum. ASFR	ASFR Pattern	Cum. ASFR pattern	Adj factors	Adj. Fertility f*
5-19	0.055	0.031	0.055	0.031	1.780	0.0916
20-24	0.150	0.137	0.150	0.137	1.498	0.2186
25-29	0.114	0.226	0.114	0.226	1.411	0.1308
30-34	0.068	0.276	0.068	0.276	1.398	0.0785
35-39	0.017	0.298	0.017	0.298	1.354	0.0448
40-44	-0.003	0.307	0.009	0.307	1.307	0.0149
45-49	-0.001	0.400	0.004	0.311	1.286	0.0019
TFR						2.001

Source: Census, 2011 and 2021.

The modified values in Table 2 describes ASFR and cumulative ASFR pattern, based on age-specified data from the 2011 census of women in Province 1. As the age group increased, the corrected value was observed to be trending downward. In June 2021, the adjusted TFR value was 2.001, and the adjusted ASFR for every age group was determined by using the adjustment factor of P2/F2 values (1.498). The average age of reproductive age was 27.01 years.

Based on the data from the younger age groups, an adjustment factor was used to estimate the TFR, assuming that younger women tend to record their fertility more correctly. Age-based estimates of fertility for the 25–34 and 20–29 age groups showed significant differences from the other estimates, suggesting that women in their 20s were more trustworthy when reporting their fertility.

Conclusion

The content that is supplied talks about estimating Nepal's fertility rates using Arriaga's approach and data from the 2011 and 2021 censuses. The statement emphasizes that there

is a notable difference in lifetime fertility indices in Nepal beyond the age of 20, with women between the ages of 20 and 24 and 25 and 34 having greater fertility rates. By the end of her reproductive years, the ordinary Nepalese woman will likely have given birth to three children, bucking the tendency that begins earlier in the reproductive period.

The text acknowledges potential issues such as misreporting of children's ages and emphasizes that the accuracy of recording births in the last 12 months should be consistent across all age groups.

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