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STUDY AND ANALYSIS OF SOME REAL LIFE APPLICATIONS OF EXPONENTIAL FUNCTION

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Abstract

The exponential function as a mathematical concept plays an important role in the Corpus of mathematical knowledge, but unfortunately Students have problems grasping it. Paper exposes examples of exponential function application in a real-world (life).One of the most prevalent applications of exponential functions Involves growth and decay models. Exponential growth and decay show up in a host of natural applications. From population growth and continuously compounded Interest to radioactive decay and Newton's law of cooling, exponential functions are ubiquitous in nature. In this section, we examine exponential grown and decay in the context of some of the applications. In the preceding section, we examined a population growth problem in which the population grew at a fixed percentage each year. In that case, we found that the population can be described by an exponential function. A similar analysis will show that any process in which a quantity grows by a fixed percentage each year (or each day, hour etc.) can be modeled by an exponential function. Compound Interest is a good example of such a process. Other applications of exponential function are bacterial growth, bacterial decay, population decline, are obtained in this project.

Keywords: Mathematical modeling, the exponential function, Real world context, functional knowledge, exponential growth, exponential Decay, population growth, population decay, Radioactive decay, compound Interest, mathematics, graph and Newton's law

Introduction

Input-output analysis delves into various aspects of economic dynamics. It explores the relationships between producer and consumer surplus even in unconventional market conditions where demand may be positive, but supply is negative (Sahani, 2023). Sahani's work indicates that even in such scenarios, there exists untapped value for both producers and consumers, underscoring the resilience and complexity of economic systems. This insight is not only intriguing but also carries significant implications for economic policies.

Another study was done by the help of input-output analysis to study the international pricing of risk within the global capital market (Sahani et al., 2023). Sahani and his colleagues underscore the role of input-output analysis in unravelling the complex relationships between different sectors of the world economy and illuminating the channels through which risk is transmitted across borders. These findings bear relevance for international financial regulation, as understanding the dynamics of risk transmission can inform policies aimed at curbing financial contagion. Furthermore, input-output analysis is applied to the realm of non-linear science, where it uncovers the intricate interactions within non-linear systems, providing predictive insights into their behaviour (Sahani & Prasad, 2023). Sahani and Prasad's work highlights the adaptability of input-output analysis as a versatile tool for understanding complex systems beyond traditional economic domains. Turning our attention to Africa, input-output analysis is instrumental in dissecting the employment and job creation challenges faced by the youth in African countries (Sahani, 2023). Sahani's research identifies critical sectors like agriculture, construction, and trade as significant contributors to youth employment. Addressing the challenges in these sectors becomes imperative for fostering economic growth and harnessing the demographic dividend of Africa's youthful population (see [1-10]).

Exponential function is the high school level are first of all, hard to grasp by students, according to Goldin and Hersovics, I Goldin and Hersovics, 1991). Secondly, this type of function is the base of calculus and differential equations and lots of natural and social phenomena could be explained through the models based on exponential functions. Finally, the great importance of Knowledge about exponential functions can be explained by professor AI Barlett's quote that "The greatest shortcoming of the human race is our inability to understand the exponential function". Barlett also argues that even though the growth is the foundation of our civilization prosperity in the sense of business, economy or technology, this topic is not adequately represented in the mathematical and Physics education. The process of growth are dominating in our everyday life, and mathematics of growth is mathematics of the exponential function. It is important to understand the exponential grown because that would make us able to evaluate many situations concerning growth (Barlett, 1976). In 1700s (18th century), lennhard Euller was first mathematician who study and explored the exponential function. In this paper he extensively explored the properties of the exponential function and its relation to other mathematical concept.

He explored various concept about the exponential function. The exponential function is a mathematical function denoted by $f(x)=e^{x}$, the term generally refers to the positive-valued function of a real variable, although it can be extended to the complex numbers or generalized to other mathematical object like matrics or leo algebras. The exponential function originated from the operation of taxing powers of a number. These concepts are given by leonhard euller.

In 1798 (18th century) Thomas Robert Malthus was a British economist and demographer who studied about population growth.

He is the famous person who knows about population growth and he publish a book name "An Essay on the principle of population". He argued that populations inevitably expand until they outgrow their available food Supply, causing the population growth to be reversed by disease, famine, war or calamity. He is also known for developing an exponential formula used to forecast population growth, which is currently known as the Malthusian growth model. In the 18^{th} and 19^{th} centuries ,Some philosophers believed firmly that human society would continue to improve and tilt toward a Utopian idea. Malthus countered this belief, arguing that Segments of the general population have invariably been poor and miserable, effectively slowing population growth. Based on his observation of condition in England in the early in 1800s, Malthus argued that the available formland was insufficient to feed the increasing population. More specifically, he stated that the human population increases geometrically while food production increases arithmetically. In

1838 Pierre-Francois verhulst is a belgian mathematician who studied about logistic function.

A typical application of the logistic equation is a common model of population growth (see also population dynamics), originally due to Pierre-Francois Verhulst in 1838, where the rate of reproduction is proportional to both the existing population and the amount of available resources, all else being equal. The Verhulst equation was published after verhulst had read Thomas Malthus an Essay on the principle of population, which describes the Malthusian growth model of simple (unconstrained) exponential growth. Verhulst derived his logistic equation to describe the self-limiting growth of a biological population.

1864 Herbert Spencer is a famous English philosopher and sociologist, propounded the biological theory of population in his book. The principles of Biology, Spencer argued that fecundity decreases when the complexity of life increases.

According to him, changes in the growth of population occur due to natural change in the reproductive capacity of human beings. Therefore, his theory has been known as a natural theory of population which is similar to the theory of Salder and Doubleday. Spencer believed that "there exists antagonism between individuation (survival) and genesis (reproduction)". when any individual does hard work for his personal development at his work place, the desire for reproduction decreases.

In other words, when more energy has been utilized for one's self development, the energy available for reproduction will be less and consequently the population grown will be less. Thus, with the development of Society and for one's success and survival (individuation), life becomes more complex which results inreduction in the capacity of reproduction.

This is observed from the fact that fertility is more in rular individuals whose life is not complex, whereas fertility is low in an industrial Society where life is more complex, the pressure of education is more and the brains are overtaking.

1896 Henri Becquerel, is a mathematician who Study about radioactive decay.

In this paper he studied about radioactive decay (also known as nuclear decay, radioactivity, radioactive disintegration, or nuclear disintegration). According to him, radioactive decay is the process by which an unstable atomic nucleus loses energy by radiation. A material containing unstable nuclei is considered radioactive. Three of the most common types of decay are alpha, beta, and gamma decay. The weak force is the mechanism that is responsible for beta decay, while the other two are governed by the electromagnetism and nuclear force.

Radioactive decay is a stochastic (i.e, random) process at the level of single atoms. According to quantum theory, it is impossible to predict when a particular atom will decay, regardless of how long the atom has existed. However, for a significant number of identical atoms, the overall decay rate can be expressed as a decay constant or as half-life. The half-lives of radioactive atoms have a huge range; from nearly instantaneous to far longer than the age of the universe.

The decaying nucleus is called the parent radionuclide (or parent radioisotope), and the process produces at least one daughter nuclide. Except for gamma decay or interval conversion from a nuclear excited State, the decay is a nuclear transmutation resulting in a daughter containing a different number of protons or neutrons (or both). when the number of protons changes, an atom of a different chemical element is created.

In 2004 Donella, was the mathematician who studied about limit to growth

In this paper Donella highlighted the consequences of exponential economic and population growth on a finite planet. Her concept emphasized the interconnectedness of Various factors such as population, resources, pollution, and technology, within a global System. Donella Meadous and her colleagues used computer models to simulate different scenarios, illustrating how unchecked growth could lead to environmental and societal collapse.

Definitions

(a) Unlimited growth function:

Unlimited growth function is also known as exponential grown function. The function modeled by the equation $f(t)=ae^{rt}$, where a and r are constants, is caned unlimited growth function or exponential grown function. Investment and some models of population growth are the examples of unlimited growth function, unlimited growth function is a mathematical function that growth without bound as the Independent variable Increases.

(b) Unlimited decay function:

Unlimited decay function is also known as exponential decay function. The function modeled by the equation $f(t) = ae^{-rt}$, where a and r are constants is called unlimited decay. function or exponential decay function. Unlimited decay function is a mathematical function that decreases without as the independent variable increases.

(c) Limited grown function:

The function modeled by the equation $f(t)=m(l-e^{-rt})$, where M and r are constants, is called limited growth function .consumptions functions, sales with advertising, etc are Some examples of limited growth function.

"Limits to Growth" refers to a concept introduced in the book of the same name, published in 1972 by a team of researchers at the Massachusetts Institute of Technology (MIT). The book used computer models to explore the consequences of exponential economic and population growth within a finite world with limited resources. It argued that if current trends continued, the world would face ecological and economic collapse within the next century. The concept emphasize the need for Sustainable of considering environmental constraints in long-term planning.

(d) Logistic growth function:

The function modeled by the equation $f(t) = m/l+ae^{-mrt,}$ where m, a and are constants, is called logistic growth function. Constrained population growth of epidemic, Sales, etc are the examples of logistic grown function.

A logistic growth function is a mathematical model that describes how a population grows over time when it is limited by resources and reaches a carrying capacity.

Discussion

Problem-(i): The Value of aeroplane depreciated Continuously in exponential model as $S=f(t)=ve^{-it}$ and if original price of the aeroplane , V is RS 1,00,00,000, then find its scrap values after 9 years when rate of compound depreciation being 12% p.a.

Problem-(ii): The population of a certain city is 10,00,000 and suppose the city is not affected by any pandemic. The population increases continuously at a rate of 5% per year, what will be population after 10 years. Calculate the population with the help of exponential function model as $Pt = Pe^{it}$

Problem-(iii): A Sum Of RS.5000 Invested in a saving account which pays interest at the rate of 12% p.a. compounding continuously. Find the amount of after 15 years using the exponential model as $A = pe^{it}$

Problem (iv): The population of a rare species of a bird is declining by the formula $p = 750000 \quad e^{-0.0035t}$ estimated the population of the species as the function of time measured in years, Determine the expected population after 5 years.

Problem-(v): How long will it take for a machine to reduce to half of its original cost price if the value decreases exponentially at 10%p.a.

Problem-(vi): The population of a country is changing according to the equation $p = se^{-0.001t}$ where t is in years, P is in millions. calculate the number of population at the end of 1o years.

Problem-(vii): The price of 10 gm gold is Rs.60,000 and the rate of price increases exponentially by 40% per year. what will be its price after 3 years ?

Problem-(viii): The educational people of a country in 2020 are 2000000, the changing in those people is 5%p.a.. what will be the educational people found in 2030 ?

Problem-(ix): The price of land is RS.4,00,000 and the Value of land appreciated 10% p.a. then what will be the price of land after 3 Years ?

Problem-(x): If, the salaries of a employees is RS.50,000 and the company promises to increase in salaries at 5% per year, then what will be the salaries of those employees after 5 years ?

Solution

Problem-(i):

given,

$s=F(t)= ve^{-it}$

v=RS.1,00,00,000

t= 9 years

r = 12% p.a.

l=r/100=12/100=0.12

SO,

$S = ve^{-it}$

$= (1,00,00,000)*e^{-0.12*9}$

$= (1,0000,000)*0.34$

=RS. 3395955.256

Problem-(ii):

given,

$P_t = Pe^{it}$

t=10years

r=5% p.a.

i=5/100=0.05

p=10,00,000

SO,

$P_t= pe^{it}$

$= (10,00,000)*e^{0.05x10}$

$= (10,00,000)\times 1.649$

=1648721.27

=1648721

Problem-(iii):

given,

p = 5,000

t= 15 years

r=12% p.a.

i=r/100 = 12/100=0.12

SO,

$A = pe^{it}$

$= (5000)*e^{0.12*15}$

$= 5,000*6.05$

$= 30248.237$

Problem-(iv):

given,

t=5 years

declining model,

$P=750000*e^{-0.0035t}$

So,

Put t=5 years,

$P=750000*e^{-0.0035*5}$

$=750000*0.983$

$=736989.177$

$=736989$

Problem-(v):

let,

original value=v

Rate percentage(r) = 10%

i =r/100 =10/100 = 0.1

Time period (n)= ?

S=v/2

we know that,

$S\text{-}v_0e^{in}$

Or, $v/2= v*e^{0.1*n}$

Or, $0.5= e^{0.1n}$

Or, log(0.5)= 0.1n

Or, 0.1n= 0.6930

Or, n=0.6930/0.1

Therefore, n=6.93 years.

problem-(vi):

given,

$P=5*e^{-0.0014t}$

t = 10 years

$p =5*e^{-0.001*10}$

$=5*e^{-0.01}$

$= 5/e^{0.01}$

=4.950249169 million

=4.950249169*1000000

=4950249.169

=4950249

Problem-(vii):

Original value (V_o) = RS.60,000

r= 40% p.a.

i= r/100 = 40/100 = 0.4

n= 3 years

s= ?

we have,

$s=v_0e^{in}$

$=60000*e^{0.4*3}$

$=60000*e^{1.2}$

$=60000*3.3201$

=RS.199,206

Problem-(viii):

P=2000000

time (t)= 2030-2020 = 10 years.

Rate (r) = 5% p.a.

i = r/100 = 5/100 = 0.05

we have,

$P_t = Pe^{it}$

$= 2000000*e^{0.005*10}$

$= 2000000*e^{0.5}$

$= 2000000*1.649$

= 3297442.541

= 3297442

Problem-(ix):

V=RS:400000

t= 3 years

R=10%

I = R/100 = 10/100 = 0.1

we have,

$s = ve^{it}$

$=400000*e^{0.1*10}$

$=400000*e^{1}$

=400000*2.718

=1087312.731

=RS.1087312.731

Problem-(x):

original Salaries (V_o) = RS.50000

t = 5 years

r = 5% p.a.

i=5/100=0.05

we have,

$V_t = V_o e^{rt}$

$= 50000 * e^{0.05x5}$

= 50000 * 1.284

= 64201.271

= RS.60201.271

Conclusion

Analysing the above problem, we concluded, in problem (i) the scrap value of aeroplane after 9 years is depreciated than face Value or purchase cost, in problem (ii) the population of a certain city is appreciated annually, in problem(iii) we concluded the saving amount how does grow and the saving amount after 15 years, in problem (iv) we concluded the population of rare species of a bird after 5 years how does decline, in problem (v) we concluded that what time will be take to reduce half amounts of its original cost prices, in problem (vi) how does the population of a country changes with various factors, in problem (vii) we concluded how to increase value in log m gold and we obtain after 3 years its price, In problem (viii) we concluded how does educational people grow and what will the educational people found after a certain time period, concluded the problem (ix) we obtain the land appreciated price after 3 years and lastly concluded the problem (x) we obtain the employees salaries (i.e, additional salaries after increasing 5% per year) after 5 years.

Revealed the above problem and Solutions we concluded that the exponential function project provided Valuable Insights into the behavior and applications of exponential function. Through data analysing and mathematical modeling, we get a deeper understanding of exponential growth and decay phenomena. Furthermore this project underscored the significance of exponential function in several field served as practical demonstration of the power and versatility of exponential functions in real-world scenarious.

The exponential function finds numerous real life applications across various fields. From modeling Population growth and decay in biology to describing the behaviour of radioactive decay in physics, exponential functions provide valuable insights into dynamic processes characterized by rapid change overtime. Additionally, in finance, exponential functions are used to model compound interest, predicting future values of investments. Moreover, in technology, exponential growth pattern are observed in the adoption of new technologies and the growth of digital networks. Understanding and applying exponential functions is essential for analyzing and predicting dynamic phenomena in the real world.

In summary, the ubiquitous presence of exponential functions in real-life applications underscores their significance in understanding and predicting dynamic processes. whether it's in economics, biology, Physics, or technology, exponential functions offer powerful tools for modeling growth, decay and change over time. Harnessing the insights provided by exponential functions enables better decision- making in fields ranging from investment planning to resource management. Embracing the mathematical elegance of exponential functions empowers us to navigate and thrive in a world where change is often exponential.

References

[1]Sahani, S.K. (2023). The Effective Theory of Producer and Consumer Surplus; Positive Demand, Negative Supply, Central Asian Journal of Mathematics Theory and Computer Sciences,Vol.4,Issue 8,70-76.

[2]Sahani, S.K. et al. (2023). The International Pricing of Risk; An Empirical Investigation of the World Capital Market Structure, American Journal of Science and Learning Development,Vol,2,No.7,49-65.

[3]Sahani, S.K. and Prasad, K.S. (2023). Study and Analysis of Input| Output Analysis to Non-Linear Science,ACCST Research Journal,Vol. XXL,No.1,22-30.

[4]Sahani, S.K. (2023). Youth on Employment and Job Creation in African Countries; Opportunities, Challenges, Characteristics Determinants and Performance of Self-Employment Among Them the Youth in African Countries, Central Asian Journal of Mathematics Theory and Computer Sciences, Vol.04,No.7,27-41.

[5]Sahani, S.K. et al. (2023). Economic Insights Unveiled: A Journey Through Input-Output Analysis in Non-Linear Mathematics, MIKAILALSYS Journal of Multidisciplinary Science, Vol.1, Issue 1, 240-259. https://doi.org/10.58578/mikailalsys.v1i3.1865

[6]Sahani, S.K. et al. (2023). Unraveling the Interdependence of Input and Output Analysis in Business Sector; A Case Study, International Journal of Education, Management, and Technology, Vol.1,Issue 1, 27-45. https://doi.org/10.58578/ijemt.v1i1.1911

[7]Sharma, S., Sahani, S., Chaudhary, B., Sahani, K., & Sah, N. (2023). Asan, The Traditional Market of Kathmandu Valley: Analyzing Market Trend and Consumer Buying Behavior of Such Market.EDUMALSYS Journal of Research in Education Management, 1(2), 215-226. https://doi.org/10.58578/edumalsys.v1i2.2048

[8]Sah, P., Sah, R., Sahani, S. K., & Sahani, K. (2024). Study and Analysis of Education System in Nepal and Its Challenges Along with Its Solution. *Mikailalsys Journal of Advanced Engineering International*, *1*(1), 33-42. https://doi.org/10.58578/mjaei.v1i1.2792

[9]Karn, P., Sahani, S. K., & Sahani, K. (2024). Study and Analysis of Some Practical Life Uses and Applications of Exponential Function. *Mikailalsys Journal of Advanced Engineering International*, *1*(1), 43-56. https://doi.org/10.58578/mjaei.v1i1.2793

[10]Sah, K. K., Sahani, S. K., Sahani, K., & Sah, B. K. (2024). A Study and Examined of Exponential Function: A Journey of Its Applications in Real Life. *Mikailalsys Journal of Advanced Engineering International*, *1*(1), 23-32. https://doi.org/10.58578/mjaei.v1i1.2791