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Thermo-Magnetic Interaction and Mixed Convection Dynamics of Casson Fluid over a Stretching Surface

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Abstract

Although thermo-magnetic Casson fluid flow over stretching surfaces has received increasing attention in previous studies, the combined influence of mixed convection, magnetic field strength, and boundary-layer control mechanisms on heat and mass transfer characteristics remains underexplored. This study aims to analyze the interactive effects of magnetic induction, Casson parameter, and convection-related factors on the thermal and velocity profiles of a Casson fluid subjected to thermo-magnetic forces. A quantitative computational approach was employed, in which the governing partial differential equations were transformed into coupled nonlinear ordinary differential equations and solved numerically using MATLAB's bvp4c solver. The model incorporated key dimensionless parameters, including the magnetic field intensity, Eckert number, Prandtl number, porosity, and inclination angle, to capture mixed convection and thermo-magnetic effects over a porous stretching surface. The findings indicate that increasing magnetic parameter

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values suppress the velocity profile while enhancing the thermal boundary-layer thickness, reflecting the retarding Lorentz force and associated thermal buildup. Similarly, higher Eckert numbers intensify viscous dissipation, leading to increased temperature fields, whereas larger Prandtl numbers reduce temperature distribution due to diminished thermal diffusivity. These results contribute to the theoretical development of magnetohydrodynamic Casson fluid dynamics and extend understanding of thermo-magnetic interactions in non-Newtonian heat transfer systems. The study concludes that magnetic field modulation and convective parameters play crucial roles in controlling Casson fluid behavior and boundary-layer structure, and recommends that future models incorporate nanoparticle effects and biological considerations to improve prediction accuracy. The implications span applied mathematics, heat transfer modeling, and industrial fluid engineering, with potential applications in cooling systems, polymer processing, and energy devices.

Keywords: Casson Fluid; Mixed Convection; Thermo-Magnetic Flow; Magnetohydrodynamics (MHD); Non-Newtonian Heat Transfer.

INTRODUCTION

The study of magnetohydrodynamic (MHD) Casson fluid flow over a vertical stretching porous surface under the influence of mixed convection has considerable relevance in engineering and industrial processes, including polymer extrusion, metallurgical operations, and cooling of electronic devices. The behavior of such non-Newtonian fluids is significantly influenced by the combined effects of the stretching velocity of the surface, buoyancy forces, variable wall temperature, and applied magnetic fields. Although extensive research has been conducted on Newtonian and some non-Newtonian fluid flows, the combined impact of nonlinear porous surfaces, variable surface velocity, temperature-dependent effects, and vertical magnetic field interactions on Casson fluids is still not fully understood. The study of the heat transfer over a nonlinear inclined extended surface which is a non-Newtonian fluids has receive much attention in the last few decades due to its numerous application in industries and applied sciences such as aerodynamics, paper work, coating ,polymer processing etc ,and also Magnetohydrodynamic(MHD) flows are essential in a wide range of industrial and technical usages, such as MHD generators, the design of the nuclear reactor, and flow meters. Magnetohydrodynamics (MHD) is the science that investigates the flow of electrically conducting solutions in the presence of a magnetic field. Numerous innovative

and expected investigations on ordinary electrically conducting flows show that the application of a magnetic field significantly alters their transportation and heat transfer properties. The analysis of magnetohydrodynamics (MHD) has several significant things, the cooling of nuclear power plants using liquid sodium and the induction pressure gauge, which is relied on fluid potential differences perpendicular to motion and the magnetic field, among other things analyzed the numerical simulation of heat transfer in MHD stagnation point flow of Cross fluid model towards a stretched surface. Studied the analysis of dynamic and thermal boundary layers. Further evolution, with a vertical flat plate, as well by using similarity solutions. discussed the numerical analysis of the stagnation point flow of radiative magnetomicropolar liquid past a stretching sheet. Swain et al presented the hybrids nano particles made of MWCNT and Fe_3O_4 have an impact on an exponentially porous shrinking sheet with chemical reaction and slip solvable specific.Furthermore, the inclusion of radiation term in the energy equation makes the equation to be more highly non linear .However ,some reasonable simplification are used to make system solvable .To be specific ,one of this simplifications is the assumption of optically dense medium in which radiation travels only short distance before being absorbed or scattered.This assumption physically means that the local radiation intensity at a point is assumed to emerge only within the neighborhood of that point. Several works on the interaction of forced convection with thermal radiation have increased greatly because of its importance in many practical applications. Radiation heat transfer becomes more significant with rising temperature levels and may be totally dominant over mix conduction and convection at very high temperature and thermal radiation is important in combustion application (furnaces, rocket, nozzles, engines etc) in nuclear reactors. In some recent research the variation of temperature and thermal parameters were observed.

METHODS

Consider a thermo-magnetic Casson fluid flowing over a nonlinear porous stretching sheet in the region. The stretching velocity and wall temperature are given by: $u_{w(x)} = bx^n$ and varying wall temperature $T_w = T_\infty + \delta x^n$ where δ is a positive constant. A spatially varying vertical magnetic field acts on the fluid $B(x) = B_0 x^{\frac{(m-1)}{2}}$ The induced magnetic and electric fields are neglected due to the low magnetic Reynolds

number. The flow is governed by the continuity, momentum, and energy equations for a Casson fluid:

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1.0)$$

Momentum equation

$$\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \gamma\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\gamma}{k}\left(1 + \frac{1}{\beta}\right)u - \frac{\sigma B^2(x)u}{\rho} + g(\beta\tau_w(\tau_w - \tau_\infty) + g\beta^2 T_1(T - T_\infty)^2 + g\beta C_w(C - C_\infty) + g\beta^2 C_1(C - C_\infty)^2 cos\varphi \quad (2.0)$$

Energy equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial y^2}\right) + \frac{Q_0}{\rho C_P}(T - T_W) + \tau\left[D_B\left(\frac{\partial T \partial C}{\partial y \partial y}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] + \frac{V}{C_P}\left(1 + \frac{1}{\beta}\right) \quad (3.0)$$

Concentration equation

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\left(\frac{\partial^2 C}{\partial y^2}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right) - R(C - C_\infty) \quad (4.0)$$

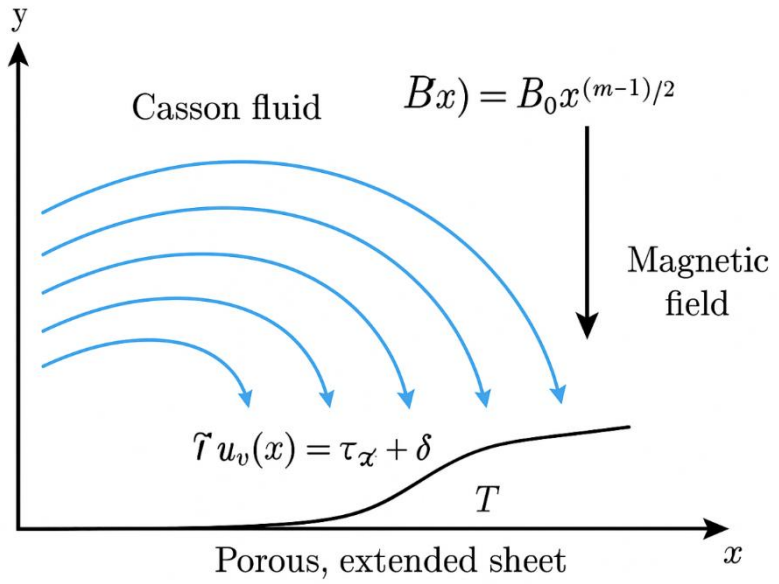


Fig. 1.0 Geometry of the problem

Using the system of non-dimensional transformations forms non-dimensional equations as follows:

$$\left(1 + \frac{1}{\beta}\right)f'''(\eta) - nf'(\eta)^2 + \left(\frac{n+1}{2}\right)f(\eta)f''(\eta) - (M + N)f'(\eta) + G_{r_1}\theta(\eta) + G_{r_2}\theta(\eta)^2 + G_{m_1}\phi(\eta) + G_{m_2}\phi(\eta)^2 cos\varphi = 0 \quad (5.0)$$

$$\frac{1}{P_r}\theta''(\eta) + \left(\frac{n+1}{2}\right) f(\eta)\theta'(\eta) + \lambda\,\theta(\eta) + N_b\theta'(\eta)\phi'(\eta) + N_t\theta'(\eta)^2 = 0 \quad (6.0)$$

$$\theta''(\eta) + \left(\frac{n+1}{2}\right) S_c f(\eta)\phi'(\eta) + \frac{N_b}{N_t}\theta''(\eta) - RS_c\phi(\eta) = 0 \quad (7.0)$$

Boundary conditions will be:

$$u = u_{w(x)},\ v = v_w,\ T = T_w = T_\infty + ax^n,\ D_B\left(\frac{\partial C}{\partial y}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right) = 0 \ \text{at}\ y = 0$$

$$U = ax^n f'(\eta) = 0,\ \theta(\eta) = \frac{T-T_W}{T_{W-T_\infty}},\ (\eta) = \frac{C-C_W}{C_{W-C_\infty}},\ \eta = \sqrt{\frac{a}{\gamma}}x^{\left(\frac{n-1}{2}\right)} y f'(\eta)$$

$$U \to 0,\ T \to T_\infty,\ C \to C_\infty,\ as\ \to \infty$$

Parameters used are given below:

$$S_c = \frac{\gamma}{D_B}\ ,\ \frac{N_b}{N_t} = \frac{D_T(T_W-T_\infty)}{D_B T_\infty (C_W-C_\infty)}\ ,R = \frac{R_0}{D_B a x^{(n-1)}}$$

$$\lambda = \frac{Q_0}{\rho C_P a x^{n-1}},\ P_r = \frac{\gamma}{\alpha},\ \frac{1}{P_r} = \frac{\alpha}{\gamma},\ N_b = \frac{\tau D_B(C_W-C_\infty)}{\gamma} \qquad N_t =$$

$$\tau\frac{D_T(T_W-T_\infty)}{T_\infty\gamma}\ E_C = \frac{a^2x^{2n}}{C_P(T_W-T_\infty)}$$

$$M = \sigma\frac{B_0{}^2}{\rho} \qquad K = \frac{\gamma}{k}\left(1+\frac{1}{\beta}\right)\frac{1}{ax^{(2n-1)}} \qquad G_{r_1} = \frac{g(\beta\tau_W(\tau_W-\tau_\infty)}{a^2x^{(2n-1)}}$$

$$G_{r_2} = \frac{g\beta^2T_1[\tau_W-\tau_\infty]^2}{a^2x^{(2n-1)}} \qquad G_{m_1} = \frac{g\beta C_W(C_W-C_\infty)}{a^2x^{(2n-1)}} \qquad G_{m_2} = \frac{g\beta^2C_1(C_W-C_\infty)}{a^2x^{(2n-1)}}$$

Numerical Solution

Equation1.0 was identically satisfied, hence the continuity equation is satisfied.

However the ordinary differential equations 1.0, 2.0, 3.0 and 4.0 were solved using finite difference methods with the aid of Maple 18.0 Software package.

RESULTS

The numerical results are presented. Graphical representations were used to show the result.

The result presented in figures 4.1 to 4.10. Figure 4.1, 4.2, 4.3, 4.4 depict the variation of the velocity profile in terms of the Casson fluid constraint parameter, Power law index, Porosity parameter, magnetic parameter respectively. Figure 4.5, 4.6 and 4.7

depict the variation of thermal field in terms of Thermophoresis, Brownian motion, and Eckert number respectively. Figure 4.8, 4.9 and 4.10 portray the variation of the inclined angle in terms of the Casson fluid constraint, Brownian motion and Thermophoresis respectively.

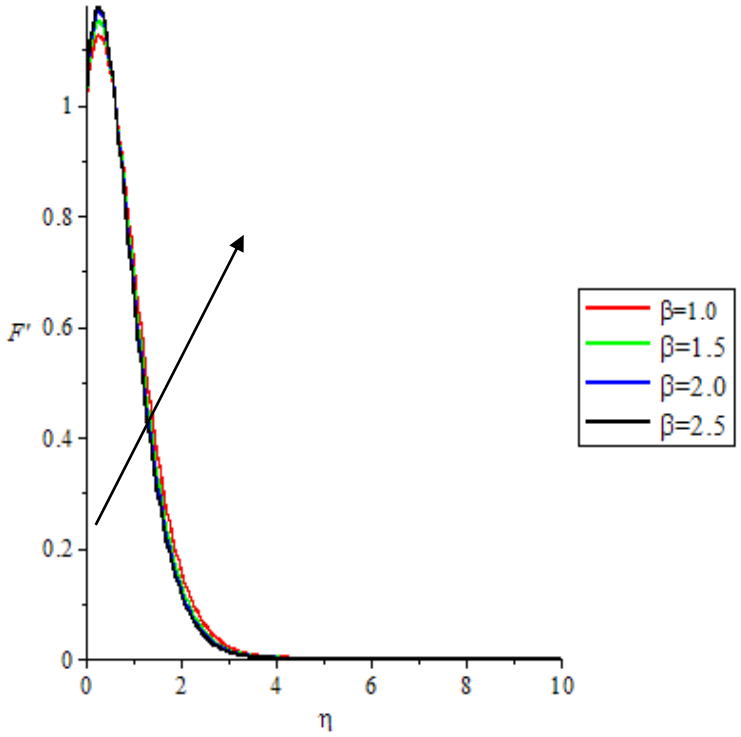


Figure 4.1: Graph of F' (η) for various values of β.

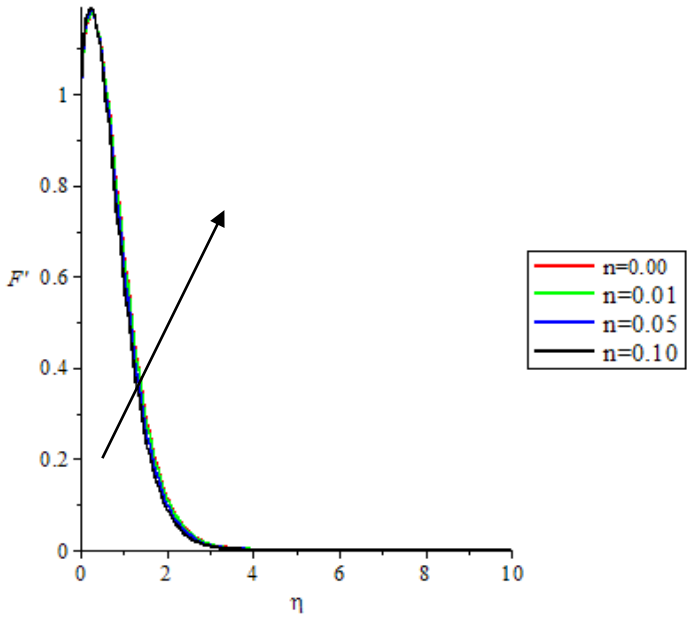


Figure 4.2: Graph of F' (η) for various values of n.

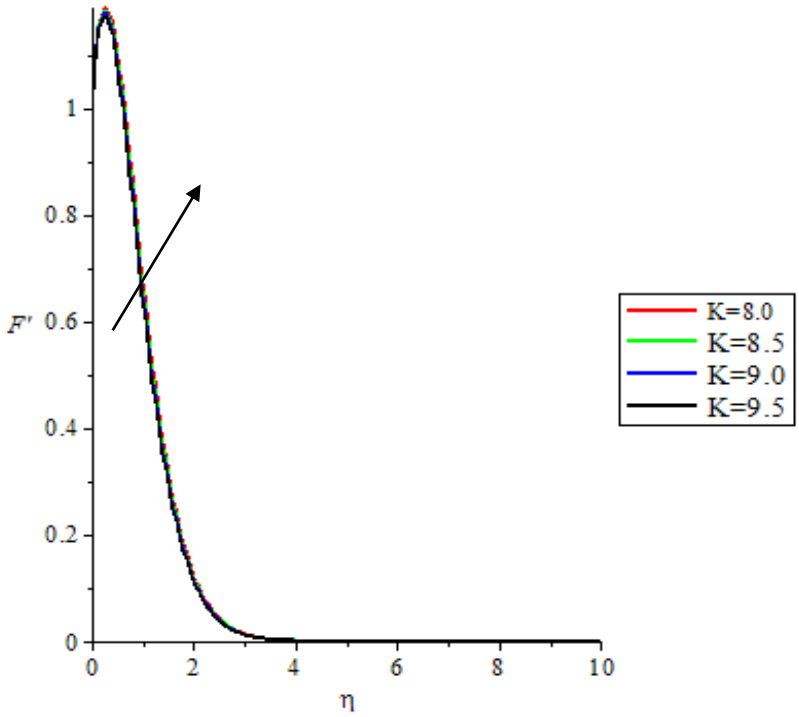


Figure 4.3: Graph of F' (η) for various values of K.

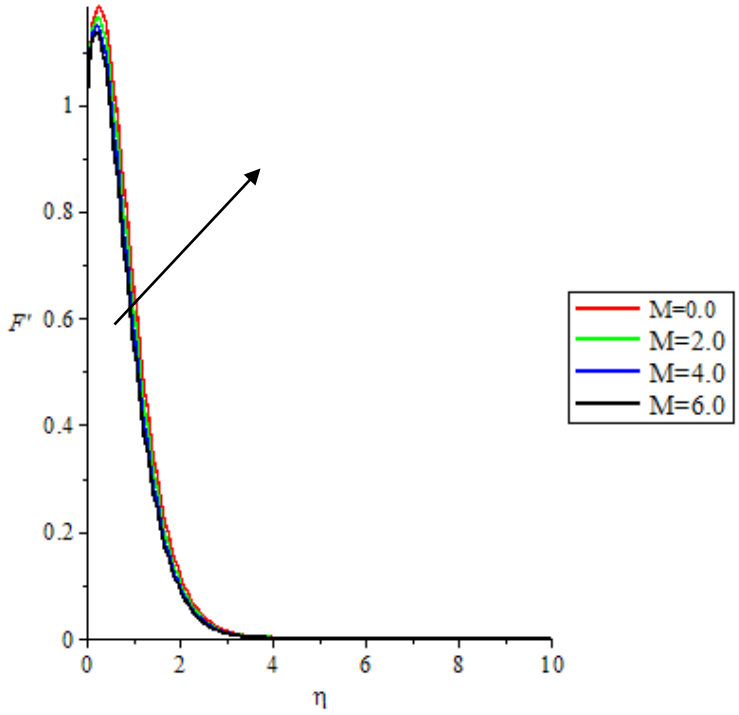


Figure 4.4: Graph of F' (η) for various values of M.

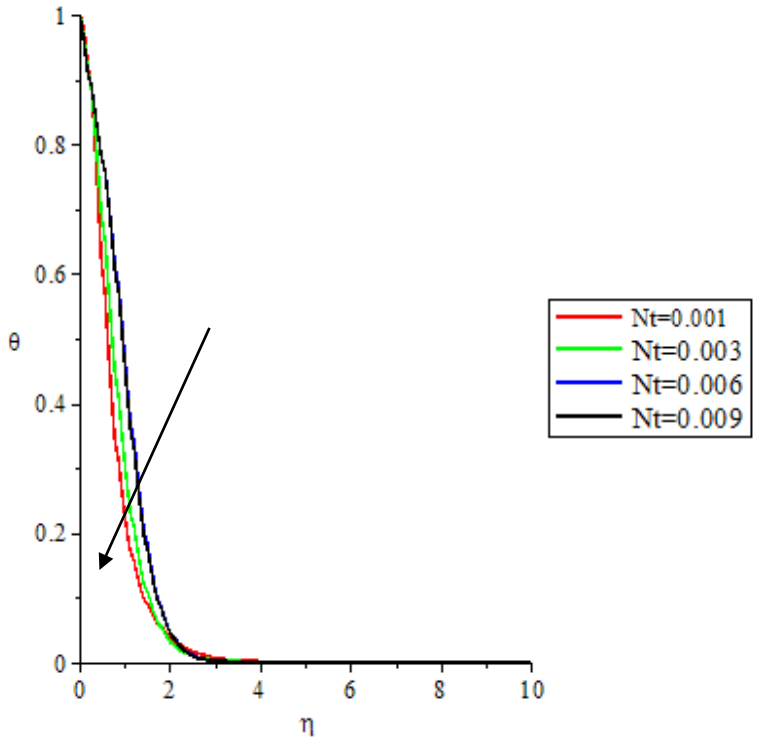


Figure 4.5: Graph of θ (η) for various values of Nt.

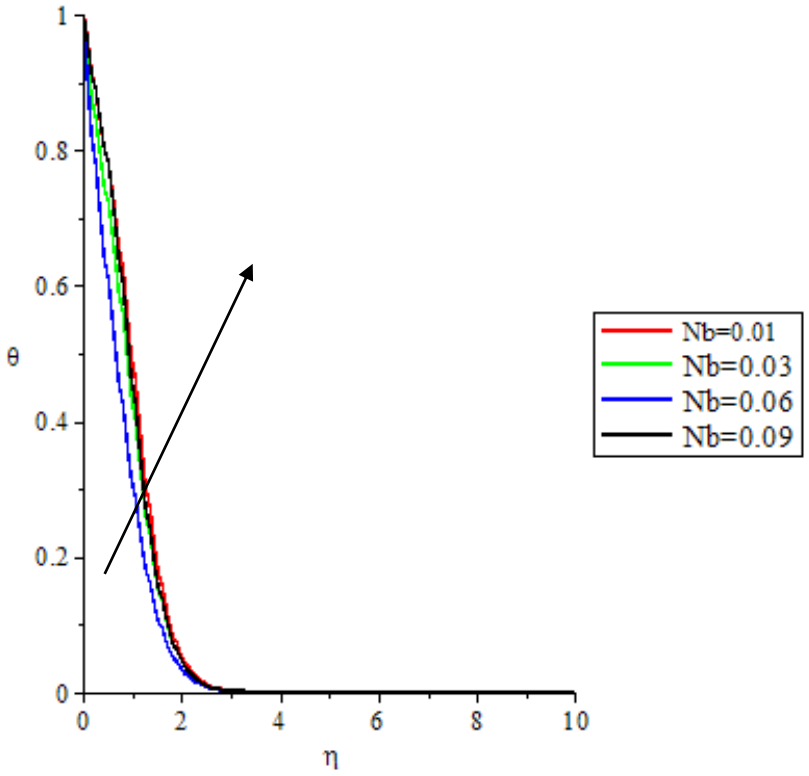


Figure 4.6: Graph of θ (η) for various values of Nb.

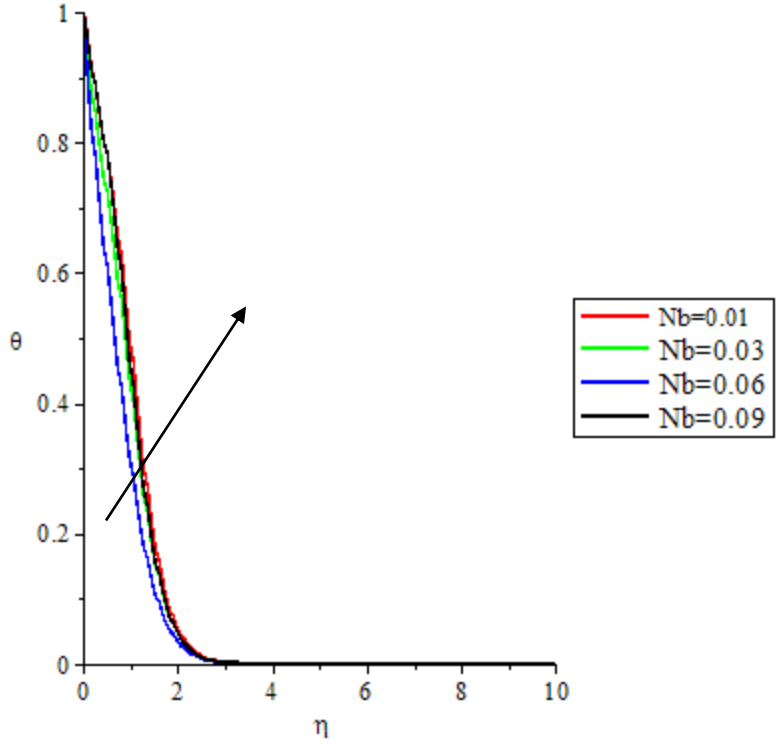


Figure 4.7: Graph of θ (η) for various values of Ec.

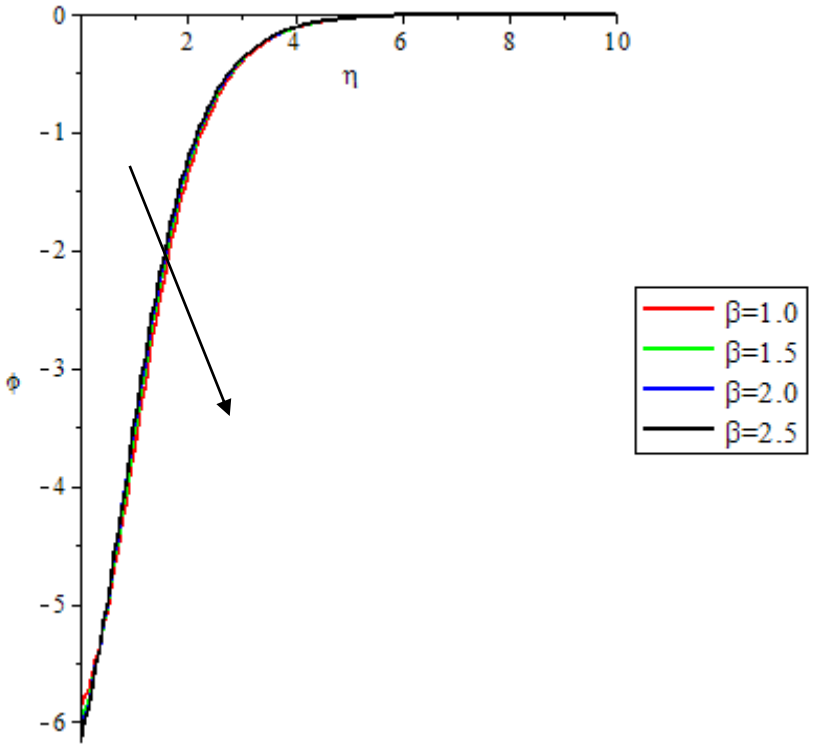


Figure 4.8: Graph of φ (η) for various values of β.

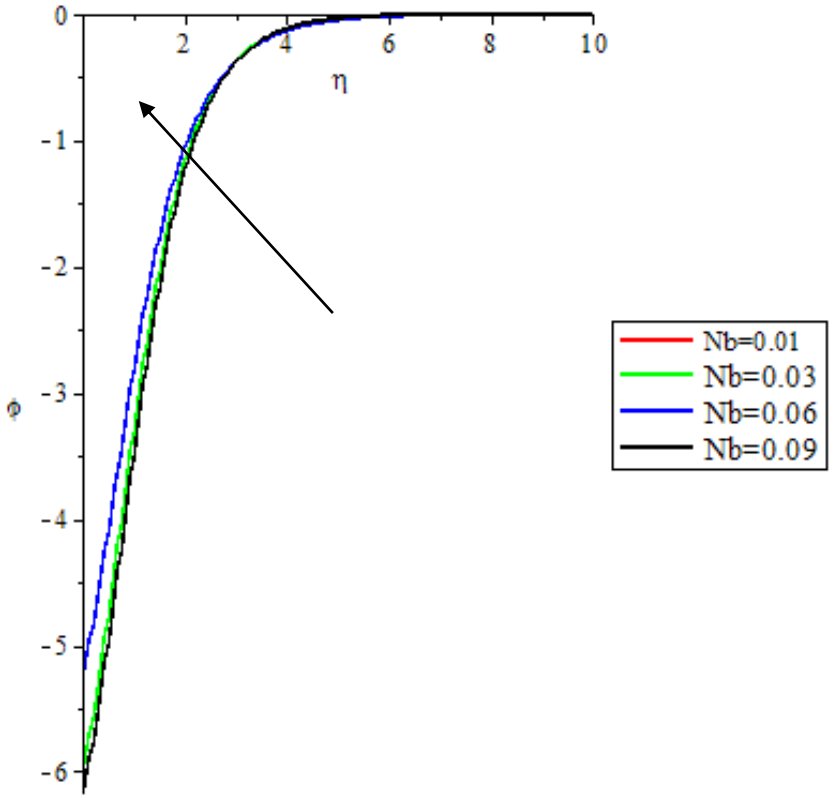


Figure 4.9: Graph of φ (η) for various values of Nb.

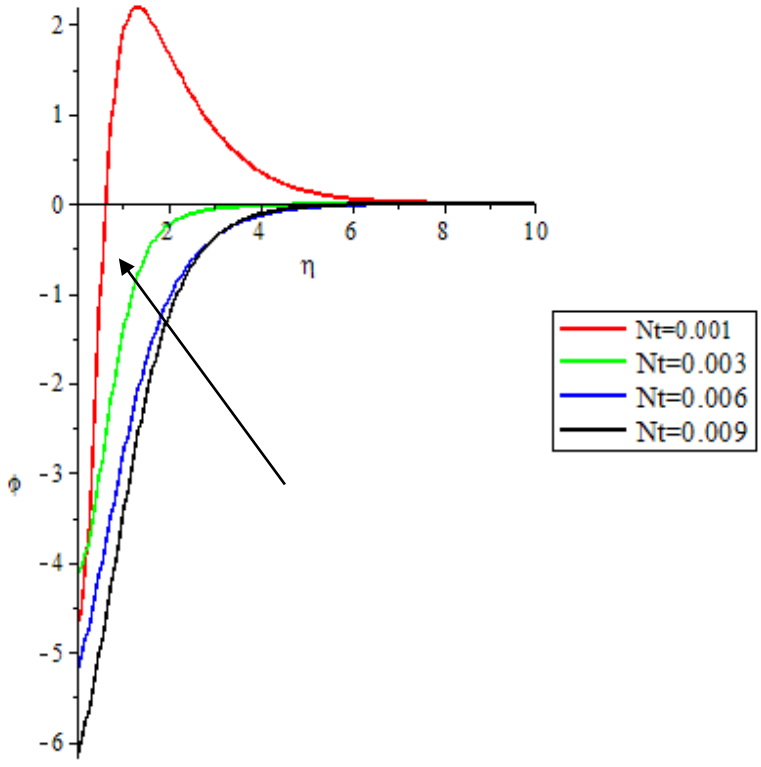


Figure 4.10: Graph of φ (η) for various values of Nt.

CONCLUSION

From the above study of impact of mixed convection on MHD casson fluid and its effects on a stretching plate the following important remarks can be summarized:

The computational analysis has been made on the impact of mixed convection rate of MHD Casson fluid flow over an inclined plate. Mixed convection parameter, Magnetic parameter, Porosity parameter and non-linear stretching rate. Parameters are the major contribution of this project. Based on the result obtained from this work the following conclusion were made;

i. It was observed that the magnetic parameter decays its velocity profile, while it increases the thermal fluid.

ii. The velocity profile is enhanced as the mixed convection parameter increases.

iii. Also a reduction is noticed on the velocity profile as the permeability parameter increases.

iv. Eckert number and Prandtl number are both dimensionless physical properties of the thermal fluid.

v. The thermal fluid is noticed to increase with increases in the Eckert number while the temperature profile decreases with increase in the Prandtl number. This is because the Prandtl number is inversely proportional to the thermal conductivity of the medium.

Recommendation

Based on the result obtained in this problem, it is hereby recommended that researchers can rebuild the model by adding micro organism profile to the system.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared toInfluence the work reported in this paper.

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REFERENCES

Abbasbandy, S., Hayat, T., Alsaedi, A., & Rashidi, M. M. (2014). Numerical and analytical solutions for Falkner-Skan flow of MHD Oldroyd-B fluid. *International Journal of Numerical Methods for Heat & Fluid Flow, 24*(2), 390–401. https://doi.org/10.1108/hff-05-2012-0096

Akindele, A. O., Ajala, O. A., Adegbite, P., & Ogunsola, A. W. (2021). Convective flow of nanofluids using Blasius-Rayleigh-Stoke variable with slip effect. *IOSR Journal of Mathematics, 17*(3), 1–8.

Akindele, A. O., & Ogunsola, A. W. (2021). A study of non-isothermal permeable flow of nano-fluid in a stretchable rotating disk system. *J. Math. Comput. Sci., 11*(2), 1486–1498. https://scik.org/index.php/JMCS/article/view/5954/3047

Bhattacharyya, K., Hayat, T., & Alsaedi, A. (2013). Analytic solution for magnetohydrodynamic boundary layer flow of Casson fluid over a stretching/shrinking sheet with wall mass transfer. *Chinese Physics B, 22*(2), 024702. https://doi.org/10.1088/1674-1056/22/2/024702

Gireesha, B. J., Ganesh Kumar, K., Krishnamurthy, M. R., Manjunatha, S., & Rudraswamy, N. G. (2019). Impact of ohmic heating on MHD mixed convection flow of Casson fluid by considering cross diffusion effect. *Nonlinear Engineering, 8*(1), 380–388. https://doi.org/10.1515/nleng-2017-0144

Khan, I., Ali, F., Samiulhaq, & Shafie, S. (2013). Exact solutions for unsteady magnetohydrodynamic oscillatory flow of a Maxwell fluid in a porous medium. *Zeitschrift für Naturforschung A, 68*(10–11), 635–645. https://doi.org/10.5560/zna.2013-0040

Kodi, R., & Mopuri, O. (2022). Unsteady MHD oscillatory Casson fluid flow past an inclined vertical porous plate in the presence of chemical reaction with heat absorption and Soret effects. *Heat Transfer, 51*(1), 733–752. https://doi.org/10.1002/htj.22327

Kumar, K. K., Baby Rani, C. H., & Papa Rao, A. V. (2021). MHD Casson fluid flow along inclined plate with Hall and aligned magnetic effects. *Frontiers in Heat and Mass Transfer, 17*, 2. https://doi.org/10.5098/hmt.17.2

Mehmood, U. O., Mustapha, N., & Shafie, S. (2014). Nonlinear peristaltic flow of Walter's B fluid in an asymmetric channel with heat transfer and chemical reactions. *Thermal Science, 18*(4), 1095–1107. https://doi.org/10.2298/tsci110921096m

Mukhopadhyay, S., De, P. R., Bhattacharyya, K., & Layek, G. C. (2013). Casson fluid flow over an unsteady stretching surface. *Ain Shams Engineering Journal, 4*(4), 933–938. https://doi.org/10.1016/j.asej.2013.04.004

Nawaz, M. (2020). Numerical study on the impact of variable diffusion coefficients and chemical reaction on transport phenomenon in nonlinear axisymmetric flow. *Physica Scripta, 95*(1), 015203. https://doi.org/10.1088/1402-4896/ab4853

Nawaz, M., Arif, U., & Qureshi, I. H. (2019). Impact of temperature-dependent diffusion coefficients on heat and mass transport in viscoelastic liquid using generalized Fourier theory. *Physica Scripta, 94*(11), 115206. https://doi.org/10.1088/1402-4896/ab1cec

Obalalu, A. M., Ajala, O. A., Abdulraheem, A., & Akindele, A. O. (2021). The influence of variable electrical conductivity on non-Darcian Casson nanofluid flow with first-

and second-order slip conditions. *Partial Differential Equations in Applied Mathematics, 4*, 100084. https://doi.org/10.1016/j.padiff.2021.100084

Oyinkansola, S., Martins Obalalu, A., Ajala, O. A., Bakare, T. B., Ogunsola, A. W., & Akindele, A. O. (2023). Thermal radiation effect on non-Newtonian Casson fluid through a porous material over a magnetic field with buoyancy. *International Journal of Thermofluid Science and Technology, 10*(1).

Parvin, S., Mohamed Isa, S. S. P., Arifin, N. M., & Md Ali, F. (2021). The inclined factors of magnetic field and shrinking sheet in Casson fluid flow, heat and mass transfer. *Symmetry, 13*(3), 373. https://doi.org/10.3390/sym13030373

Prasad, D. K., Chaitanya, G. K., & Raju, R. S. (2019). Double diffusive effects on mixed convection Casson fluid flow past wavy inclined plate in presence of Darcian porous medium. *Results in Engineering, 3*, 100019. https://doi.org/10.1016/j.rineng.2019.100019

Rajakumar, K. V. B., Umasankara Reddy, M., Balamurugan, K. S., & Raja Ram, K. V. B. S. (2020). Steady MHD Casson ohmic heating and viscous dissipative fluid flow past an infinite vertical porous plate in the presence of Soret, Hall, and ion-slip current. *Heat Transfer, 49*(3), 1583–1612. https://doi.org/10.1002/htj.21680

Raveendra, N., Veena, P. H., & Pravin, V. K. (2017). Mass transfer and radiative heat transfer flow of MHD Casson fluid with temperature gradient-dependent heat sink and internal mass diffusion in a vertical channel with stretching porous walls. *Chem Process Engin Res, 47*, 34–43.

Rawi, N. A., Ilias, M. R., Lim, Y. J., Isa, Z. M., & Shafie, S. (2017). Unsteady mixed convection flow of Casson fluid past an inclined stretching sheet in the presence of nanoparticles. *Journal of Physics: Conference Series, 890*(1), 012048. https://doi.org/10.1088/1742-6596/890/1/012048

Saeed, A., Algehyne, E. A., Aldhabani, M. S., Dawar, A., Kumam, P., & Kumam, W. (2022). Mixed convective flow of a magnetohydrodynamic Casson fluid through a permeable stretching sheet with first-order chemical reaction. *PLOS ONE, 17*(4), e0265238. https://doi.org/10.1371/journal.pone.0265238

Sadeghy, K., Khabazi, N., & Taghavi, S. M. (2007). Magnetohydrodynamic (MHD) flows of viscoelastic fluids in converging/diverging channels. *International Journal of Engineering Science, 45*(11), 923–938. https://doi.org/10.1016/j.ijengsci.2007.05.007

Santhosh, N., Radhakrishnamacharya, G., & Chamkha, A. J. (2015). Flow of a Jeffrey fluid through a porous medium in narrow tubes. *Journal of Porous Media, 18*(1), 71–78. https://doi.org/10.1615/jpormedia.v18.i1.50

Sharada, K., & Shankar, B. (2015). MHD mixed convection flow of a Casson fluid over an exponentially stretching surface with the effects of Soret, Dufour, thermal radiation, and chemical reaction. *World Journal of Mechanics, 5*(9), 165–177. https://doi.org/10.4236/wjm.2015.59017

Venkata Ramana, K., Gangadhar, K., Kannan, T., & Chamkha, A. J. (2022). Cattaneo–Christov heat flux theory on transverse MHD Oldroyd-B liquid over nonlinear stretched flow. *Journal of Thermal Analysis and Calorimetry, 147*, 2749–2759. https://doi.org/10.1007/s10973-021-10568-x

Zakaria, M. N., Hussanan, A., Khan, I., & Shafie, S. (2013). The effects of radiation on free convection flow with ramped wall temperature in Brinkman-type fluid. *Jurnal Teknologi (Sciences & Engineering), 62*(3), 33–39. https://doi.org/10.11113/jt.v62.1886