APPLICATIONS OF EXPONENTIAL FUNCTION IN OUR REAL LIVES

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Article Info:
Submitted: Mar 17, 2024
Revised: Apr 10, 2024
Accepted: Apr 13, 2024
Published: Apr 17, 2024

Abstract

This project delves into the real-life applications of exponential functions, analysing their significance in various fields. Through a comprehensive study, this project aims to provide insights into the practical implications of exponential functions and their role in modelling natural phenomena. This work is motivated by the works of [1-15,22].

Keywords: Exponential functions, Real-life applications, Modelling, Growth, Decay, Bitcoin valuation

Introduction

Brief overview of exponential functions and their significance in mathematics.

Exponential functions are a fundamental concept in mathematics that describe how things grow or shrink at an ever-increasing or decreasing rate. In simpler terms, they show how something multiplies rapidly or diminishes rapidly over time. Imagine you have a bank account with compound interest, where your money grows not just based on what you
originally put in, but also on the interest that accumulates over time. That's an example of exponential growth. On the flip side, if you have a substance that decays over time, like a radioactive element, the rate at which it breaks down gets faster and faster, which is an example of exponential decay.

These functions are significant in mathematics because they help us understand and predict many natural phenomena, from population growth to the spread of diseases, from the decay of materials to the appreciation of investments. They provide us with powerful tools for modeling real-world situations and making informed decisions in various fields of science, economics, and engineering (see [17-21]).

Definition:

1. **Exponential function**: it refers to the function of form \( F(x) = a^x \) where \( a \neq 1, a > 0 \) and \( x \) belongs to real number. For example, \( 2^x, 3^x, 4^x \) etc. following are the basic form of exponential function. The special form of exponential function is \( e \) whose value is equal to \( 2 = 7182818284 \).

\[
e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n:
\]

This \( e \) is a special type of exponential function which is basically derived from compound interest. However, it can be used for different purpose: This exponential function \( e \) helps to find out the growth of anything we can say that compound growth. Similarly, it also helps in finding compound depreciation.

Some definitions can be made with help of exponential function are shown below;

i. **Limitless growth function**: A function of the form \( y = ae^{rt} \) in which \( a \) and \( r \) are considered as constant is known as limitless growth function.

ii. **Limitless decay function**: A function of the form \( y = ae^{-rt} \) where \( a \) and \( r \) are considered as constant is known as limitless decay function.

iii. **Limited growth function**: The function of the form \( y = M(1-e^{-rt}) \), where \( M \) and \( r \) are considered constant, is known as limited growth function.

**Discussions**

Before going up to the problem I’d like to give you the information and concept about Bit coin Valuation.
What is exactly Bit coin valuation?

Bit coin valuation refers to the process of determining the worth of Bit coin, a decentralized digital currency.

Let’s summarize the Bit coin valuation with the help of a hypothetical example:

Suppose the price of Bit coin follows an exponential growth model, where the value of Bit coin doubles every 6 months. If the current price of Bit coin is $50,000, what would be its projected value after 2 years?

Solution:

Let’s denote:

- \( P_0 \) as the initial price of Bit coin ($50,000 in this case)
- \( r \) as the Growth rate (which is doubling every 6 month, so \( r = 100\% = 1 \) in decimal form)
- \( t \) as the time in years

The exponential growth formula is given by:

\[
P(t) = P_0 \times (1 + r)^t
\]

Given that the value doubles every 6 months, we can express the time in terms of years:

2 years + 4 half-years

So, substituting the values into the formula:

\[
P(2) = 50,000 \times (1 + 1)^4
\]

\[
P(2) = 50,000 \times 2^4
\]

\[
P(2) = 50,000 \times 16
\]

\[
P(2) = 800,000
\]

Therefore, the projected value of bit coin after 2 years would be $800,000.

Problem: 1

An investor purchased 10 Bitcoin’s in 2017 at a price of $5,000 per Bitcoin. If the average annual growth rate of Bitcoin's value is 200% over the next four years, what is the projected value of the investor's Bitcoin holdings in 2021? Furthermore, how does inflation affect the real value of Bitcoin investments?

Solution 1:
Using the formula for compound interest:

$$A = P \left(1 + r\right)^n$$

- $A$ = the future value of the investment
- $P$ = the principal investment amount (initial value of Bitcoin holdings)
- $r$ = the annual interest rate (growth rate of Bitcoin’s value)
- $n$ = the number of years

Given:

- $P = 10$ Bitcoins
- $r = 200\% = 2$ (expressed as a decimal)
- $n = 4$ years

Calculating the future value ($A$) of the investment:

$$A = 10 \left(1 + 2\right)^4$$
$$A = 10 \left(1 + 2\right)^4$$
$$A = 10 \left(1 + 16\right)$$
$$A = 10 \times 17$$

Therefore, the project value of the investor’s Bitcoin holdings in 2021 is 170 Bitcoins.

Tosses the impact of inflation on the real value of Bitcoin investments, we need to compare the purchasing power of Bitcoin overtime. Since Bitcoin often consider ahead against inflation, its real value may increase relative of at currencies experiencing inflationary pressures.

Conclusion:

Exponential functions provide valuable insights into the growth trajectory of Bitcoin value, allowing investors to make informed decisions about crypto currency investments. By understanding the principles of compound interest and inflation, investors can assess the real value of Bitcoin holdings and mitigate risks associated with market volatility. As the crypto currency market continues to evolve, advanced understanding of exponential functions and their applications in Bitcoin valuation is essential for navigating this dynamic investment landscape.
Problem 2:

A person invests 0.5 Bitcoin in 2022 when its value is 10,000,000 rupees. The Bitcoin market price is expected to grow exponentially by 20% annually. Evaluate the value of the Bitcoin after 3 years and its real value after considering inflation rates.

Solution 2:

Chart for inflation in Nepal:

<table>
<thead>
<tr>
<th>Year</th>
<th>Inflation rate</th>
<th>Approximate rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>2022</td>
<td>8%</td>
<td>7.5%</td>
</tr>
<tr>
<td>2023</td>
<td>7%</td>
<td>6.5%</td>
</tr>
<tr>
<td>2024</td>
<td>6%</td>
<td></td>
</tr>
</tbody>
</table>

Bitcoin Appreciation:

Initial investment: 0.5 Bitcoin

2022: 0.5 Bitcoin * (1 + 20%) = 0.6 Bitcoins

2023: 0.6 Bitcoins * (1 + 20%) = 0.72 Bitcoins

2024: 0.72 Bitcoins * (1 + 20%) = 0.864 Bitcoins

After three years, the Bitcoin is worth 0.864 Bitcoins.

Inflation Impact:

Value of 10,000,000 rupees in 2022 due to inflation:

2023: 10,000,000 + 7.5% = 10,750,000 rupees

2024: 10,750,000 + 6.5% = 11,462,500 rupees

By 2024, the value of 10,000,000 rupees from 2022 is equivalent to 11,462,500 rupees.

Comparison:

Bitcoin value after 3 years: 0.864 Bitcoins

Value of 10,000,000 rupees after 3 years due to inflation: 11,462,500 rupees

Surplus amount considering inflation: 1,462,500 rupees.
Problem 3:
A person invests 3,000,000 rupees in stocks in 2020. The stock portfolio appreciates by 18% annually. Compute the value of the stock after 3 years and its real value after considering inflation rates.

Details:

Chart for inflation in Nepal:

<table>
<thead>
<tr>
<th>Year</th>
<th>Inflation rate</th>
<th>Approximate rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>2020</td>
<td>5%</td>
<td>5.05%</td>
</tr>
<tr>
<td>2021</td>
<td>4%</td>
<td>4.09%</td>
</tr>
<tr>
<td>2022</td>
<td>8%</td>
<td></td>
</tr>
</tbody>
</table>

Solution 3:

Stock Appreciation:
Initial investment: 3,000,000 rupees

2020: 3,000,000 + 18% = 3,540,000 rupees
2021: 3,540,000 + 18% = 4,183,200 rupees
2022: 4,183,200 + 18% = 4,932,896 rupees

After three years, the stock is worth 4,932,896 rupees.

Inflation Impact:
Value of 3,000,000 rupees in 2020 due to inflation:

2021: 3,000,000 + 5.05% = 3,151,500 rupees
2022: 3,151,500 + 8% = 3,405,220 rupees

By 2022, the value of 3,000,000 rupees from 2020 is equivalent to 3,405,220 rupees.

Comparison:
Stock value after 3 years: 4,932,896 rupees
Value of 3,000,000 rupees after 3 years due to inflation: 3,405,220 rupees
Surplus amount considering inflation: 1,527,676 rupees.
Problem 4:
If Bitcoin's value doubles every year and is currently valued at $50,000, what will its value be in 5 years?

Solution 4:
Using the formula for exponential growth:
\[ \text{Value} = \text{Initial Value} \times (1 + \text{Growth Rate})^{\text{Number of Years}} \]

\[ \text{Value} = 50,000 \times (1 + 1)^5 \]
\[ \text{Value} = 50,000 \times 32 \]
\[ \text{Value} = 1,600,000 \]

Problem 5:
A person invests 3 Bitcoins in 2024 when its value is 15,000,000 rupees. The Bitcoin market price is expected to grow exponentially by 22% annually. Compute the value of the Bitcoin after 2 years and its real value after considering inflation rates.

Details:
Chart for inflation in Nepal:

<table>
<thead>
<tr>
<th>Year</th>
<th>Inflation rate</th>
<th>Approximate rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>2024</td>
<td>6%</td>
<td>5.5%</td>
</tr>
<tr>
<td>2025</td>
<td>5%</td>
<td>4.5%</td>
</tr>
<tr>
<td>2026</td>
<td>4%</td>
<td></td>
</tr>
</tbody>
</table>
Solution 5:

Bitcoin Appreciation:

Initial investment: 3 Bitcoins

2024: 3 Bitcoins * (1 + 22%) = 3.66 Bitcoins

2025: 3.66 Bitcoins * (1 + 22%) = 4.47412 Bitcoins

2026: 4.47412 Bitcoins * (1 + 22%) = 5.46722544 Bitcoins

After two years, the Bitcoin is worth 5.46722544 Bitcoins.

Inflation Impact:

Value of 15,000,000 rupees in 2024 due to inflation:

2025: 15,000,000 + 5.5% = 15,825,000 rupees

2026: 15,825,000 + 4.5% = 16,554,750 rupees

By 2026, the value of 15,000,000 rupees from 2024 is equivalent to 16,554,750 rupees.

Comparison:

Bitcoin value after 2 years: 5.46722544 Bitcoins

Value of 15,000,000 rupees after 2 years due to inflation: 16,554,750 rupees

Surplus amount considering inflation: 1,554,750 rupees.

References

4. Sahani, S.K. (2023). Youth on Employment and Job Creation in African Countries; Opportunities, Challenges, Characteristics Determinants and Performance of Self-


