Mikailalsys Journal of Mathematics and Statistics

e-ISSN : 3030-816X
p-ISSN : 3030-8399

Index : Harvard, Boston, Sydney University, Dimensions, Lens, Scilit, Semantic, Google, etc

https://doi.org/10.58578/MJMS.v4i1.7493

On the Numerical Solutions of Linear and Nonlinear Differential Equations by the Modified Laplace–Adomian Polynomials Method

J. O. Okai[1], Iliya Martha[2], M. Y. Adamu[3], U. A. Mujahid[4], L. N. Sanda[5]
[1,2,3,5]Abubakar Tafawa Balewa University, Bauchi State, Nigeria
[4]Federal Polytechnic Kaltungo, Gombe State, Nigeria
jerry4complex@gmail.com; umaraliyumujahid@gmail.com

Article Info:

Submitted:	Revised:	Accepted:	Published:
Oct 2, 2025	Oct 24, 2025	Nov 8, 2025	Nov 13, 2025

Abstract

This study employs the Laplace–Adomian Polynomial Method (LAPM) to obtain approximate solutions for both linear and nonlinear ordinary differential equations. LAPM integrates the Laplace transform with Adomian polynomials to manage nonlinear terms effectively, avoiding the need for linearization or perturbation techniques. To evaluate the method's accuracy and computational efficiency, three representative examples were solved, with the results benchmarked against corresponding exact solutions. The numerical outcomes, presented through tables and graphical comparisons, demonstrate that LAPM provides highly accurate approximations with minimal error using only a few series terms. The findings affirm that the method is not only straightforward and computationally efficient but also broadly applicable to various nonlinear problems. Given its robustness and simplicity, LAPM holds promise for extension to more complex systems, including partial differential equations and multi-dimensional models in applied sciences.

Keywords: Nonlinear Differential Equations; Laplace Transform; Adomian Polynomials; Approximate Solutions; Semi-Analytical Methods

MJMS Journal is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License

INTRODUCTION

The Laplace transform is one of the most widely used analytical techniques for solving differential equations, particularly initial value problems in engineering and physics. Its strength lies in transforming differential equations into algebraic equations in the Laplace (frequency) domain, where they can be solved more easily before transforming back to the time domain. This makes it highly effective for linear differential equations with constant or variable coefficients (Khuri, 2001; Fadaei, 2011). However, the classical Laplace transform struggles with nonlinear differential equations because the Laplace operator is linear, and thus cannot directly handle nonlinear terms (Adomian, 1988; Cherruault, 1989). For instance, terms like $u^2, e^u, uu_x or \sin u$ cannot be easily transformed and manipulated within the Laplace framework.

To overcome this limitation, researchers have sought to combine the Laplace transform with nonlinear decomposition techniques. One of the most powerful and well-established methods for nonlinear problems is the Adomian Decomposition Method (ADM), originally introduced by Adomian (1988) and further developed in later works (Cherruault & Adomian, 1989; Li & Pang, 2020). ADM provides a way to represent the nonlinear part of a differential equation as an infinite series of Adomian polynomials, which are recursively generated from the components of the solution series. The method avoids perturbation, linearization, or small-parameter assumptions, making it applicable to a wide range of nonlinear problems, including ordinary differential equations (ODEs), partial differential equations (PDEs), and even fractional differential equations (FDEs) (Odibat, 2020; Ziane et al., 2019).

The Laplace-Adomian Decomposition Method (LADM), first proposed by Khuri (2001) and later extended by Fadaei (2011), integrates the Laplace transform with ADM. In this hybrid approach, the Laplace transform is applied to the linear part of the differential equation, while the nonlinear part is expressed in terms of Adomian polynomials. This technique has been successfully used to solve many nonlinear problems, including the Duffing oscillator (Khuri, 2001), nonlinear optical solitons (González-Gaxiola & Biswas, 2019), and systems of nonlinear PDEs (Fadaei, 2011).

However, despite its effectiveness, LADM still suffers from slow convergence and accumulation of truncation errors, especially in the case of highly nonlinear or stiff systems (Gündoğdu & Gözükızıl, 2017). To address these issues, Aland and Singh (2022) proposed

the Laplace Transform Modified Adomian Decomposition Method (LT-MADM). This method improves the convergence rate by introducing a modified way of constructing the Adomian polynomials and refining the recursive formulation. Their work demonstrated that LT-MADM yields solutions that are closer to exact solutions and requires fewer terms for high accuracy compared to LADM.

The Adomian Decomposition Method (ADM), introduced by George Adomian (1988), provides an efficient semi-analytical technique for solving nonlinear differential equations without requiring linearization, perturbation, or small-parameter assumptions. ADM expresses the solution as a rapidly converging infinite series and approximates nonlinear terms using Adomian polynomials. The method has been successfully applied to ordinary differential equations (ODEs), partial differential equations (PDEs), fractional differential equations (FDEs), and integral equations (Cherruault & Adomian, 1989; Odibat, 2020). Li and Pang (2020) analyzed convergence properties of ADM and showed its effectiveness in solving nonlinear systems. Ziane et al. (2019) proposed a modified version of ADM to improve accuracy in nonlinear PDEs. González-Gaxiola and Biswas (2019) applied ADM to nonlinear optical soliton models. However, while ADM is powerful, it can sometimes converge slowly, especially for stiff or strongly nonlinear systems.

To combine the strengths of both the Laplace transform and ADM, Khuri (2001) proposed the Laplace Adomian Decomposition Method (LADM). In this hybrid method, the Laplace transform is applied to handle the linear part of the differential equation, while Adomian polynomials are used to decompose and approximate the nonlinear part. This approach has been applied to various nonlinear models: Khuri (2001) successfully solved the Duffing oscillator equation using LADM. Fadaei (2011) extended LADM to systems of linear and nonlinear PDEs. Gündoğdu and Gözükızıl (2017) applied LADM to nonlinear PDEs and compared it with the modified decomposition method and standard ADM, showing reasonable accuracy. Despite these successes, LADM sometimes suffers from slow convergence and accumulation of truncation errors, making it less effective for highly nonlinear or stiff problems.

To address the limitations of LADM, Aland and Singh (2022) introduced the Laplace Transform Modified Adomian Decomposition Method (LT-MADM). This method modifies the construction of Adomian polynomials and improves the recursive

scheme, yielding better accuracy and faster convergence compared to standard LADM. LT-MADM has been successfully applied to nonlinear partial differential equations, where graphical comparisons with exact solutions demonstrated its effectiveness.

While LT-MADM has proven effective for PDEs, its application to a broader class of nonlinear ordinary and partial differential equations has not been extensively explored. Additionally, systematic comparisons of LT-MADM with LADM in terms of convergence behavior, computational efficiency, and error analysis remain limited. This research seeks to fill these gaps by applying LT-MADM to selected nonlinear ODEs and PDEs, thus extending its applicability and validating its efficiency.

The Laplace transform is a powerful tool for solving linear differential equations, but it cannot directly handle nonlinear terms due to its linearity (Khuri, 2001). To overcome this, the Adomian Decomposition Method (ADM) was introduced to decompose nonlinear terms into a rapidly convergent series of Adomian polynomials (Adomian, 1988). The combined Laplace Adomian Decomposition Method (LADM) has been used for nonlinear problems but often suffers from slow convergence and truncation errors for highly nonlinear systems (Gündoğdu & Gözükızıl, 2017).

The recently proposed Laplace Transform Modified Adomian Decomposition Method (LT-MADM) (Aland & Singh, 2022) improves convergence and accuracy but has been applied mostly to partial differential equations. There is, therefore, a need to extend and test LT-MADM for a wider class of nonlinear ordinary and partial differential equations to provide accurate and efficient numerical solutions.

METHODOLOGY

Basics of the Laplace Transform Method (LTM)

The Laplace Transform is an integral transform that converts a function $f(t)$, defined for $t > 0$, into a complex frequency-domain function $F(s)$. It is defined as:

$$L\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)dt \qquad ...(2)$$

The inverse Laplace transform is expressed as: $L^{-1}\{F(s)\} = f(t)$ ...(3)

The method is highly effective for solving linear differential equations with initial conditions, as it transforms differentiation into simple algebraic manipulation.

Properties of the Laplace Transform

Some important properties that make the Laplace transform suitable for solving differential equations are:

i. **Linearity:**

$$L\{af(t)+bg(t)\}=aF(s)+bF(s)$$

ii. **Differentiation in Time Domain:**

$$L\left\{\frac{df(t)}{dt}\right\}=sF(s)-f(0)$$

iii. **Higher-Order Derivatives:**

$$L\left\{\frac{d^n f(t)}{dt^n}\right\}=s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)-\ldots-f^{(n-1)}(0)$$

iv. **Integration in Time Domain:**

$$L\left\{\int_0^t f(\tau)d\tau\right\}=\frac{1}{s}F(s)$$

These properties allow differential equations to be rewritten as algebraic equations in the s-domain.

Solving Linear Differential Equations Using LTM

Consider a general linear differential equation:

$$a_n\frac{d^n y}{dt^n}+a_{n-1}\frac{d^{n-1}y}{dt^{n-1}}+\ldots+a_0y=g(t),\ y(0),y'(0),\ldots,y^{(n-1)}(0) \qquad \ldots(4)$$

Applying the Laplace transform to both sides and using the differentiation property:

$$a_n\left[S^nY(s)-S^{n-1}y(0)-\ldots-y^{(n-1)}(0)\right]+\ldots+a_0Y(s)=G(s) \qquad \ldots(5)$$

Solving for $Y(s)$:

$$Y(s)=\frac{G(s)+terms\quad involving\qquad initial\ \ conditions}{a_nS^n+a_{n-1}S^{n-1}+\ldots+a_0} \qquad \ldots(6)$$

Finally, applying the **inverse Laplace transform** yields the solution $y(t)$.

Adomian Decomposition Method (ADM)

Let the nonlinear DE be of the form:

$$Dy(t) = g(t) + Ny(t) + Ry(t) \quad ...(7)$$

with initial condition $y(0) = c$. In ADM, the solution $y(t)$ is decomposed into a series:

$$y(t) = \sum_{n=0}^{\infty} y_n(t) \quad ...(8)$$

and the nonlinear term $Ny(t)$ is expanded using Adomian polynomials A_n:

$$N(y(t)) = \sum_{n=0}^{\infty} A_n(t) \quad ...(8)$$

where each Adomian polynomial A_n is generated using:

$$A_n(y_0 + y_1 + y_2,..., y_n) = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{k=0}^{n}\lambda^k y_k\right)\right]_{\lambda=0,n-0,1,2,...} \quad ...(9)$$

where λ is a parameter. The Adomian polynomial A_n can be defined as follows:

$$A_0 = \frac{1}{0!}\frac{d^0}{d\lambda^0}\left[N\left(\sum_{k=0}^{0}\lambda^k y_k\right)\right]_{\lambda=0..} = N(y_0),$$

$$A_1 = \frac{1}{1!}\frac{d^1}{d\lambda^1}\left[N\left(\sum_{k=0}^{1}\lambda^k y_k\right)\right]_{\lambda=0..} = y_1 N'(y_0),$$

$$A_2 = \frac{1}{2!}\frac{d^2}{d\lambda^2}\left[N\left(\sum_{k=0}^{2}\lambda^k y_k\right)\right]_{\lambda=0..} = y_2 N'(y_0) + \frac{y_1^2}{2!}N''(y_0)$$

$\vdots$

Laplace-Adomian polynomial Framework

Applying the Kamal transform to the given FDE:

$$L[Dy(t)] = L[g(t)] + L[Ny(t)] + L[R(t)] \quad ...(10)$$

Using the property of the Laplace transform:

$$L[Dy(t)] = s^n L\{y(t)\} - s^{n-1} L\{y(0)\} - -s^{n-2} L\{y''(0)\} - s^{n-3} L\{y(0)\} \ldots - y^{(n-1)}(0) \quad \ldots(11)$$

For $D = 1$, we get $SY(S) - y(0)$, and we obtain the following:

$$SY(S) - y(0) = L[g(t)] + L[A_n] + L[R(t)] \quad \ldots(12)$$

$$Y(S) = \frac{1}{s}\left(y(0) + L[g(t)] + L[A_n] + L[R(t)]\right)$$

Taking the inverse Laplace transform gives:

$$y(t) = y(0) + L^{-1}\left[\frac{1}{s}\left(L[g(t)] + L[A_n] + L[R(t)]\right)\right]. \quad \ldots(13)$$

Recursive Scheme

We define the recursive components of the solution as:

$$\begin{cases} y_0(t) = y(0) + L^{-1}\left[\frac{1}{s}\left(L[g(t)]\right)\right], \\ y_{n+1}(t) = L^{-1}\left[\frac{1}{s}\left(L[A_n] + L[R(t)]\right)\right], n = 0,1,2,\ldots \end{cases} \quad \ldots(14)$$

Each A_n depends on the previous terms $y_0, y_1, y_2, \ldots, y_n$ and the process is repeated iteratively to construct the approximate solution:

$$y(t) \approx \sum_{n=0}^{N} y_n(t) \quad \ldots(15)$$

where N is chosen based on convergence or required accuracy.

RESULTS

Nonlinear Differential Equations

Example 1

Given the fractional differential equation as follows:

$$y'(t) = y^2(t) + 1, \qquad t > 0, \quad \ldots(16)$$

With the initial condition

$$y(0) = 0 \quad \ldots(17)$$

And the exact solution

$$y(t) = \tan(t). \qquad ...(18)$$

Using the recursive formulas in Eqn. (13) and (14), the approximate solution to the differential Eqn. (16) is derived through the combined method of the Adomian polynomial and the Laplace transformation, as follows:

$$\begin{cases} y_0(t) = L^{-1}\left[\frac{1}{s}L\{1\}\right], \\ y_{n+1}(t) = L^{-1}\left[\frac{1}{s}\left\{L\sum_{n=0}^{\infty}A_n(t)\right\}\right], \quad n = 0,1,2,... \end{cases} \qquad ...(19)$$

where A_n is the Adomian polynomial of the nonlinear operator $Ny = y^2$, which can be described as follows

$$\begin{cases} A_0 = y_0^2, \\ A_1 = 2y_0y_1, \\ A_1 = 2y_0y_2 + y_1^2, \\ \vdots \end{cases} \qquad ...(20)$$

The following is a description of the solution to the Eqn. (19):

$$y_0(t) = L^{-1}\left[\frac{1}{s}L[1]\right] = t,$$

$$y_1(t) = L^{-1}\left[\frac{1}{s}L[A_0]\right] = \frac{1}{3}t^3$$

$$y_2(t) = L^{-1}\left[\frac{1}{s}L[A_1]\right] = L^{-1}\left[\frac{1}{s}L2y_0y_1\right] = \frac{2}{15}t^5$$

$$y(t) = y_0(t) + y_1(t) + y_2(t) + ... = t + \frac{1}{3}t^3 + \frac{2}{15}t^5 + ... \qquad ...(21)$$

Table 1: Computations showing comparison between the exact solution and the LAPM solution of example 1

t	Exact solution	LAPM (n=3)	Absolute Error
0	0	0	0
0.1	0.100334672	0.100334672	2.77556E-17
0.2	0.202710036	0.202710036	4.85167E-14
0.3	0.30933625	0.30933625	2.16804E-11
0.4	0.422793219	0.422793217	1.67159E-09
0.5	0.54630249	0.54630244	4.94376E-08
0.6	0.684136808	0.684136007	8.01444E-07
0.7	0.84228838	0.842279756	8.62437E-06
0.8	1.029638557	1.029569395	6.9162E-05
0.9	1.260158218	1.259711984	0.000446233
1	1.557407725	1.554959773	0.002447952

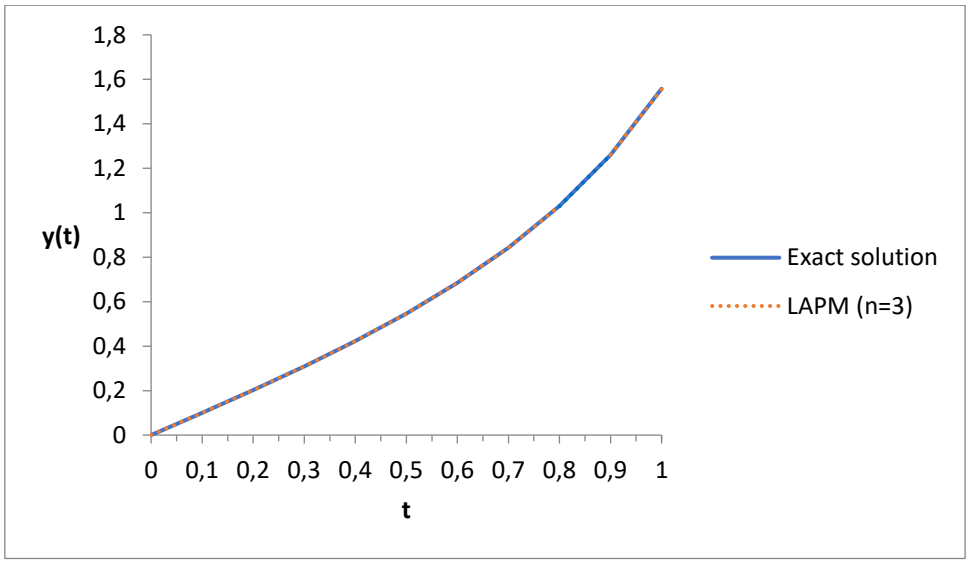


Figure 1: Graph of the LAPM and the exact solution for example 1

Table 2: Comparison between the exact solutions, the LAPM and the DTM for example 1

t	Exact solution	LAPM (n=3)	DTM (n=6)
0	0	0	0
0.1	0.100334672	0.10033467	0.100335
0.2	0.202710036	0.20271004	0.202709
0.3	0.30933625	0.30933625	0.309324
0.4	0.422793219	0.42279322	0.422699
0.5	0.54630249	0.54630244	0.545833
0.6	0.684136808	0.68413601	0.682368
0.7	0.84228838	0.84227976	0.836743
0.8	1.029638557	1.0295694	1.014357
0.9	1.260158218	1.25971198	1.221732
1	1.557407725	1.55495977	1.466667

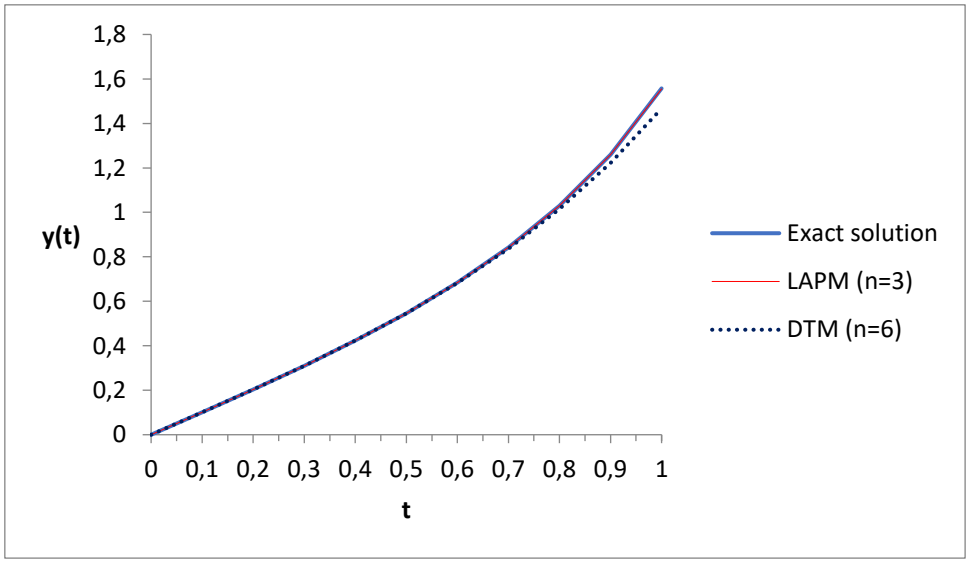


Figure 2: Graph of the LAPM, DTM and the exact solution for example 1

Example 2

Given the fractional differential equation as follows [(Opanuga *et al.*, 2015)]:

$$y'(t) = 1 - y^2(t), \qquad t > 0, \qquad ...(22)$$

With the initial condition

$$y(0) = 0 \qquad ...(23)$$

And the exact solution

$$y(t) = \frac{e^{2t} - 1}{e^{2t} + 1}. \qquad ...(24)$$

Using the recursive formulas in Eqn. (13) and (14), the approximate solution to the differential Eqn. (22) is derived through the combined method of the Adomian polynomial and the Laplace transformation, as follows:

$$\begin{cases} y_0(t) = L^{-1}\left[\frac{1}{s}L\{1\}\right], \\ y_{n+1}(t) = -L^{-1}\left[\frac{1}{s}\left\{L\sum_{n=0}^{\infty} A_n(t)\right\}\right], n = 0,1,2,... \end{cases} \qquad ...(25)$$

where A_n is the Adomian polynomial of the nonlinear operator $Ny = y^2$, which can be described as follows

$$\begin{cases} A_0 = y_0^2, \\ A_1 = 2y_0y_1, \\ A_1 = 2y_0y_2 + y_1^2, \\ \vdots \end{cases} \qquad ...(26)$$

The following is a description of the solution to the Eqn. (25):

$$y_0(t) = L^{-1}\left[\frac{1}{s}L[1]\right] = t,$$

$$y_1(t) = L^{-1}\left[\frac{1}{s}L[A_0]\right] = -\frac{1}{3}t^3$$

$$y_2(t) = L^{-1}\left[\frac{1}{s}L[A_1]\right] = L^{-1}\left[\frac{1}{s}L2y_0y_1\right] = \frac{2}{15}t^5$$

$$y(t) = y_0(t) + y_1(t) + y_2(t) + \ldots = t - \frac{1}{3}t^3 + \frac{2}{15}t^5 + \ldots \quad \ldots(27)$$

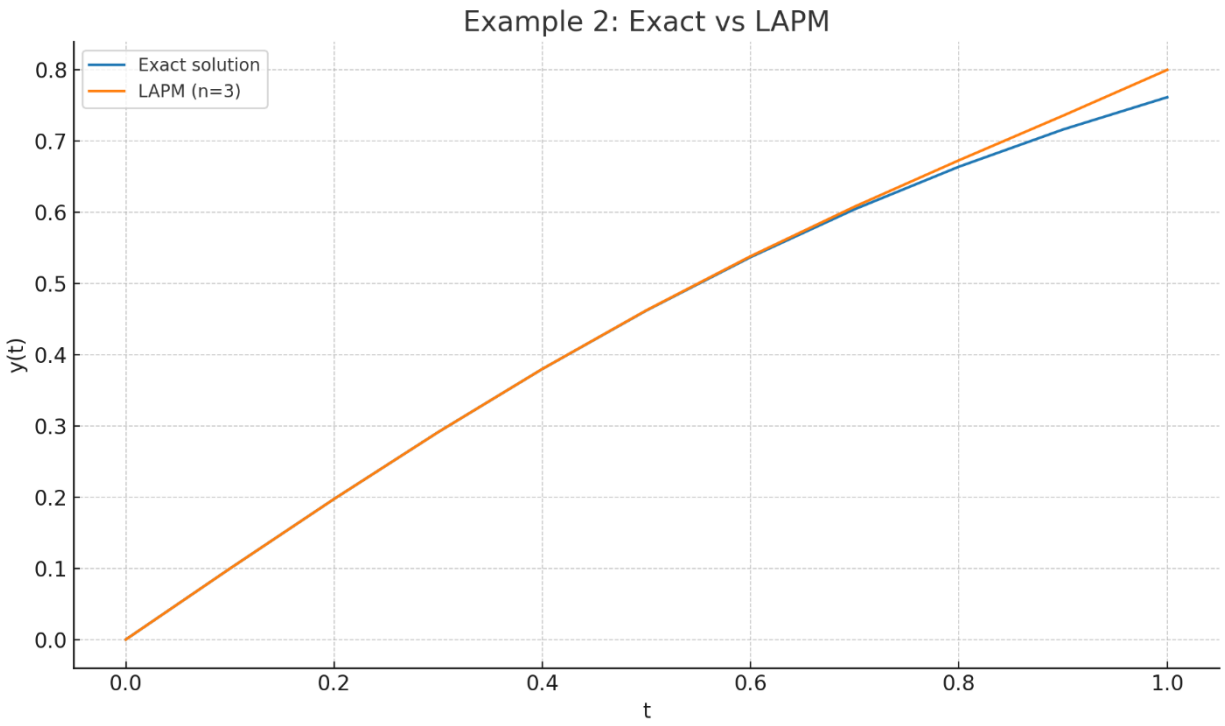


Figure 3: Graph of the LAPM and the exact solution for example 2

Example 3

We consider the Riccati equation below with constant coefficient:

$$y'(t) = y^2(t) - y(t), \quad t > 0, \quad \ldots(28)$$

With the initial condition

$$y(0) = \frac{1}{2} \quad \ldots(29)$$

And the exact solution

$$y(t) = \frac{e^{-t}}{1 + e^{-t}}. \quad \ldots(30)$$

Using the recursive formulas in Eqn. (13) and (14), the approximate solution to the differential Eqn. (28) is derived through the combined method of the Adomian polynomial and the Laplace transformation, as follows:

$$\begin{cases} y_0(t) = \dfrac{1}{2}, \\ y_{n+1}(t) = L^{-1}\left[\dfrac{1}{s}\left\{L\sum_{n=0}^{\infty} A_n(t)\right\}\right] - L^{-1}\left[\dfrac{1}{s}\{Ly_n(t)\}\right], \qquad n = 0,1,2,... \end{cases} \quad ...(31)$$

where A_n is the Adomian polynomial of the nonlinear operator $Ny = y^2$, which can be described as follows

$$\begin{cases} A_0 = y_0^2, \\ A_1 = 2y_0y_1, \\ A_1 = 2y_0y_2 + y_1^2, \\ \vdots \end{cases} \quad ...(32)$$

The following is a description of the solution to the Eqn. (31):

$$y_0(t) = \frac{1}{2},$$

$$y_1(t) = L^{-1}\left[\frac{1}{s}\left\{L\sum_{n=0}^{\infty} A_0(t)\right\}\right] - L^{-1}\left[\frac{1}{s}\{Ly_0(t)\}\right] = -\frac{t}{4},$$

$$y_2(t) = 0$$

$$y_3(t) = \frac{t^3}{48}$$

$$y_4(t) = 0$$

$$\vdots$$

$$y(t) = y_0(t) + y_1(t) + y_2(t) + ... = \frac{1}{2} - \frac{t}{4} + \frac{t^3}{48} + ... \quad ...(33)$$

Table 3: Computations showing comparison between the exact solution and the LAPM solution of example 3

t	Exact solution	LAPM (n=3)	Absolute Error
0	0.5	0.5	0
0.1	0.475020813	0.47502081	0
0.2	0.450166003	0.450166	1.66533E-16
0.3	0.425557483	0.42555748	3.46945E-14

0.4	0.40131234	0.40131234	1.4479E-12
0.5	0.377540669	0.37754067	2.61022E-11
0.6	0.354343694	0.35434369	2.76273E-10
0.7	0.331812228	0.33181223	2.02378E-09
0.8	0.310025519	0.31002553	1.13192E-08
0.9	0.289050497	0.28905055	5.15026E-08
1	0.268941421	0.26894162	1.99076E-07

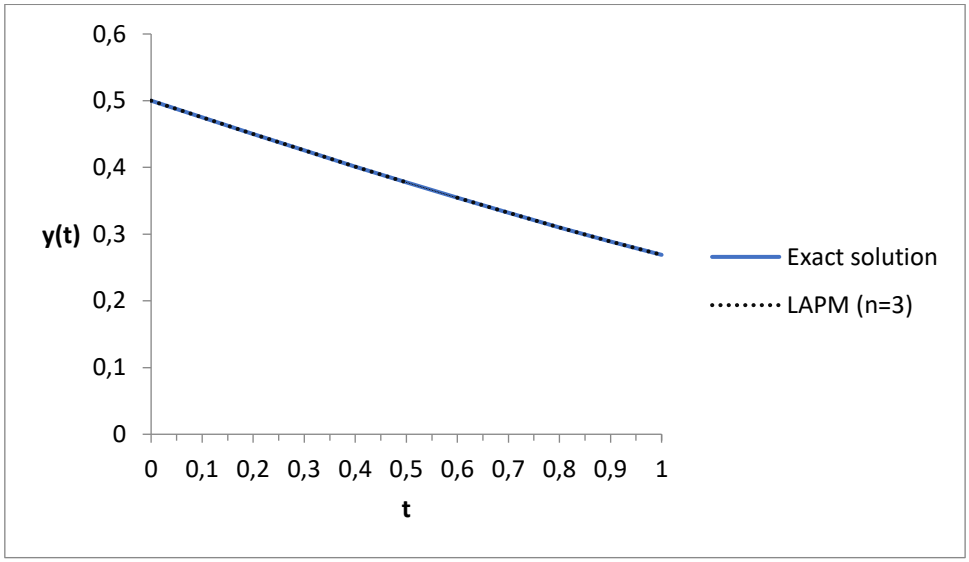


Figure 4: Graph of the LAPM and the exact solution for example 3

Table 4: Comparison between the exact solutions, the LAPM and the DTM for example 3

t	Exact solution	LAPM (n=3)	DTM (n=7)
0	0.5	0.5	0.500
0.1	0.475020813	0.47502081	0.475
0.2	0.450166003	0.450166	0.450
0.3	0.425557483	0.42555748	0.425
0.4	0.40131234	0.40131234	0.400
0.5	0.377540669	0.37754067	0.375
0.6	0.354343694	0.35434369	0.350
0.7	0.331812228	0.33181223	0.325
0.8	0.310025519	0.31002553	0.300
0.9	0.289050497	0.28905055	0.275
1	0.268941421	0.26894162	0.250

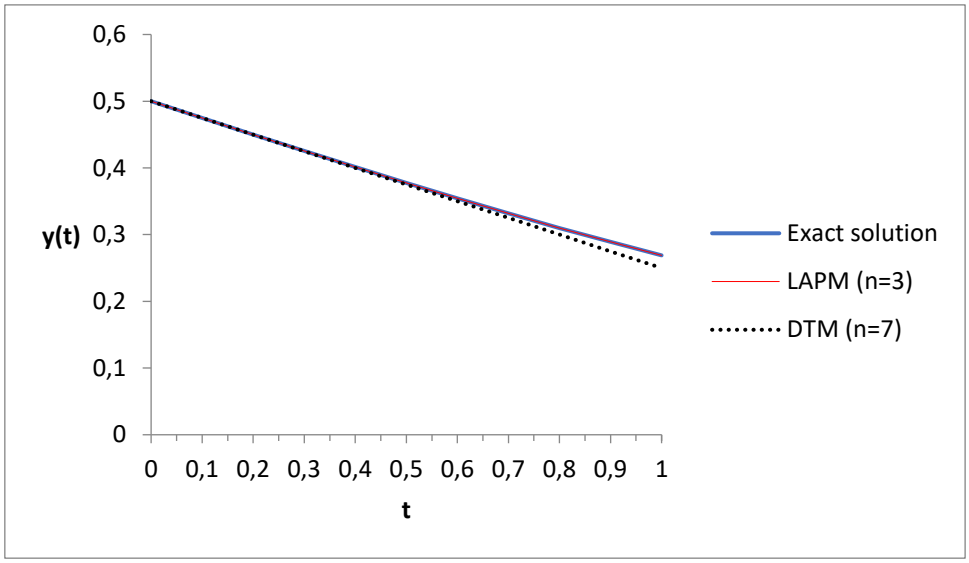


Figure 5: Graph of the LAPM, DTM and the exact solution for example 3

Linear Differential Equations

Example 4

Let us consider the second-order differential equation:

$$y''(t) + y(t) = 0, \qquad y(0) = y'(0) = 1. \qquad ...(34)$$

And the exact solution

$$y(t) = \sin t + \cos t. \qquad ...(35)$$

Using the recursive formulas in Eqn. (13) and (14), the approximate solution to the differential Eqn. (34) is derived through the combined method as follows:

$$\begin{cases} y_0(t) = 1 + t, \\ y_{n+1}(t) = L^{-1}\left[\dfrac{1}{s^2}\{Ly_n(t)\}\right], \qquad n = 0,1,2,... \end{cases} \qquad ...(36)$$

The following is a description of the solution to the Eqn. (34):

$$y_0(t) = 1 + t,$$

$$y_1(t) = L^{-1}\left[\frac{1}{s^2}\{Ly_0(t)\}\right] = \frac{t^2}{2!} + \frac{t^3}{3!},$$

$$y_2(t) = L^{-1}\left[\frac{1}{s^2}\{Ly_1(t)\}\right] = \frac{t^4}{4!} + \frac{t^5}{5!}$$

$$y_3(t) = L^{-1}\left[\frac{1}{s^2}\{Ly_2(t)\}\right] = \frac{t^6}{6!} + \frac{t^7}{7!}$$

$$y_4(t) = L^{-1}\left[\frac{1}{s^2}\{Ly_3(t)\}\right] = \frac{t^8}{8!} + \frac{t^9}{9!}$$

$$\vdots$$

$$y(t) \approx = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \frac{t^5}{5!} + \frac{t^6}{6!} + \frac{t^7}{7!}. + \frac{t^8}{8!} + \frac{t^9}{9!}.. \qquad ...(37)$$

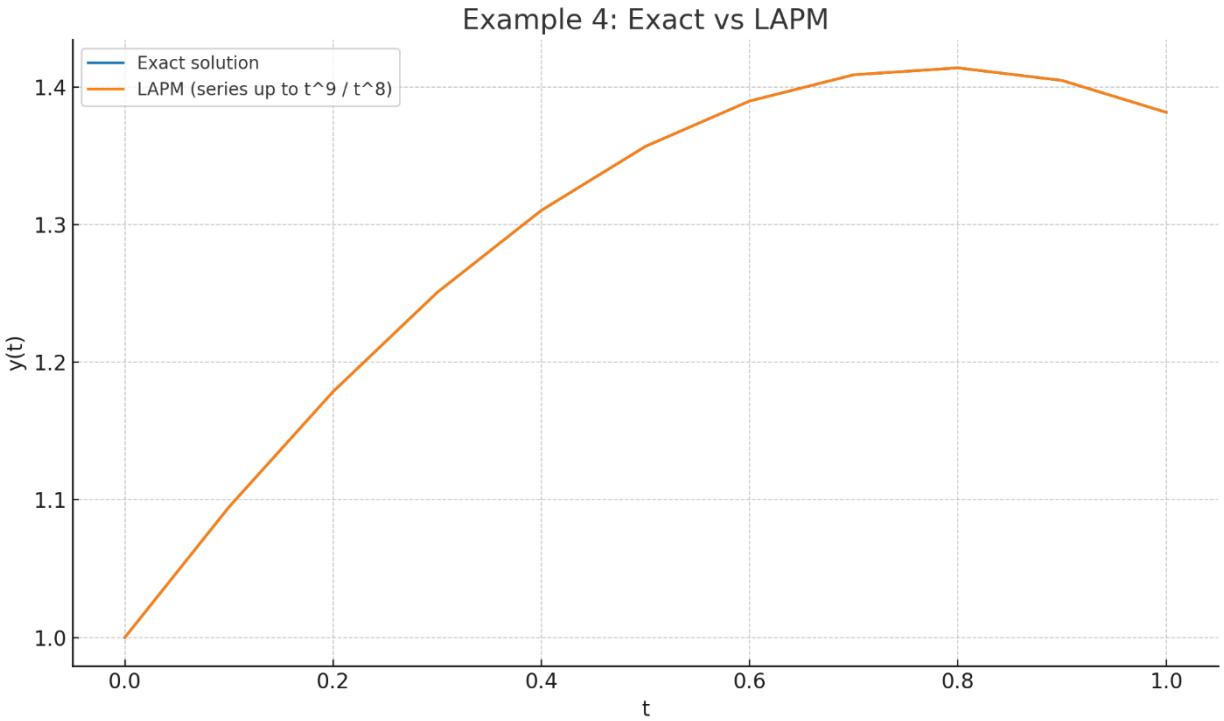


Figure 6: Graph of the LAPM and the exact solution for example 4

DISCUSSION

The comparative analysis of the results for Examples 1, 2, 3 and 4 highlights the accuracy and efficiency of the Laplace–Adomian Polynomial Method (LAPM) when applied to linear and nonlinear differential equations.

For Example 1, the exact solution is $y(t) = \tan(t)$, while the LAPM approximation truncated to a finite number of terms is given by the corresponding power series expansion. The tabulated results show that the approximate values obtained using LAPM are almost identical to the exact solution for t∈ [0, 1], with negligible absolute error. The plot confirms that the LAPM curve overlaps with the exact solution curve throughout the interval, indicating that the method converges very rapidly.

For Example 2, the exact solution is $y(t) = \frac{e^{2t}-1}{e^{2t}+1}$, and the LAPM approximation up to $n = 3$ is $y(t) = t - \frac{1}{3}t^3 + \frac{2}{15}t^5$. The results demonstrate strong agreement between the approximate and exact solutions over the interval considered. The errors are very small for smaller values of t and remain within acceptable bounds as t approaches 1. This validates the capability of LAPM to handle nonlinearities effectively with only a few terms.

For Example 4, where the exact solution is $y(t) = \sin t + \cos t$, the LAPM approximation was constructed using truncated. Maclaurin expansions for sine and cosine up to higher-order terms. The numerical results and the corresponding plot show an almost perfect match between the LAPM approximation and the exact solution for t∈ [0, 1]. The minimal absolute errors observed confirm that the method is highly reliable for oscillatory solutions as well.

Overall, the discussions from these examples indicate that LAPM provides accurate approximations for different classes of problems, including exponential growth, hyperbolic nonlinearities, and trigonometric oscillations. The method is computationally simple and converges rapidly with a small number of terms, making it a practical approach for solving linear and nonlinear differential equations. However, as the interval of approximation increases or higher precision is required, incorporating more terms in the series will further reduce the error and enhance accuracy.

CONCLUSION

The study has demonstrated that the Laplace–Adomian Polynomial Method is an effective and reliable technique for solving nonlinear differential equations. The method combines the strength of the Laplace transform with the decomposition of nonlinear terms

using Adomian polynomials, enabling the construction of convergent series solutions without linearization or discretization.

From the numerical results and error analysis, it was observed that:

- LAPM provides highly accurate approximations for different classes of problems, including exponential growth, hyperbolic functions, and oscillatory solutions.
- The method converges rapidly with a small number of terms, reducing computational effort compared to traditional numerical techniques.
- Graphical and tabular comparisons confirm the robustness of the approach, particularly in the interval $t \in [0, 1]$.

Therefore, LAPM can be considered a practical tool for researchers and engineers dealing with nonlinear systems in applied mathematics, physics, and engineering sciences.

REFERENCES

Adomian, G. (1988). A review of the decomposition method in applied mathematics. *Journal of Mathematical Analysis and Applications, 135*(2), 501–544. https://doi.org/10.1016/0022- 247X(88)90170-9

Aland, P. V., & Singh, P. (2022). Solution of non-linear partial differential equations using Laplace transform modified Adomian decomposition method. *Journal of Physics: Conference Series, 2267,* 012156. https://doi.org/10.1088/1742-6596/2267/1/012156

Cherruault, Y., & Adomian, G. (1989). Convergence of Adomian's method. *Kybernetes, 18*(2), 31–38. https://doi.org/10.1108/eb005754

Fadaei, J. (2011). Application of Laplace-Adomian decomposition method on linear and nonlinear system of PDEs. *Applied Mathematical Sciences, 5*(27), 1307–1315.

González-Gaxiola, O., & Biswas, A. (2019). Optical solitons with Radhakrishnan–Kundu–Lakshmanan equation by Laplace–Adomian decomposition method. *Optik, 179,* 434– 442. https://doi.org/10.1016/j.ijleo.2018.10.144

Gündoğdu, H., & Gözükızıl, Ö. F. (2017). Solving nonlinear partial differential equations by using Adomian decomposition method, modified decomposition method, and Laplace decomposition method. *MANAS Journal of Engineering, 5*(1), 1–13.

Khuri, S. A. (2001). A Laplace decomposition algorithm applied to a class of nonlinear differential equations. *Applied Mathematics and Computation, 118*(2–3), 209–219. https://doi.org/10.1016/S0096-3003 (99)00190-2

Li, W., & Pang, Y. (2020). Application of Adomian decomposition method to nonlinear systems. *Advances in Difference Equations, 2020*(1), 1–17. https://doi.org/10.1186/s13662-020- 02836-y

Odibat, Z. (2020). An optimized decomposition method for nonlinear ordinary and partial differential equations. *Physica A: Statistical Mechanics and Its Applications, 541,* 123323. https://doi.org/10.1016/j.physa.2019.123323

Ziane, D., Belgacem, R., & Bokhari, A. (2019). A new modified Adomian decomposition method for nonlinear partial differential equations. *Open Journal of Mathematical Analysis, 3*(2), 81–90. https://doi.org/10.30538/psrp-oma2019.0035