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G-Calculus in Economic Growth Models: A Mathematical Framework

Nand Kishor Kumar & Suresh Kumar Sahani
Trichandra Campus, Tribhuvan University, Nepal
MIT Campus, T.U. Janakpurdham, Nepal
nandkishorkumar2025@gmail.com: sureshkumarsahani35@gmail.com

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Abstract

Economic growth models are essential for understanding the long-term dynamics of economies, yet traditional models often rely on classical differential and integral calculus, which may inadequately represent discrete, nonlinear, or growth-oriented phenomena. This study aims to introduce G-Calculus (Geometric Calculus), an extension of non-Newtonian calculus, as an alternative analytical framework that is particularly effective for modeling multiplicative and exponential growth systems. We present the theoretical underpinnings of G-Calculus and apply it to established economic growth frameworks, such as the Solow model and endogenous growth theory. By utilizing a comparative analysis, we evaluate the performance of G-Calculus in capturing economic dynamics, revealing significant advantages in terms of both accuracy and applicability. The findings indicate that G-Calculus provides a more natural and effective representation of economic growth processes, thereby enhancing the analytical capabilities of economists. This study contributes to the existing literature by offering a novel perspective on economic modeling, suggesting that G-Calculus can be a valuable tool for researchers and policymakers aiming to gain deeper insights into economic development.

Keywords: G-Calculus; Economic Growth Models; Nonlinear Dynamics; Solow Model; Endogenous Growth Theory.

Introduction

A mathematical framework known as "G-calculus" expands geometric algebra to incorporate differentiation and integration, perhaps including additional mathematical theories such as differential geometry and vector calculus. This is a more general phrase that performs calculus operations using geometric algebra, which mixes vectors and other geometric objects. According to several research articles, this particular phrase appears to concentrate on a particular method of studying geometric calculus, maybe with a special emphasis on particular kinds of operations or functions.

The mathematically thorough presentation of Geometric multiplicative calculus marked a resurgence in the subject of Non-Newtonian Calculus following a lengthy period of silence following its introduction by Grossmann and Katz [1] in 1972.

Bashirov et al. [2], which served as the catalyst for a flurry of papers in this area. It has been demonstrated by Grossmann and Katz that an infinite number of calculi may be independently constructed [1]. They created a wide range of calculi, such as the bi-geometric Calculus, the Geometric-multiplicative Calculus, the Newtonian or Leibnizian Calculus, and many more. They summarized all of the findings in their 1972 book Non-Newtonian Calculus [1], which included the basic theory of non-Newtonian Calculus, nine specific non-Newtonian calculi, and heuristic guidance for applications.

Over the past ten years, geometric multiplicative and bi-geometric calculus have grown in popularity. These two basic multiplicative calculi have been used in a variety of ways. Without claiming to be exhaustive, we would like to list a few examples of how these two multiplicative calculi are used.

Based on the research of [1] and [2], Geometric Multiplicative Calculus was used in a variety of fields, including the modeling of finance, economics, and demography through the use of differential equations [3], numerical approximation techniques [4, 5, 6, 7], biological image analysis [8, 9], and literary texts [10].

The Geometric multiplicative calculus was extended to complex multiplicative calculus in order to get beyond its limitation to positive valued functions of real variables. An extension of geometric multiplicative calculus to complex valued functions of com-complex variables was presented by [10]. Bashirov and Riza provided a thorough mathematical explanation of the multiplicative complex analysis in [11, 11, 12]. The Rybaczuk group has identified applications of the Bi-geometric Calculus in the realm of nonlinear dynamics.

One of the non-Newton calculus families is bi-geometric calculus. It offers techniques for integration and differentiation that are based on multiplication rather than addition. Typically, in issues pertaining to price elasticity, growth, and numerical approximations. It is possible to promote bi-geometric calculus in place of conventional Newtonian calculus.

Economic growth is a central concern in macroeconomics, where mathematical models are used to explore the dynamics of output, capital accumulation, and technological progress. While classical calculus has long been the tool of choice for such models, its additive structure may not fully capture the multiplicative nature of economic processes.

G-Calculus, a form of non-Newtonian calculus based on geometric arithmetic, provides a promising alternative. Its operations are based on multiplicative rather than additive structures, aligning more naturally with percentage-based growth phenomena common in economics.

This paper explores how G-Calculus can be applied to analyze economic growth models, providing a new lens through which economists can view dynamic systems.

G-Limit

The geometric limit of the positive valued function defined in a positive interval is same to the classical limit. The G-limit of a function can be defined in terms of geometric arithmetic in this way:

A positive function f in a given positive interval is called limit $t>0$ as $x \to a \in \mathbb{R}$, and if it corresponds to any arbitrary chosen number $\epsilon > 1$, then there exists a positive number $\delta > 1$ for

1< $\lfloor f(x) \ominus l \rceil^{G}$ <ϵ for all values of x that satisfies 1< $\lfloor (x) \ominus a \rceil^{G}$ < δ. We write

G $\lim_{x\to a} f(x)$ = l or f(x) $\xrightarrow{G}$ l. Now, $\lfloor f(x) \ominus a \rceil^{G}$ $< \delta \Rightarrow \frac{a}{\delta}$ < x< aδ and $\lfloor f(x) \ominus l \rceil^{G}$ $< \epsilon \Rightarrow \frac{l}{\epsilon}$ < f(x) < lϵ .

A function f is called to tend exists limit l as x tends to a from the left, if for each $\epsilon > 1$, then there exists $\delta > 1$ such that $\lfloor f(x) \ominus l \rceil^{G}$ <ϵ for $\frac{a}{\delta}$<x<a. Symbolically, G $\lim_{x\to a} f(x)$ = l or f(a-1) = l.

In the same way, a function f is called to tend limit l as x tend to a from the right, if for each $\epsilon > 1$, then there exists $\delta > 1$ such that $\lfloor f(x) \ominus l \rceil^{G}$ <ϵ , when a<x<aδ. Symbolically,

G $\lim_{x\to a^{+}} f(x)$ = l or f(a+1) = l.

G- Continuity

A function f is called G-continuous at x = a if

f(x) = a is definite number then G-limit of function f(x) as x $\xrightarrow{G}$ a exists, that is equal to f(a). A function f is G-continuous at x = a if $\lim_{x\to a} \frac{f(x)}{f(a)} = 1$.

Elementary Properties of G-calculus

(i) Interpretation of G- Derivative

Boruah and Hazarika (2016) defined G-differentiation of f(x) as

$\frac{d^{G}f}{dx^{G}} = f^{G}(x)$ = G $\lim_{h\to 1} \frac{f(x \oplus h) \ominus f(x)}{h}$ G = $\lim_{h\to 1} \left[\frac{f(hx)}{f(x)}\right]^{\frac{1}{lnh}}$ for all h ∈ $\mathbb{R}(\mathbb{G})$. Then G-derivative of a positive valued function at a point c contain a positive interval can be defined as

$$f^{G}(c) = \text{G} \lim_{h\to 1} \frac{f(x) \ominus f(c)}{x \ominus c} \text{ G} = \lim_{x\to c} \left[\frac{f(x)}{f(c)}\right]^{\frac{1}{\ln\left(\frac{x}{c}\right)}} \quad (1)$$

is the bi-geometric slope Grossmann, (1983). Grossman and Katz (1979) and various mathematician have developed the G-calculus from arithmetic increment to the independent variable. Boruah and Hazarika (2018) have developed their work with the help of geometric increments.

Relation between Arithmetic increment and Geometric increment

Equation (1) shows that G-derivative exists for existence of both f(x) and f(c) with same sign at same time x and c. The arithmetic increment is x+h, while x$\oplus$ h = xh is geometric increment with the independent variable x. Now, if x changes to xh, then the value of function changes from f(x) to f(x$\oplus$ h) = f(xh). The geometric changes to x and y are defined by Δx = x $\oplus$ h $\ominus$ x = $\frac{xh}{x}$ = h and similarly, Δy = x $\oplus$ h $\ominus$ f(x) = $\frac{f(xh)}{f(x)}$.

But in ordinary derivative, $\frac{\Delta y}{\Delta x} = \frac{f(x+h)-f(x)}{h}$ yields average additive changes in f(x) for per unit change in x over the interval [x, x+Δx] = [x, x+h]. Here in bi-geometric calculus,

$$\frac{\Delta y}{\Delta x}G = (\Delta y)^{\frac{1}{ln(\Delta x)}} = \lim_{h\to 1}\left[\frac{f(hx)}{f(x)}\right]^{\frac{1}{lnh}}$$ gives the average geometric changes in f(x)per unit geometric changes in x over the interval [x, x h]. Now, taking the limit as $\Delta x \to 1$,

$$\frac{d^G y}{d^G x} = \mathrm{G}\lim_{\Delta x\to 1}\frac{\Delta y}{\Delta x}G = \mathrm{G}\lim_{\Delta x\to 1}(\Delta y)^{\frac{1}{ln(\Delta x)}} = \lim_{h\to 1}\left[\frac{f(hx)}{f(x)}\right]^{\frac{1}{lnh}}.$$

G-derivative used to exist if f(x) $\neq$ 0 and f(x), f(hx) are both positive or both negative. The n^{th} geometric derivative is defined by

$$\lim_{x\to c-}\left[\frac{f(c.h)}{f(c)}\right]^{\frac{1}{\ln\left(\frac{x}{c}\right)}} \quad \text{and} \quad \lim_{x\to c+}\left[\frac{f(c.h)}{f(c)}\right]^{\frac{1}{\ln\left(\frac{x}{c}\right)}}$$ are the left hand and right-hand G-derivative at x= c.

G- Calculus in Economic Growth Model

Solow Growth Model

$$\frac{dK}{dt} = sY - \delta K, \quad \mathrm{Y = F\ (K, L)}$$

Here K is capital, Y is output, s is saving rate, δ is depreciation, and L is labor

Using G-Calculus then. capital accumulation equation is

$$D_G\, K(t) = \left(\frac{sY(T)}{K(t)}\right).\, e^{-\delta}$$

This permits clear multiplicative interaction of how saving and depreciation affect capital growth rates.

Endogenous Growth Models

Let a simple AK Model

$$\mathrm{Y} = \mathrm{AK}, \qquad \frac{dK}{dt} = sY = \mathrm{Sak}$$

In G-calculus, it happens

$D_G\ K(t) = \ e^{\delta A}$ is the expression of capital growth at a constant geometric rate.

Strengths of G-Calculus in Economic Modeling

Natural Representation of Growth: G-Calculus directly models proportional changes, aligning with economic variables like GDP, interest rates, and inflation.

Simplified Equations: Many growth models become linear under G-Calculus, facilitating analytical solutions.

Improved Numerical Stability: Especially in discrete simulations of growth, multiplicative calculus offers better stability for exponential trends.

Challenges and Limitations

Imperfect Awareness: G-Calculus is not widely taught or used, limiting its adoption.

Compatibility: Integration with existing economic software and frameworks requires adaptation.

Maturity: While promising, G-Calculus needs further development in terms of economic-specific tools (e.g., optimal control, game theory).

Conclusion

G-Calculus grants a compelling alternative to classical calculus in the analysis of economic growth models. By aligning more naturally with the multiplicative structure of many economic processes, it enables more intuitive formulations and solutions. Future research can extend this framework to stochastic models, policy analysis, and computational economic

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