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CONTRACTION TYPE EXPANSIVE MAP ON COMPLEX VALUED METRIC SPACE WITH FIXED POINTS

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Abstract

According to the present paper, the two self mappings that satisfy contraction type conditions of expansive in complete complex valued metric space reveals that these mappings have some common fixed points. Furthermore, the paper provides generalizations and extensions of well-known results from the existing literature which further expands our understanding of this topic. Some illustrative examples are given to help us obtain results.

Keywords: Expansive Mapping, Stationary (Fixed) Point, Common Stationary Point, Complete Complex Valued Metric Space

INTRODUCTION

A significant result regarding contractions in complete metric spaces was established by Banach, S. (1922). Initially, Machuca (1967) developed a model of fixed points in expansive mappings, later extended by Jungck (1976) to address numerous other types of expansive mappings. Wang et al. (1984) used expanding mappings to demonstrate theorem of fixed point under complete metric spaces. As early , Daffer and Kaneko (1992) proved a

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pair of mappings by defining expanding status as under complete metric spaces at a common stationary(or fixed) point.

Recently, Azam et al. (2011) and Rauzkard and Imdad (2012) explored the field of complex-valued metric spaces (CVMS). By studying sequence properties and using contraction maps to establish theorems for common stationary points, they were able to gain fresh insights into fixed points within these spaces. Previous research in this field has resulted in a number of broad conclusions, as cited in sources [Ahmad and Kumar, P.(2013), Abbas, M. et al. (2013), Ahmad, J. et al.(2013), Kutbi, M. A. et al.(2013), Klin-Cam, C. and C. Suanoom, (2013). Sintunavarat W., and Kumar, P. (2012), Sintunavarat, W., Cho, Y. J. and Kumar, P. (2013), Sitthikul, K., and Saejung, S. (2012), Senthil T. Kumar and Jahirhuss, R. (2014), Tiwari, S.K., and Sahu, T. (2014), Tiwari, S. K. and Dharmendra Das (2017), Sonant, B. and Tiwari, S.K., (2021). Tiwari, S. K. and Sonant, B., (2022)].

Furthermore, the manuscript provides generalizations and extensions of well-known results from the existing literature which further expands our understanding of this topic.

PRELIMINARY NOTES

As a starting point, let's discuss the features of cone-metric spaces, including their definition.

Presume that, ξ_1, ξ_2 be a any two complex numbers with their the set $\mathbb{C}$ and $\leq$ be ordered partially under $\mathbb{C}$, then consider as below:

$$\xi_1 \preccurlyeq \xi_2 \text{ if an only if } \mathbb{R}(\xi_1) \leq \mathbb{R}(\xi_2) \text{ and } Im(\xi_1) \leq Im(\xi_2)$$

As a result, we are able to deduce $\xi_1 \preccurlyeq \xi_2$ whenever anyone circumstances are accomplished

(i) $Re(\xi_1) = Re(\xi_2), Im(\xi_1) < Im(\xi_2);$

(ii) $Re(\xi_1) < Re(\xi_2), Im(\xi_1) = Im(\xi_2);$

(iii) $Re(\xi_1) < Re(\xi_2), Im(\xi_1) < Im(\xi_2);$

(iv) $Re(\xi_1) = Re(\xi_2), Im(\xi_1) = Im(\xi_2);$

When $\xi_1 \preccurlyeq \xi_2$ and $\xi_1 \neq \xi_2$, then condition of (i), (ii) and (iii) are gratified , and if (iii) is only rewarding , then correspond to $\xi_1 \prec \xi_2$. Take note of the fact that,

(a) If $0 \precsim \xi_1 \precnsim \xi_2$, then $|\xi_1| < |\xi_2|$,

(b) if $\xi_1 \precsim \xi_2$ and $\xi_2 \prec \xi_3, then \xi_1 \prec \xi_3$,

(c) $\alpha, \beta \in \mathbb{R}$ and $\alpha \le \beta$, then $\alpha\xi \precsim \beta\xi, \forall \xi \in \mathbb{C}$.

Definition 1[3]**:** Setting a operator as function $d: Y \times Y \to \mathbb{C}$ such that gratify some circumstances as below:

(i) $0 \precsim d(a,b), \forall\ a,b \in Y$, and $(a,b) = 0 \Leftrightarrow a = b$;

(ii) $d(a,b) = d(b,a), \forall a,b \in Y$;

(iii) $d(a,b) \precsim d(a,c) + d(c,b). \forall a,b,c \in Y$.

Then we can say pair (Y,d) is well-known CVMS where d be CVM on Y.

Example 2: Presuming that an operator $d: Y \times Y \to \mathbb{C}$ on (Y,d) and $Y = \mathbb{C}$ specified by

$$d(\xi_1,\xi_2) = e^{it}|\xi_1 - \xi_2|, \text{ where } \xi_1 = (a,b) \text{ and } \xi_2 = (c,d) \text{ at } \left[0,\frac{\pi}{2}\right].$$

As condition (Y,d) be a CVMS.

Definition 3 [3]**:** Assume we have $\{\lambda_k\}$ been a sequence on (Y,d) and $\lambda \in Y$.Thenwe can say that

(i) $\{\lambda_k\}$ is convergent to λ, through $c \in \mathbb{C}$ with $0 \prec c$, we have $k \in \mathbb{N}$ such that $d(\lambda_k,\lambda) < c. \forall\ \lambda > k$, Presented by $\lambda_k \to \lambda$ as $k \to \infty$ or $\lim_{k\to\infty} \lambda_k = \lambda$;

(ii) Additionally, $\{\lambda_k\}$ is sequence of Cauchy , through $c \in \mathbb{C}$ and $0 \prec c$, we have $k \in \mathbb{N}$ such that , $d(\lambda_k,\lambda_{k+l}) < c, \forall \lambda > k$ and $l \in \mathbb{N}$;

(iii) When each Cauchy sequence converges, in (Y,d), so, (Y,d) will be considered Complete CVMS.

Lemma 4 [3]: Presume that $\{\lambda_k\} \in Y$ is a sequence on(Y,d), then sequence $\{\lambda_k\}$ converges to λ if that's the case $|d(\lambda_k,\lambda)|$, like $k \to \infty$.

Lemma 5 [23]: Presume that $\{\lambda_k\} \in$ Y is a sequence on(Y,d), then sequence $\{\lambda_k\}$ is a Cauchy sequence if and only if $|d(\lambda_k,\lambda_{k+l})| = 0$, where $l \in \mathbb{N}$.

Lemma 6 [20] If $\{\lambda_k\}$ converges to $\lambda \in Y$, and $\{\delta_k\}$ converges to $\delta \in Y$ over (Y,d), then

$$\lim_{k\to\infty} d(\lambda_k,\delta_k) = d(\lambda,\delta);\ \lim_{k\to\infty} |d(\lambda_k,\delta_k)| = |d(\lambda,\delta)|.$$

In particular, for any fixed element $u \in Y$, the following holds

$$\lim_{k\to\infty} d(\lambda_k,u) = d(\lambda,u);\ \lim_{k\to\infty} |d(\lambda_k,u)| = |d(\lambda,u)|.$$

Lemma 7: Assume that $\{\lambda_k\}$ be a Cauchy sequence on (Y, d). If $\exists\ 0 \leq h < 1$ through for all $k \in N$

$$d(\lambda_{k+1}, \lambda_k) \leq hd(\lambda_k, \lambda_{k-1}).$$

MAIN RESULTS

Theorem 8: Suppose that the two continuous onto mappings L_1 and $L_2 : Y \to Y$ on complete CVMS (Y, d) . Suppose $\eta \geq -1$, $\beta + \gamma \geq 0, \alpha + \gamma \geq$ and ½ $< \gamma \leq 1$, are constants, with $\alpha + \beta + \gamma + \eta > 1$. For all $\lambda, \delta \in Y$, the condition holds as follows

$$d(L_1\lambda, L_2\delta) + \alpha d(\lambda, L_1\lambda) + \beta d(\delta, L_2\delta) + \gamma[\, d(\lambda, L_1\lambda) + d(\delta, L_2\delta)] \succcurlyeq \eta\, d(\lambda, \delta)\ldots$$
(8.1)

Then, it can be determining that L_1 and L_2 are common stationary point in Y as unique .

Proof: Consider $\lambda_0 \in Y$. We also have two onto function L_1 and L_2, $\exists\ \lambda_1, \lambda_2 \in X$ such that

$$L_1(\lambda_1) = \lambda_0, \text{and } L_2(\lambda_2) = \lambda_1.$$

From here, we can define two sequences$\{\lambda_{2k}\}$ and $\{\lambda_{2k+1}\}$ by

$$\lambda_{2k} = L_1\lambda_{2k+1}, \text{ some } k = 0,1,2,3 \ldots$$

and

$$\lambda_{2k+1} = L_2\lambda_{2k+2}, \text{ some } k = 0,1,2,3 \ldots$$

Remark that, $\lambda_{2k} = \lambda_{2k+1}$ for $k \geq 1$, then it is fixed point of L_1 and L_2.

Currently put $\lambda = \lambda_{2k+1}$ and $\delta = \lambda_{2k+2}$, in (8.1) we gain

$$(\eta - \beta - \gamma)d(\lambda_{2k+2}, \lambda_{2k+1}) \succcurlyeq 0.$$

Hence,

$|d(\lambda_{2k+2}, \lambda_{2k+1})| \geq 0 \Rightarrow d(\lambda_{2k+2}, \lambda_{2k+1}) = 0$. Since $\beta + \gamma > \eta$. So, $\lambda_{2k+2} = \lambda_{2k+1}$.

Thus $L_1\,\lambda_{2k+2} = \lambda_{2k} = \lambda_{2k+1} = L_2\lambda_{2k+2} = L_2\lambda_{2k+1}$. Implies that λ_{2k+1} is common fixed point of $L_1$and L_2.

If there exists k such that $\lambda_{2k+2} = \lambda_{2k+1}$, then we put $\lambda = \lambda_{2k+3}, \delta = \lambda_{2k+2}$ in (8.1.) we get

$$d(\lambda_{2k+2}, \lambda_{2k+1}) + \alpha d(\lambda_{2k+3}, \lambda_{2k+2}) + \beta d(\lambda_{2k+2}, \lambda_{2k+1})$$
$$+\gamma[(\lambda_{2k+3}, \lambda_{2k+2}) + (\lambda_{2k+2}, \lambda_{2k+1})] \succcurlyeq \eta d(\lambda_{2k+3}, \lambda_{2k+2})$$

This implies $(\eta - \alpha - \gamma)d(\lambda_{2k+3}, \lambda_{2k+2}) \succcurlyeq 0$, hence $|d(\lambda_{2k+3}, \lambda_{2k+2})| \geq 0$.

Since $\beta + \gamma > \eta$. Therefore, $|d(\lambda_{2k+3}, \lambda_{2k+2})| = 0 \Rightarrow d(\lambda_{2k+3}, \lambda_{2k+2}) = 0$,

$i.e\ \lambda_{2k+3=}\lambda_{2k+2}$. Hereby , $L_2\ \lambda_{2k+2} = \lambda_{2k+1} = \lambda_{2k+2} = L_1\lambda_{2k+3} = L_1\lambda_{2k+2}$.

Implies that, λ_{2k+2} is common fixed point of $L_1$and L_2.

Hence from now on, we presume that, $\lambda_{2k} \neq \lambda_{2k+1}$, $\forall k = 0,1,2, \dots \dots$

Now taking $\lambda = \lambda_{2k+1}$, $\delta = \lambda_{2k+2}$ in (8.1), we get

$$d(\lambda_{2k+1}, \lambda_{2k}) \succcurlyeq \frac{\eta-\beta-\gamma}{1+\alpha+\gamma}\ d(\lambda_{2k+1}, \lambda_{2k+2})\dots$$

Hence,

$$|d(\lambda_{2k+1}, \lambda_{2k+2})| \leq \frac{1+\alpha+\gamma}{\eta-\beta-\gamma}\ |d(\lambda_{2k}, \lambda_{2k+1})| \tag{8.2}$$

Similarly, we can obtain that

$$|d(\lambda_{2k+3}, \lambda_{2k+2})| \leq \frac{1+\beta+\gamma}{\eta-\beta-\gamma}\ |d(\lambda_{2k+2}, \lambda_{2k+1})| \tag{8.3}$$

Now let $h = max\left\{\frac{1+\beta+\gamma}{\eta-\alpha-\gamma}, \frac{1+\alpha+\gamma}{\eta-\beta-\gamma}\right\}$. Then $0 < h < 1$, and from (8.2) and (8.3)

$$|d(\lambda_{2k+2}, \lambda_{2k+1})| \leq h\ |d(\lambda_{2k+1}, \lambda_{2k})| \tag{8.4}$$

Hence, by Lemma 7, $\{\lambda_{2k}\}$ corresponds to the Cauchy sequence over (Y, d) as well as also complete $\exists\ \lambda^* \in Y$ such that

$\lambda_{2k} \to \lambda^*$, as $k \to \infty$, given L_2 is continuous and onto mapping there exists a point λ^{**} in X by through

$$\lambda^{**} \in L_2^{-1}(\lambda^*)\ i.e.\ \lambda^* = L_2(\lambda^{**}), \lambda_{2k+1} \to L_2\lambda^{**} \text{ and } \lambda_{2k} \to L_2\lambda^{**}.$$

Taking $\lambda = \lambda_{2k+1}$ and $\delta = \lambda^{**}$ in (8.1.) we get

Now Consider

$$d(\lambda_{2k}, L_2\lambda^{**}) + \alpha\ d(\lambda_{2k+1}, \lambda_{2k}) + \beta d(\lambda^{**}, L_2\lambda^{**}) + \eta[d(\lambda_{2k+1}, \lambda_{2k}) + d(\lambda^{**}, L_2\lambda^{**})]$$
$$\succcurlyeq \eta d(\lambda_{2k+1}, \lambda^{**})$$

Hence $|d(\lambda_{2k}, L_2\lambda^{**})| + \alpha\ |d(\lambda_{2k+1}, \lambda_{2k})| + \beta|d(\lambda^{**}, L_2\lambda^{**})|$

$$+\eta[|d(\lambda_{2k+1}, \lambda_{2k}) + d(\lambda^{**}, L_2\lambda^{**})|]$$
$$\succcurlyeq \eta|d(\lambda_{2k+1}, \lambda^{**})|$$

Let $k \to \infty$, then by Lemma 6, the above inequality becomes

$(\beta + \gamma - \eta)|d(L_2\lambda^{**}, \lambda^*)| \geq 0$ .Since $\eta \leq \beta + \gamma$. so, $|d(L_2\lambda^{**}, \lambda^*)| = 0$.

$\Rightarrow d(L_2\lambda^{**}, \lambda^*) = 0 \Rightarrow L_2\lambda^{**} = \lambda^*$. In similar manner, we can prove that $L_1\lambda^{**} = \lambda^*$.

As results, $L_1\ \lambda^{**} = \lambda^* = L_2\ \lambda^{**}$. Thus both L_1 and L_2 have λ^* as their common stationary points.

Now to demonstrate uniqueness, assume that, $v = L_1 v = L_2 v$ holds, where v be additional common fixed point of L_1 & L_2. Now Taking $\lambda = \lambda^{**}$ and $\delta = v$ in (8.1), we get

$$1-(\eta+\alpha+\beta)d(\lambda^{**}, v) \succcurlyeq 0.$$

Therefore, $1-(\eta+\alpha+\beta)|d(\lambda^{**}, v)| \geq 0$. Because$(\eta+\alpha+\beta) < 1$. So, $|d(\lambda^{**}, v)| = 0$.

$\Rightarrow d(\lambda^{**}, v) = 0 \Rightarrow \lambda^{**} = v$. Thus both L_1 and L_2 have λ^* as their common stationary points with unique. Complete proof of this theorem.

The following Corollary obtain, If $\alpha = \beta = 0$ and $\gamma = \alpha$ and $\eta = \beta$ in Theorem 8.

Corollary 9: Suppose that the two continuous onto mappings L_1 and $L_2 : X \to X$ on (Y, d). Suppose $\alpha \geq 0$, is constants, and $\alpha + \beta > 1$. For all $\lambda, \delta \in Y$, the condition holds as follows

$$d(L_1\lambda, L_2\delta) + \alpha[\, d(\lambda, L_1\lambda) + d(\delta, L_2\delta)] \succcurlyeq \beta\, d(\lambda, \delta) \ldots \quad (9.1)$$

Then, It has been determined that there exists a unique stationary point in Y that is common to L_1 and L_2 .

The following Corollary obtain, if $\alpha = 0$ in corollary 9.

Corollary 10: Suppose that the two continuous onto mappings L_1 and $L_2 : Y \to Y$ on (Y, d) Suppose $\eta \geq -1$. For all $\lambda, \delta \in Y$, the condition holds as follows

$$d(L_1\lambda, L_2\delta) \succcurlyeq \eta\, d(\lambda, \delta) \ldots \quad (10.1)$$

Then, It has been determined that there exists a unique stationary point in Y that is common to L_1 and L_2 .

To clarify the result mentioned earlier, we provide an instance.

Example 11: Suppose $Y = [0, \infty)$ with function $d: Y \times Y \to \mathbb{C}$ define by

$$d(\lambda, \delta) = |\lambda - \delta| e^{i\theta}, \theta = tan^{-1}\left|\frac{\delta}{\lambda}\right|.$$

Then (Y, d) represent to be metric space with complex valued. Now considering

$$\eta(\lambda, \delta) = \begin{cases} 1, \text{ if } \lambda, \delta \in [0,1] \\ \frac{3}{2}, \quad \text{Otherwise.} \end{cases}$$

Now, describe a function $L_1, L_2: Y \to CB(Y)$ by

$$\begin{cases} [0, \frac{\lambda}{5}], \text{ if } \lambda, \delta \in [0,1] \end{cases}$$

$L_1(\lambda) =$

$[2\lambda, 3\lambda]$, Otherwise

and

$$L_2(\lambda) = \begin{cases} [0, \frac{\lambda}{10}], & \text{if } \lambda, \delta \in [0,1] \\ [3\lambda, 4\lambda], & \text{Otherwise} \end{cases}.$$

We prove that all condition of our Corollary 9 and 10 with main Theorem 8 are satisfied.

If $\lambda, \delta \in [0,1]$. The main theorem for expanding type contractive condition becomes easy to understand under $\lambda = 0 = \delta$.

Assume, with sacrificing generalization, that every instance of $\lambda, \delta \geq 0$ and $\lambda < \delta$.Then

$d(\lambda, \delta) = |\delta - \lambda| e^{i\theta}$, $d(\lambda, L_1\lambda) = \left|\lambda - \frac{\lambda}{5}\right| e^{i\theta}$, $d(\delta, L_2\delta) = \left|\delta - \frac{\delta}{10}\right| e^{i\theta}$, and

$d(L_1\lambda, L_2\delta) = \left|\frac{\lambda}{5} - \frac{\delta}{10}\right| e^{i\theta}$. Clearly for $\eta = \frac{1}{5}$, we have

$\left|\frac{\lambda}{5} - \frac{\delta}{10}\right| \geq \frac{1}{5} |\lambda - \delta|$.Thus $d(L_1\lambda, L_2\delta) \geq \eta d(\lambda, \delta)$.

Hence, Corollary 10 and the other criteria of Corollary 9 are satisfied, along with inequality 8.1 of Theorem 8.

CONCLUSION

We discuses and explain Theorems 3.11 of the literature Yong- Jie, Piao, (2015) for a contractive mapping of expansive type on CVMS with a common stationary (fixed) point as unique. Some important corollaries are obtained under this contractive condition. And some illustrative examples are given to help us obtain results.

REFERENCES

Azam, A., Fisher, B. Khan, M., Common fixed point theorems in complex valued metric Spaces, Numer. Fun. Anal. Opti., Vol. 32, No. 3, 2011, 243-353 https://doi.org/10.1080/01630563.2011.533046.

Ahmad, A. J Klin-Eam, C., Azam, A. (2023). Common fixed point theorems for multi valued mapping in complex valued metric spaces with applications, Abstract and Applied Analysis, Vol.2013, Article ID 854965, 1-12. https://doi.org/10.1155/2013/854965

Abbas, M. Arshad, M. and. Azam, M. (2013). Fixed points of asymptotically regular mappings in complex valued metric spaces, Georgian Maths. J. 20(2), 213-221. https://doi.org/10.1515/gmj-2013-0013

Banach S. Surles Operation Dans Les Ensembles Abstraits Et Leur Application A Equations Integrals. Fund. Math. 3, 1922. P. 133-181. https://doi.org/10.4064/fm-3-1-133-181.

Daffer P.Z. and Kaneko, H.(1992). On expansive mappings, Math Japon, 37, 1992.

Jungck, G. (1976).Commuting mappings and fixed points. Amer. Math. Monthly, 83, 261-263. https://doi.org/10.1080/00029890.1976.11994093.

Kutbi, M. A. Azam, A., Ahmad, J. and Bari Di, C. (2013) Some common coupled fixed point results or generalized contractions in complex-valued metric spaces,
J. Appl. Math.2013, Article ID 352927. https://doi.org/10.1155/2013/352927.

Klin-Cam, C. and C. Suanoom, (2013).Some common fixed point theorems for generalized contractive type mapping a complex-valued metric space, Abstr.
Appl. Anal. 2013, Article ID 604215. DOI:10.1155/2013/604215

Machuca, R.(1967). A coincidence theorem. Amer. Month. Monthly, 74, 1967.

Rauzkard, F. and Imdad, M., Some Common fixed point theorem on complex valued
Metric spaces, Compt. Math. Appl. 64(6), 2012, 1866-1874. doi:10.1016/j.camwa.2012.02.063

Sintunavarat, W. and Kumam, P. (2012). Generalized common fixed point theorems in Complex valued metrics spaces an applications, J. Ine. App. 2012 (84). doi:10.1186/1029-242X-2012-84.

Sintunavarat, W., Cho, Y. J and Kumam, P. (2013) Urysohn, Integral equations approach by common fixed points in complex-valued metric spaces, Adv. Differ. Equ. 2013 (49),1- 14.

Sitthikul, K. and Saejung, S.(2012). Some fixed point theorems in complex-valued metric spaces, Fixed Point Theory Appl. 2012, 189.

Senthil T. Kumar and Jahirhuss, R. (2014). Fixed point and Common fixed Theorems generalized contractive type mapping in complex-valued metric spaces, J. Math. Comp. Sci. 4 (2014), 639-648. doi:10.1186/1687-1812-2012-189.

Sonant, B. and Tiwari, S.K., (2021).Unique common fixed point results for rational contraction in complex valued metric space,Int. J. of math. and Comp. Research., Volume-09 (11), 2493-2505. DOI: 10.47191/ijmcr/v9i11.04.

Tiwari, S. K. and Sonant, B., (2022), complex valued metric spaces for Das and Gupta contraction and fixed point and common fixed point theorems, JP Journal of Fixed Point Theory and Applications, Volume 17, 2022, Pages 11-38. .http://www.pphmj.com, http://dx.doi.org/10.17654/0973422822002.

Tiwari, S.K., and Sahu,T.(2014). Expansive mapping theorems in cone metric spaces, Int. Journal of Innovative Science, Engi.and Technology, 1(6), 260-267. ww.ijiset.com

Tiwari,S. K. and Dharmendra Das.(2017). Some Common Fixed Point Theorems for Expanding Mapping in Complete Cone Metric Space", International Journal of Mathematics Trends and Technology (IJMTT), 41(3), 269-271.http//www.ijmttjournal.org , doi: 10.14445/22315373/IJMTT-V41P524.