

CLASSICAL STUDY OF REAL LIFE APPLICATIONS OF EXPONENTIAL FUNCTION

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Abstract

One of the most prevalent applications of exponential function involves growth and decay modes. Exponential growth and decay show up in a host of applications. From population growth and continuously compounded interest to radioactive decay and Newton's law of cooling exponential function are ubiquities in nature. In this section, we examine exponential growth and decay in the context of some of the application. In the preceding section, we examined a population growth problem in which the population grew at a fixed percentage each year. In that case, we found that the populations can be described by an exponential function. A similar analysis will show that any process in which a quality grows by a fixed percentage each year (or each day, hours, etc) can be modeled by an exponential function. Compound interest is a good example of such a process. Other applications of exponential function are bacterial growth, bacterial decay, population decline are obtained in this project (see [1-27])

Keywords: Mathematical Modeling, The Exponential Function, Real World Context, Functional Knowledge, Population Decay, Radioactive Decay, Compound Interest, Mathematical Graph, Newton's Law

Introduction

An **exponential function** is a Mathematical function in the form $f(x) = a^x$, where “x” is a variable and “a” is a constant which is called the base of the function and it should be greater than 0. The most commonly used exponential function base is the transcendental number e, which is approximately equal to 2.71828.

- An exponential function is defined by the formula $f(x) = a^x$, where the input variable x occurs as an exponent. The exponential curve depends on the exponential function and it depends on the value of the x.

The exponential function is an important mathematical function which is of the form of $f(x) = a^x$

Where $a > 0$ and a is not equal to 1. x is any real number.

If the variable is negative, the function is undefined for $-1 < x < 1$.

2.1700S (18th century):- Leonhard Euler was first mathematician who study and explored the exponential function. In this paper he extensively explored the properties of the exponential function e^x and its relation to other mathematical report.

- was a Swiss mathematician, physicist, astronomer, geographer, logician, and engineer who founded the studies of graph theory and topology and made pioneering and influential discoveries in many other branches of mathematics such as analytic number theory, complex analysis, and infinitesimal calculus. He introduced much of modern mathematical terminology and notation, including the notion of a mathematical function. He is also known for his work in mechanics, fluid dynamics, optics, astronomy, and music theory.
- He explored various concept about the exponential. The exponential function is a mathematical function denoted by $f(x) = e^x$, the term generally to the positive valued functions of real variable, although it can be extended to complex number.

1798(18th century):- Thomas Robert Malthus was a British economist and demographer who studied about population growth.

- He is a famous person who know about population growth and he published anonymously the first edition of *An Essay on the Principle of Population as It Affects the Future Improvement of Society, with Remarks on the Speculations of Mr. Godwin, M. Condorcet, and Other Writers*. The work received wide notice. Briefly, crudely, yet strikingly,

Malthus argued that infinite human hopes for social happiness must be vain, for population will always tend to outrun the growth of production. The increase of population will take place, if unchecked, in a geometric progression, while the means of subsistence will increase in only an arithmetic progression. Population will always expand to the limit of subsistence. Only “vice” (including “the commission of war”), “misery” (including famine or want of food and ill health), and “moral restraint” (i.e., abstinence) could check this excessive growth.

Based on his observation of England in the early in 1800s, Malthus argued that the available farmland was insufficient to feed the increasing population. More specifically, he stated that human population increases arithmetically.

1838 Pierre:-

Pierre-Francois Verhulst, is a Belgian mathematician and demographer Pierre-François Verhulst, best known for the conceptualization and specification of the logistic curve, was born in Brussels to wealthy parents.

- The verhulst equation was published after verhulst had read Thomas Malthus, An essay on the principle of population, which describes the Malthusian growth modes of simple exponential growth. Verhulst derived his logostic equation to describe the self limiting growth of a biological population.

1864(Herbert Spencer):-

Herbert Spencer is a famous English philosopher and sociologist, propounded the biological theory of population in his book. The principles of biology, spencer argued that fertility decrease when the complexity of life increases.

- According to him, changes in the growth of population occurs due to natural change in the reproductive capacity in human being. Therefore, his theory has been known as a natural theory of population which is similar to the theory of swider and double day.

1896(Henry Becquerel):-

Becquerel is a famous Mathematician who study about radioactive decay.

- Becquerel's earliest work was concerned with the plane polarization of light, with the phenomenon of phosphorescence and with the absorption of light by crystals (his doctorate thesis). He also worked on the subject of terrestrial magnetism.
- His previous work was overshadowed by his discovery of the phenomenon of natural radioactivity. Following a discussion with Henri Poincare on the radiation which had recently been discovered by Rontgen (X-rays) and which was accompanied by a type of phosphorescence in the vacuum tube, Becquerel decided to investigate whether there was any connection between X-rays and naturally occurring phosphorescence.

2004(Donella):-

- Donella (Dana) Meadows (March 13, 1941 – February 20, 2001) was an American scientist, teacher, and writer. She is probably best known as lead author of the influential book *The Limits to Growth* which was the first time her pioneering work on Systems Science had reached a wide audience.

Definitions

a.) Unlimited growth function -

Unlimited growth function is also known as exponential growth function. The function modeled by the equation $f(t) = a e^{rt}$, where a and r are constants is called unlimited growth function or exponential growth function.

b.) Unlimited decay function:-

Unlimited decay function is also known as exponential decay function. The function modeled by the equation $f(t) = a e^{-rt}$, where a and r are constant are called unlimited decay function.

c.) Limited growth function:-

The function modeled by the equation $f(t) = m(1 - e^{-rt})$, where m and r are constant are called limited growth function. Consumption functions, sales with advertising, etc are some example of limited growth function.

d.) Logostic growth function:-

The function modeled by the equation $f(t)=m$,where m and r are constant logostic growth function. A logostic function is a mathematical model that describes how a population grows over time when it is limited by resources and reaches a causing capacity.

Problems:-

1. The value of machine depreciated continuously exponential model as $S=f(t)= Ve^{-it}$ and if original price of the machine, V is Rs 5000000 then find its scrap value S after 15 year when rate of compound depreciation being 10% p.a.

Solution:-

$$S=f(t)=Ve^{-it}$$

$$V=Rs5000000$$

$$t= 15 \text{ years}$$

$$R= 10\% \text{ p.a}$$

$$i= R/100=10/100=0.1$$

We have,

$$S=Ve^{-it}$$

$$= (5000000)e^{-0.1*15}$$

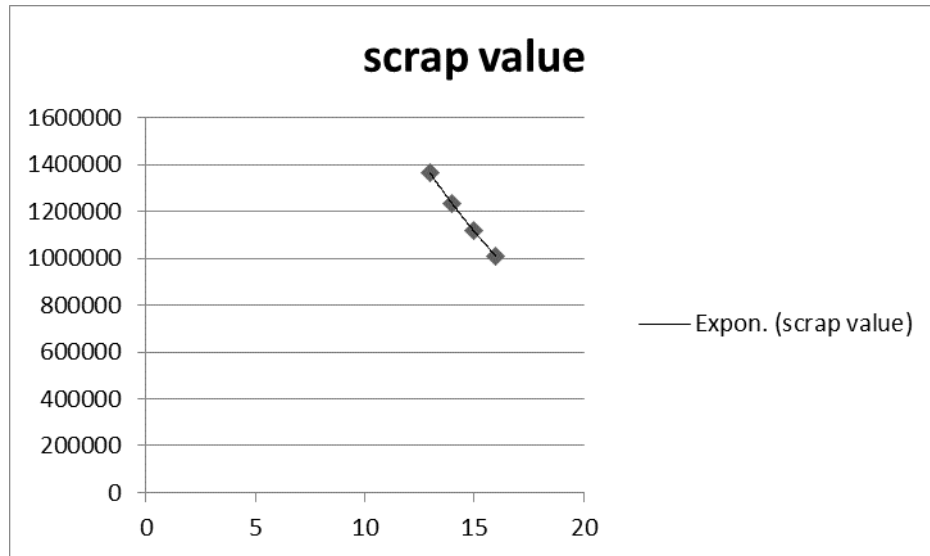
$$= 5000000*0.2231301601$$

$$= 1115650.801$$

Since the scrap value of the machine after 15 years is 1115650.801

Representing the above in diagram way.

Stating: X-axis represent the time and Y-axis represents scrap value



2. The population of a city is estimated to increase by 20% per year. The population today is 40000. make a graph of the population function and find out what the population will be in 15 years from now.

Solution:-

$$P = 40000$$

$$R = 20\%$$

$$T = 15 \text{ years}$$

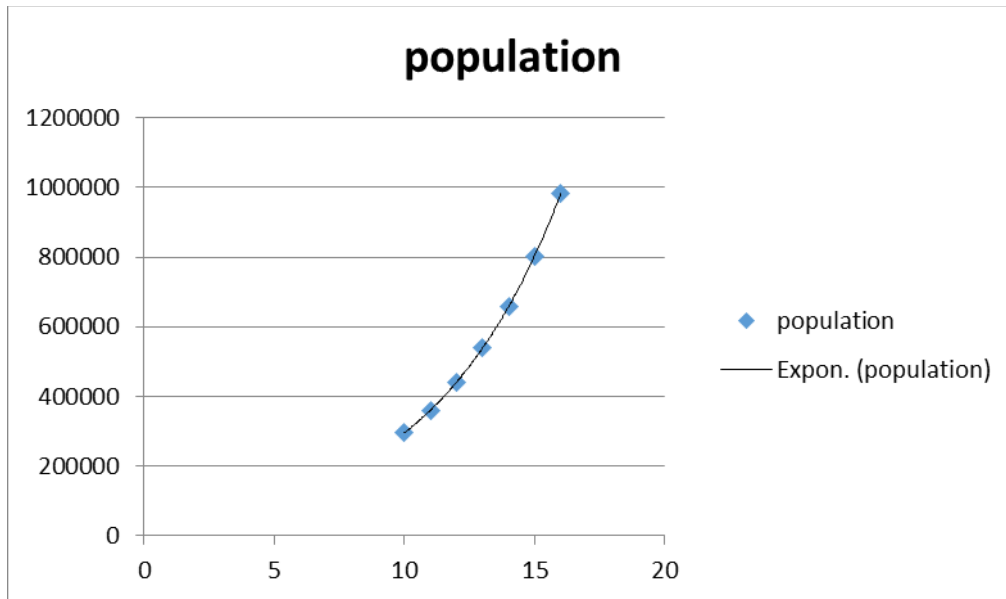
we have,

$$P_t = Pe^{it}$$

$$= 40000 * e^{0.2 * 15}$$

$$P_{15} = 803421.4769$$

Since the population after 15 years is 803421.4769 representing the above information in diagram way.



Stating: x-axis represents the time and Y-axis represents the populations.

3. A sum of 60000 invested in a saving account which pays interest at the rate of 15% p.a. compounding continuously. find the amount after 20 years using the exponential model as $A=Pe^{it}$.

Solution:-

$$P= \text{Rs } 60000$$

$$R=15\%$$

$$i= R/100 = 0.15$$

$$t= 20 \text{ years}$$

We have

$$A= Pe^{it}$$

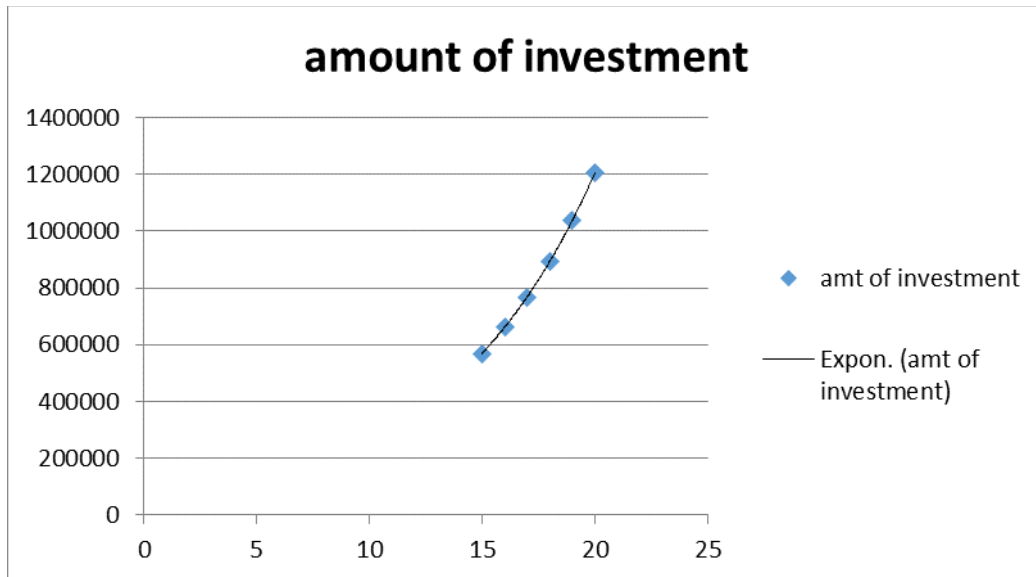
$$= \text{Rs } 60000 e^{0.15 \times 20}$$

$$= \text{Rs } 60000 \times 20.0855$$

$$= 1205132.215$$

Since the amount after 20 years will be 1205132.215

Representing the above information in diagram way.



Stating: X-axis represent the time and Y- axis represent the amount of investment.

4. A town has a population of 100000 that is increasing at the rate of 20% each year. Find the population after 15years.

Solution;

$$P = 100000$$

$$P_t = ?$$

$$T = 15\text{years}$$

$$R = 20\%$$

Now,

$$P_e = P e^{rt}$$

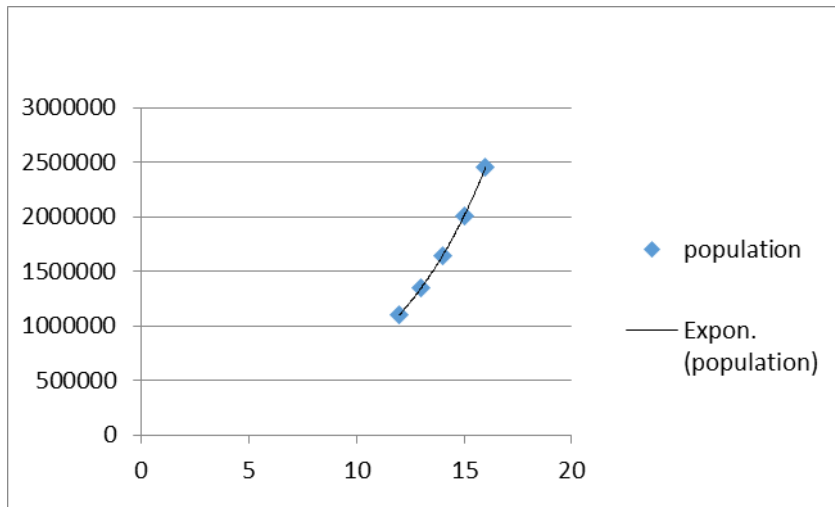
$$= 100000 * e^{(0.2 * 15)}$$

$$= 100000 * 20.085$$

$$P_{15} = 2008553.692$$

Since the population of town after 15 years is 2008553.692

Representing the above in diagram way.



Stating : X-axis represents the time and Y-axis represents the population.

Conclusion

Analysis the above problem, we have obtained in problem (i) we have found scrape value after 15 years, in problem (ii), and (iv) , we have found population after 15 years and in problem (ii) , we have found amount after 20 years.

The exponential function project provides the valuable insights into the behavior and applications of exponential function. Through data analysis and mathematical modeling, we gained a deeper understanding of exponential growth and decay phenomena. Furthermore, this project underscored the significance of exponential function in various field served as a practical demonstration of the power and versality of exponential function in real world scenarios.

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