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CLASSICAL STUDY OF EXPONENTIAL FUNCTION AND THEIR APPLICATIONS

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Abstract

The exponential function as a mathematical concept plays an important role in the corpus of mathematical knowledge, but unfortunately students have problems grasping it. Paper exposes example of exponential example of exponential function application in real world. One of the most prevalent application of exponential function involves growth and decay models. Exponential growth and decay show up in a host of natural application. From population Growth and continuously compounded interest to radioactive decay and Newton's law of cooling, exponential function are ubiquitous in nature. In this section, we examine exponential growth and decay in the context of some of the application. In the preceding section, we examined a population growth problem in which the population grew at a fixed percentage each year. In that case, we found that the population can be described by exponential function. A similar analysis will show that any process in which a quantity grows by a fixed percentage each year can be modeled by an exponential function. Compound interest is good example of such a process. This work is motivated by the works of [1-15, 22]. Other example of exponential function are bacterial growth, bacterial decay, population decline, are obtained in this project.

Keywords*; Mathematical modeling, the exponential function, Real world context, Functional knowledge, Exponential growth, Exponential Decay, Population growth, Population decay, Radioactive decay, Compound Interest, Mathematics, Graph and Newton's law*

Introduction

Exponential function is the high school level are first of all, hard to grasp by students, according to Goldin and Hersovics, I Goldin and Hersovics, 1991). Secondly, this type of function is the base of calculus and differential equations and lots of natural and social phenomena could be explained through the models based on exponential functions. Finally, the great importance of Knowledge about exponential functions can be explained by professor Al Barlett's quote that "The greatest shortcoming of the human race is our inability to understand the exponential function". Barlett also argues that even though the growth is the foundation of our civilization prosperity in the sense of business, economy or technology, this topic is not adequately represented in the mathematical and Physics education. The process of growth are dominating in our everyday life, and mathematics of growth is mathematics of the exponential function. It is important to understand the exponential grown because that would make us able to evaluate many situations concerning growth (Barlett, 1976).

(2) 1700s (18th century), lennhard Euller was first mathematician who study and explored the exponential function. In this paper he extensively explored the properties of the exponential function and its relation to other mathematical concept.

He explored various concept about the exponential function. The exponential function is a mathematical function denoted by f(x)=e, the term generally refers to the positive-valued function of a real variable, although it can be extended to the complex numbers or generalized to other mathematical object like matrics or leo algebras. The exponential function originated from the operation of taxing powers of a number. These concepts are given by leonhard euller.

(3) 1798 (18th century) Thomas Robert Malthus was a British economist and demographer who studied about population growth.

He is the famous person who knows about population growth and he publish a book name "An Essay on the principle of population". He argued that populations inevitably expand until they outgrow their available food Supply, causing the population growth to be reversed by disease, famine, war or calamity. He is also known for developing an exponential formula used to forecast population growth, which is currently known as the Malthusian growth model. In the 18th and 19th centuries, Some philosophers believed firmly that human society would continue to improve and tilt toward a Utopian idea. Malthus countered this belief, arguing that Segments of the general population have invariably been poor and miserable, effectively slowing population growth. Based on his observation of condition in England in the early in 1800s, Malthus argued that the available formland was insufficient to feed the increasing population. More specifically, he stated that the human population increases geometrically while food production increases arithmetically.

(4) 1838 Pierre-Francois verhulst is a Belgian mathematician who studied about logistic function.

A typical application of the logistic equation is a common model of population growth (see also population dynamics), originally due to Pierre-Francois Verhulst in 1838, where the rate of reproduction is proportional to both the existing population and the amount of available resources, all else being equals.

The Verhulst equation was published after verhulst had read Thomas Malthus an Essay on the principle of population, which describes the Malthusian growth model of simple (unconstrained) exponential growth. Verhulst derived his logistic equation to describe the self-limiting growth of a biological population.

(5) 1864 Herbert Spencer is a famous English philosopher and sociologist, propounded the biological theory of population in his book. The principles of Biology, Spencer argued that fecundity decreases when the complexity of life increases.

According to him, changes in the growth of population occur due to natural change in the reproductive capacity of human beings. Therefore, his theory has been known as a natural theory of population which is similar to the theory of Salder and Doubleday. Spencer believed that "there exists antagonism between individuation (survival) and genesis (reproduction)". when any individual does hard work for his personal development at his work place, the desire for reproduction decreases.

In other words, when more energy has been utilized for one's self development, the energy available for reproduction will be less and consequently the population grown will be less. Thus, with the development of Society and for one's success and survival (individuation), life becomes more complex which results inreduction in the capacity of reproduction.

This is observed from the fact that fertility is more in rural individuals whose life is not complex, whereas fertility is low in an industrial Society where life is more complex, the pressure of education is more and the brains are overtaking.

(6) 1896 Henri Becquerel, is a mathematician who Study about radioactive decay.

In this paper he studied about radioactive decay (also known as nuclear decay, radioactivity, radioactive disintegration, or nuclear disintegration). According to him, radioactive decay is the process by which an unstable atomic nucleus loses energy by radiation. A material containing unstable nuclei is considered radioactive. Three of the most common types of decay are alpha, beta, and gamma decay. The weak force is the mechanism that is responsible for beta decay, while the other two are governed by the electromagnetism and nuclear force.

Radioactive decay is a stochastic (i.e., random) process at the level of single atoms. According to quantum theory, it is impossible to predict when a particular atom will decay, regardless of how long the atom has existed. However, for a significant number of identical atoms, the overall decay rate can be expressed as a decay constant or as half-life. The half-lives of radioactive atoms have a huge range; from nearly instantaneous to far longer than the age of the universe.

The decaying nucleus is called the parent radionuclide (or parent radioisotope), and the process produces at least one daughter nuclide. Except for gamma decay or interval conversion from a nuclear excited State, the decay is a nuclear transmutation resulting in a daughter containing a different number of protons or neutrons (or both). when the number of protons changes, an atom of a different chemical element is created.

(7) In 2004 Donella, was the mathematician who studied about limit to growth

In this paper Donella highlighted the consequences of exponential economic and population growth on a finite planet. Her concept emphasized the interconnectedness of Various factors such as population, resources, pollution, and technology, within a global System. Donella Meadous and her colleagues used computer models to simulate different scenarios, illustrating how unchecked growth could lead to environmental and societal collapse.

Definition

(a) Unlimited growth function:

Unlimited growth function is also known as exponential grown function. The function modeled by the equation $f(t)= ae^{rt}$ where a and rare constants, is caned unlimited growth function or exponential grown function. Investment and some models of population growth are the examples of unlimited growth function, unlimited growth function is a mathematical function that growth without bound as the Independent variable Increases.

(b) Unlimited decay function:

Unlimited decay function is also known as exponential decay function. The function modeled by the equation $f(t) = ae^{-rt}$ where a and rare constants is called unlimited decay, function or exponential decay function. Unlimited decay function is a mathematical function that decreases without as the independent variable increases.

(c) Limited grown function:

The function modeled by the equation $f(t)=M(1-e^{-rt})$, where M and r are constants, is called limited growth function .consumptions functions, sales with advertising etc are Some examples of limited growth function.

'Limits to Growth" refers to a concept introduced in the book of the same name, published in 1972 by a team of researchers at the Massachusetts Institute of Technology (MIT). The book used computer models to explore the consequences of exponential economic and population growth within a finite world with limited resources. It argued that if current trends continued, the world would face ecological and economic collapse within the next century. The concept emphasize the need for sustainable for considering environmental constraints in long-term planning (see[16-21]).

(d) Logistic growth function: The function modeled by the equation $f(t) =$

$M/1+ae^{-rMt}$, where M, a, and r are constants, is called logistic growth function. Constrained population growth of epidemic, Sales, etc. are the examples of logistic grown function.

A logistic growth function is a mathematical model that describes how a population grows over time when it is limited by resources and reaches a carrying capacity.

Discussion:

<u>Problem-(i):</u> The price value at RS.400000000 depreciated continuously at the rate of 10% for 2 years.

Solution:

Initial value of ship (a) =RS.400000000

Rate of depreciation (r) = 10% p.a.

We know unlimited decay function $f(t) = ae^{-it}$

Scrape value of ship next two years = ?

We have,

I = r/100 = 10/100 = 0.1

Scrape value s = ae^{-it}

$= 400000000*e^{-0.1*2}$

= RS.327492301.2

Problem (2): The number of population in a country is 20000000 and it grow by 10% every year.

Solution:

Number of population in a country (p) = 20000000

Rate of growth (r) = 10% p.a.

So, the population growth in 6 years = ?

We have,

i = r/100 = 10/100 = 0.1

So, the number of population after 6 year = ae^{it}

$= 20000000*e^{0.1*6}$

=36442376.01

Problem (3): The population of tiger in 2023 in Nepal is 562 and increasing with a rate of 10%. Find the population of tiger in 2025.

Solution:

Population of tiger in 2023 (p) = 562

Rate of increasing (r) = 10% p.a.

We know,

Population of in 2025 = ae^{it}

$= 562*e^{0.1*2}$

$= 686.4283501$

$= 686$

Problem (4): Mr. Ram deposited Rs.60000 in a bank with an interest of 10% p.a. for 10 years.

Solution:

Sum (e) = Rs.60000

Invested for (t) =10 years

Rate of interest (r) = 10% p.a.

We have,

i = r/100 =10/100 = 0.1

so, the sum in 10 years = ae^{it}

$= 60000*e^{0.1*10}$

= Rs.163096.9097

Conclusion

Analyzing the above all problems we revealed and concluded that the exponential function project provided Valuable Insights into the behavior and applications of exponential function. Through data analyzing and mathematical modeling, we get a deeper understanding of exponential growth and decay phenomena. Furthermore this project underscored the significance of exponential function in several field served as practical demonstration of the power and versatility of exponential functions in real-world scenarios.

The exponential function finds numerous real life applications across various fields. From modeling Population growth and decay in biology to describing the behavior of radioactive decay in physics, exponential functions provide valuable insights into dynamic processes characterized by rapid change overtime. Additionally, in finance, exponential functions are used to model compound interest, predicting future values of investments. Moreover, in technology, exponential growth pattern are observed in the adoption of new technologies and the growth of digital networks. Understanding and applying exponential functions is essential for analyzing and predicting dynamic phenomena in the real world.

In summary, the ubiquitous presence of exponential functions in real-life applications underscores their significance in understanding and predicting dynamic processes. Whether it's in economics, biology, Physics, or technology, exponential functions offer powerful tools for modeling growth, decay and change over time. Harnessing the insights provided by exponential functions enables better decision-making in fields ranging from investment planning to resource management. Embracing the mathematical elegance of exponential functions empowers us to navigate and thrive in a world where change is often exponential.

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