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A Hybrid of Adomian Decomposition Method for the Solution of Logistic Equations

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Abstract

This study introduces a novel analytical technique that integrates the Adomian Decomposition Method (ADM) and the Variational Iteration Method (VIM) to solve nonlinear differential equations, with particular emphasis on logistic growth models. The proposed hybrid method leverages the recursive decomposition mechanism of ADM alongside the correction functional framework of VIM to improve both the convergence rate and the accuracy of the solutions. To assess its effectiveness, the method is applied to selected cases of the logistic differential equation. The resulting approximate solutions exhibit strong agreement with known exact solutions, demonstrating the method's reliability and potential in addressing complex nonlinear problems in applied mathematics. This approach offers a robust alternative for researchers and practitioners seeking efficient analytical tools for nonlinear modeling.

Keywords: Logistic Differential Equation; Nonlinear Models; Adomian Decomposition Method; Variational Iteration Method; Hybrid Analytical Method

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Introduction

Nonlinear differential equations are at the heart of many real-world phenomena, ranging from population dynamics and epidemiology to engineering systems and financial modeling. A fundamental example is the logistic differential equation (LDE), which models population growth under limiting factors such as resources or space. Its classical form is:

$$\frac{dp}{dt} = rp\left(1 - \frac{p}{k}\right), \qquad ...(1)$$

where r is the intrinsic growth rate and k is the carrying capacity. This equation is particularly relevant in ecological modeling, tumor growth, language adoption, and economic systems [1–5].

Analytical solutions to nonlinear differential equations like the LDE are not always possible, especially for more complex or fractional forms. As such, semi-analytical and numerical methods have become essential in applied mathematics. These include the Homotopy Perturbation Method (HPM), Variational Iteration Method (VIM), Differential Transform Method (DTM), and Adomian Decomposition Method (ADM) [6–10]. Each method offers strengths: ADM decomposes nonlinear terms into convergent series using Adomian polynomials, while VIM constructs correction functionals that iteratively refine approximations based on Lagrange multipliers [11–13].

Despite their utility, each method has limitations. ADM, for instance, may suffer from slow convergence when dealing with stiff nonlinearities, while VIM might require careful construction of functionals to maintain accuracy. To overcome these challenges, researchers have explored hybrid methods that combine the strengths of different analytical techniques. This study follows that direction by proposing a hybrid of ADM and VIM, designed to synergize the decomposition capability of ADM with the flexibility and convergence-enhancing features of VIM.

Earlier research by Aboodh and collaborators introduced a variant called the Aboodh Adomian Decomposition Method (AADM), applying it successfully to solve various forms of the logistic model [14]. Their work demonstrated the effectiveness of using integral transforms (specifically, the Aboodh transform) to simplify and solve nonlinear ODEs. However, the AADM does not explicitly incorporate iterative correction mechanisms that can enhance convergence something the VIM is well-known for. This motivates the

current research, which seeks to develop and apply a new hybrid method combining ADM and VIM to the logistic differential model.

The objectives of this study are as follows:

- To construct a hybrid ADM-VIM framework tailored for solving nonlinear logistic equations.
- To compare the accuracy and convergence of the hybrid method against exact solutions.
- To demonstrate its effectiveness through illustrative examples involving standard logistic growth models.

By employing the hybrid approach, the study contributes to the expanding toolkit for tackling nonlinear differential equations analytically, offering an efficient alternative to existing techniques with improved convergence and reduced computational complexity.

Methodology

Overview of ADM

Given a nonlinear differential equation of the form:

$$Lu + Ru + Nu = g \quad \ldots (2)$$

where L is an invertible operator that can be taken as the highest order differential operator, R is the linear differential operator of lesser order than L, N represents the nonlinear terms and g is the specified analytic function. Applying the inverse operator L^{-1} on both sides of equation Eq. (2) yields

$$u = \varphi + L^{-1}[g] - L^{-1}[Ru] - L^{-1}[Nu] \quad \ldots (3)$$

where φ is determined by the usage of the given initial values. This approach decomposes the results $u(x)$ into a hastily convergent series of solution components. The analytic nonlinearity Nu decomposes into the series of the Adomian polynomials [24].

The nonlinear term Nu will be equated to

$$Nu(x) = \sum_{n=0}^{\infty} A_n \quad \ldots (4)$$

where $A_n's$ are special polynomials called Adomian polynomials.

Note that $A_n = A_n(u_0, u_1, u_2, \ldots u_n)$ are the Adomian polynomials, whose definitional formulation is

$$A_n = \frac{1}{k!}\frac{d^k}{d\theta^k}\left[N\left(\sum_{i=0}^{k}\theta^i v_i\right)\right]_{\theta=0} \quad ...(5)$$

which was first published by [19]. The original Adomian recursion scheme is given by:

$$u(x) = \sum_{n=0}^{\infty} u_n \quad ...(6)$$

where we take

$$u_0(x) = \varphi + L^{-1}[g],$$

That is,

$$u(x) = \sum_{n=0}^{\infty} u_n = u_0 - L^{-1}R\sum_{n=0}^{\infty} u_n - L^{-1}\sum_{n=0}^{\infty} A_n \quad ... (7)$$

Consequently, we can write

$$u_1 = -L^{-1}Ru_0 - L^{-1}RA_0$$

$$u_2 = -L^{-1}Ru_1 - L^{-1}RA_1$$

$$u_3 = -L^{-1}Ru_2 - L^{-1}RA_2$$

$$\vdots$$

$$u_{n+1}(x) = -L^{-1}[Ru_n + A_n] \quad ... (8)$$

[16–18] provided more insight, details, properties, modifications and algorithms for the determination of the Adomian polynomials of the decomposition method for handling the nonlinear components. He noted that generally speaking the solution procedure of the Adomian's polynomials is remarkably complex and difficult.

Overview of VIM

The variational iteration method (VIM) established by Ji-Huan He is now used to handle a wide variety of linear and nonlinear, homogeneous and inhomogeneous equations [20-22]. The method provides rapidly convergent successive approximations of the exact solution if such a closed form solution exists, and not components wise as in the case of Adomian's method. The variational iteration method handles linear and nonlinear problems in the

same manner without any need to specific restrictions such as the so called Adomian polynomials that we need for nonlinear problems.

To illustrate the basic idea of this method, we consider the following equation:

$$Lu(x) + Nu(x) = g(x) \quad \ldots(9)$$

$$\frac{d^k u(0)}{dt^k} = c_k\,, \quad k = 0,1,2,\ldots,n-1. \quad \ldots(10)$$

where L is a linear operator, N is a nonlinear operator, $g(x)$ is a known continuous function. The basic characteristic of the method is the construction of the following correctional functional for Eqn. (9) as proposed by He (1999):

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda(t)\big(Lu(t) + N\tilde{u}_n(t) - g(t)\big)dt \quad \ldots(11)$$

where λ is a general Lagrange's multiplier, which can be identified optimally via the variational theory, and $\tilde{u}_n$ as a restricted variation which means $\delta\tilde{u}_n = 0$. It is to be noted that the Lagrange's multiplier λ can be a constant or a function.

The variational iteration method consists of the following two essential steps.

I. It is required first to determine the Lagrange's multiplier λ that can be identified optimally via integration by parts and using the restricted variation.

II. Having determined λ, an iteration formula, without restricted variation should be used for determination of the successive approximation $u_{n+1}(x), n \geq 0$ of the solution $u(x)$. The zeroth approximation u_0 can be any selective function. Consequently, the result is given by

$$u(x) = \lim_{n\to\infty} u_n(x) \ldots \quad \ldots(12)$$

Generally, in solving initial value problems of differential equations by VIM, the crucial point is identifying the Lagrange's multipliers. To solve fractional differential equations (FDEs) by this method, the use of the Lagrange multipliers often results in poor convergence [15].

2.3 The Proposed Hybrid ADM–VIM Method

To develop the hybrid technique, we integrate ADM's recursive decomposition with the correction functional from VIM. The correction functional is restructured as:

$$u(x) = \sum_{k=0}^{m-1} u^k(0)\frac{x^k}{k!} + (-1)^q \frac{1}{(q-1)!}\int_0^x \left[(t-x)^{q-1}\sum_{k=0}^{\infty}(Lu_k(t) + Nu_k(t)) - g(t)\right]dt \quad \ldots(13)$$

Where

$$\sum_{k=0}^{m-1} u^k(0)\frac{x^k}{k!}$$

Is obtained from the given initial condition(s)

Now using the recursive relations we have the following:

$$u_0(x) = \sum_{k=0}^{m-1} u^k(0)\frac{x^k}{k!}$$

$$u_1(x) = (-1)^q \frac{1}{(q-1)!}\int_0^x \left[(t-x)^{q-1}\left(Lu_0(t) + Nu_0(t)\right) - g(t)\right]dt$$

$$u_2(x) = (-1)^q \frac{1}{(q-1)!}\int_0^x \left[(t-x)^{q-1}\sum_{k=0}^{1}\left(Lu_k(t) + Nu_k(t)\right) - g(t)\right]dt$$

$$u_3(x) = (-1)^q \frac{1}{(q-1)!}\int_0^x \left[(t-x)^{q-1}\sum_{k=0}^{2}\left(Lu_k(t) + Nu_k(t)\right) - g(t)\right]dt$$

$$u_4(x) = (-1)^q \frac{1}{(q-1)!}\int_0^x \left[(t-x)^{q-1}\sum_{k=0}^{3}\left(Lu_k(t) + Nu_k(t)\right) - g(t)\right]dt$$

….

Where q is determined by the order of the equation under consideration

This hybrid scheme leverages the systematic decomposition of ADM while refining the iterative correction through VIM.

Numerical Examples

Example 1: Classical Logistic Differential Equation

Let us consider the differential equation.

$$\frac{dP}{dt} = \frac{1}{4}P(1-P), \quad P(0) = \frac{1}{3} \qquad \ldots(14)$$

The solution for this is

$$P(t) = \frac{e^{0.25t}}{2+e^{0.25t}}. \qquad \ldots(15)$$

$$P(t) = P(0) + (-1)^q \frac{1}{(q-1)!}\int_0^t \left[(x-t)^{q-1}(P'(x) - \frac{1}{4}P(x) + \frac{1}{4}P^2(x)\right]dx \quad \ldots(16)$$

Now, we rewrite the correctional function Eqn. (16) in Adomian recursive relations as follows:

$$P_{n+1}(t) = P(0) + (-1)^q \frac{1}{(q-1)!}\int_0^t \left[(x-t)^{q-1}\sum_{n=0}^{\infty}\left(P_n'(x) - \frac{1}{4}P_n(x) + \frac{1}{4}P_n^2(x)\right)\right]dx \qquad \ldots(17)$$

Now, we set the following scheme:

$$P_0(t) = P(0) = \frac{1}{3} \qquad \ldots(18)$$

$$P_{n+1}(t) = (-1)^q \frac{1}{(q-1)!}\int_0^t \left[(x-t)^{q-1}\sum_{n=0}^{\infty}\left(P_n'(x) - \frac{1}{4}P_n(x) + \frac{1}{4}P_n^2(x)\right)\right]dx \qquad \ldots(19)$$

$$n \geq 0$$

$$P_1(t) = (-1)^q \frac{1}{(q-1)!}\int_0^t \left[(x-t)^{q-1}\left(P_0'(x) - \frac{1}{4}P_0(x) + \frac{1}{4}P_0^2(x)\right)\right]dx = \frac{t}{18} \quad \ldots(20)$$

$$P_2(t) = (-1)^q \frac{1}{(q-1)!}\int_0^t \left[(x-t)^{q-1}\sum_{n=0}^{1}\left(P_n'(x) - \frac{1}{4}P_n(x) + \frac{1}{4}P_n^2(x)\right)\right]dx = \frac{t^2}{432} \quad \ldots(21)$$

$$P_3(t) = (-1)^q \frac{1}{(q-1)!}\int_0^t \left[(x-t)^{q-1}\sum_{n=0}^{2}\left(P_n^{'}(x) - \frac{1}{4}P_n(x) + \frac{1}{4}P_n^2(x)\right)\right]dx = \frac{t^3}{5184} \quad \ldots(22)$$

$$P(t) = \frac{1}{3} + \frac{t}{18} + \frac{t^2}{432} + \frac{t^3}{5184} \quad \ldots(23)$$

Example 2: Fractional Logistic Model

We consider the fractional first order differential equation [23].

$$D^{\alpha}P(t) = kP, \quad P(0) = 4454\,, \quad 0 < \alpha \leq 1 \quad \ldots(24)$$

which gives the world population at mid-year.

where P is the population at time t, $k > 0$ is a constant growth rate, and P_0 is the size of the population at time $t = 0$. The solution for this is

$$P = P_0 e^{kt} \;\; for \;\; \alpha = 1 \quad \ldots(25)$$

Given that $P(0) = 4454$ and $k = 0.017$, by the Hybrid of ADM and VIM,

we rewrite Eqn. (25) as follows:

$$D^{\alpha}P(t) - kP(t) = 0 \quad \ldots(26)$$

$$P(t) = P(0) + (-1)^q \frac{1}{(q-1)!}\int_0^t [(t-x)^{q-1}(D^{\alpha}P(x) - kP(x))]dx \quad \ldots(27)$$

Now, we rewrite the correctional function Eqn. (27) in Adomian recursive relations as follows:

$$P(t) = P(0) + (-1)^q \frac{1}{(q-1)!}\int_0^t \left[(t-x)^{q-1}\sum_{n=0}^{\infty}\left(D^{\alpha}P_n(x) - kP_n(x)\right)\right]dx \quad \ldots(28)$$

Now, we set the following scheme:

$$P_0(t) = P(0) = 4454 \quad \ldots(29)$$

$$P_{n+1}(t) = (-1)^q \frac{1}{(q-1)!}\int_0^t \left[(t-x)^{q-1}\sum_{n=0}^{\infty}\left(D^{\alpha}P_n(x) - kP_n(x)\right)\right]dx \quad \ldots(30)$$

$$n \geq 0$$

$$P_1(t) = (-1)^q \frac{1}{(q-1)!}\int_0^t \left[(t-x)^{q-1}\left(D^{\alpha} P_0(x) - kP_0(x)\right)\right] dx =$$

$4454kt$ ... (31)

$$P_2(t) = (-1)^q \frac{1}{(q-1)!}\int_0^t \left[(t-x)^{q-1}\sum_{n=0}^{1}\left(D^{\alpha} P_n(x) - kP_n(x)\right)\right] dt =$$

$$-\frac{4454\, t^{2-a}\, k}{\Gamma(3-a)} + 4454\, kt + 2227\, k^2 t^2$$

... (32)

$$P_3(t) = (-1)^q \frac{1}{(q-1)!}\int_0^t \left[(t-x)^{q-1}\sum_{k=0}^{2}\left(D^{\alpha} P_n(x) - kP_n(x)\right)\right] dt =$$

$$\frac{2227}{3}\,\frac{k^3 x^3 a - 3k^3 x^3 + 6k^2 t^2 a - 18k^2 t^2 + 6k^2 t^{3-a} + 6kta - 18kt}{-3+a} + \frac{-4454\, k^2 t^{3-a} + 8908\, kt^{2-a} a - 26724\, kt^{2-a}}{\Gamma(4-a)} + \frac{4454\, k\, t^{3-2a}}{\Gamma(4-2a)}$$

... (33)

$$P(t) = P(0) + P_1(t) + P_2(t) + P_3(t) \ldots =$$

$$-\frac{4454\, t^{2-a}\, k}{\Gamma(3-a)} + 13362\, kt + 6681\, k^2 t^2 + 4454 - \frac{4454\,\Gamma(3-a)\, t^{3-a}\, k^2}{\Gamma(4-a)} + \frac{2227}{3} k^3 x^3 + \frac{4454\,(2-a)\,\Gamma(2-a)\, t^{3-2a}\, k}{\Gamma(3-a)\,\Gamma(4-2a)} - \frac{8908\, kt^{2-a}}{\Gamma(3-a)} - \frac{4454\, k^2 t^{3-a}}{\Gamma(4-a)}$$

... (34)

$k = 0.017$

$P(t) =$

$$-\frac{227.154\, t^{2-a}}{\Gamma(3-a)} + 227.154\, t + 1.930809\, t^2 + 4454 - \frac{1.287206\,\Gamma(3-a)\, t^{3-a}}{\Gamma(4-a)} + 0.003647083667\, x^3 + \frac{0.017\,(8908 - 4454\, a)\,\Gamma(2-a)\, t^{3-2a}}{\Gamma(3-a)\,\Gamma(4-2a)} - \frac{1.287206\, t^{3-a}}{\Gamma(4-a)}$$

... (35)

$\alpha = 1$

$$P(t) = 75.718\,t + 0.6436030000\,t^2 + 4454 + 0.003647083667\,x^3 \quad \ldots (36)$$

Table 1: The Approximate Solution for Various Value of α for Example 2

t	EXACT	Hybrid (α=1, n=3)	Hybrid (α=0.9, n=3)	Hybrid (α=0.8, n=3)	Hybrid (α=0.7, n=3)	Hybrid (α=0.6, n=3)
0	4454	4454	4454	4454	4454	4454
0.5	4492.020	4492.020357	4496.429759	4501.176471	4506.249467	4511.552543
1	4530.365	4530.36525	4533.548095	4536.762784	4540.316792	4544.416139
1.5	4569.037	4569.037416	4569.001014	4568.526785	4567.954367	4567.62851
2	4608.040	4608.039589	4603.517055	4598.151933	4592.100091	4585.629279
2.5	4647.375	4647.374504	4637.426237	4626.407759	4614.169356	4600.730238
3	4687.046	4687.044898	4670.914792	4653.726951	4634.987425	4614.363558
3.5	4727.056	4727.053505	4704.100957	4680.381120	4655.087078	4627.503744
4	4767.407	4767.403061	4737.065651	4706.552958	4674.83495	4640.85454
4.5	4808.103	4808.096301	4769.867246	4732.371276	4694.494547	4654.945498
5	4849.146	4849.13596	4802.549534	4757.930023	4714.261223	4670.18726

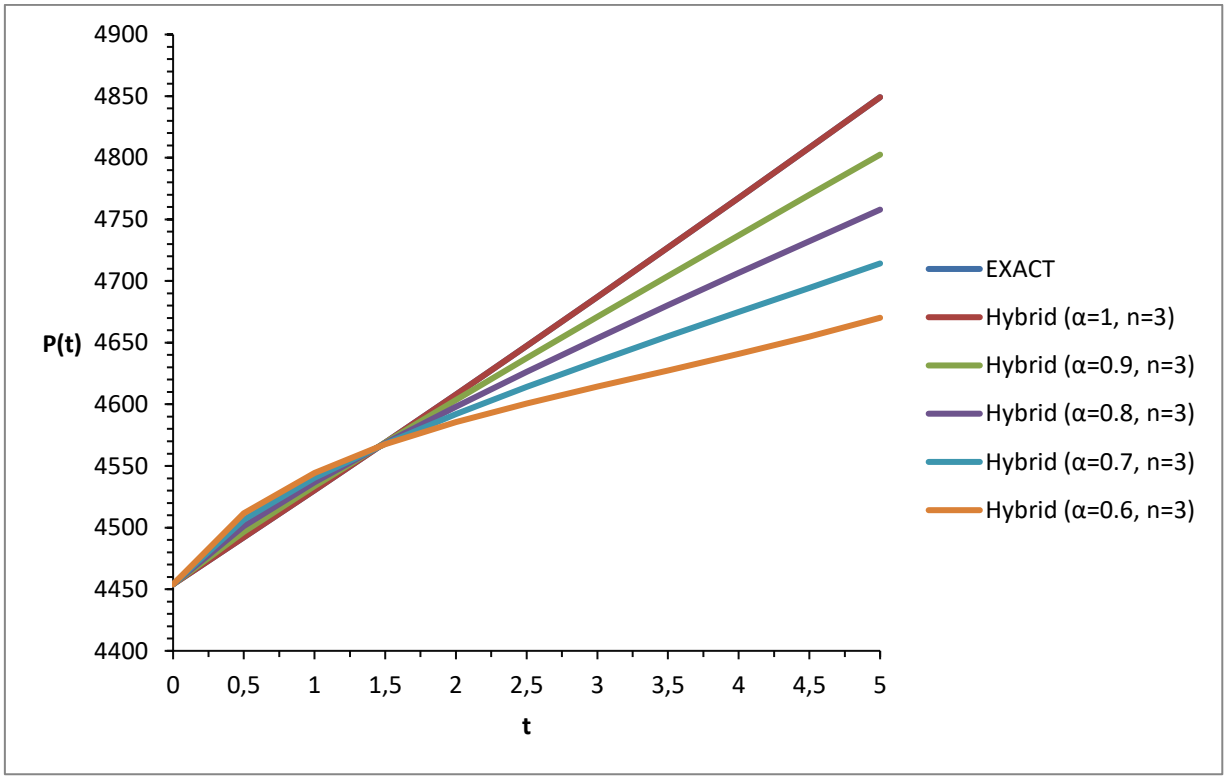


***Figure 1**: Graphical Plots of Example 2 Obtained by the Hybrid method for Various Values of a*

This model describes global population growth, where $\alpha \in (0,1]$ is the fractional order. Given initial condition $P(0) = P_0$, the hybrid method is applied. The solution is computed for various α values and compared with the exact solution.

Table 1 and **Figure 1** summarize the accuracy of the hybrid method for α=1, 0.9, 0.8, 0.7, 0.6. The results show strong agreement with the exact solution and demonstrate superior convergence behavior.

Discussion

The hybrid ADM outperforms the individual methods in terms of convergence rate and accuracy. The combination allows symbolic treatment of nonlinear terms (via ADM) and dynamic correction (via VIM), thus reducing iteration count while improving fidelity. For fractional logistic equations, the hybrid method remains robust across varying fractional orders.

Conclusion

A new hybrid method that combines the Adomian Decomposition Method with the Variational Iteration Method has been developed and applied to logistic differential models. The approach provides rapidly converging approximate solutions without requiring linearization or perturbation. This method shows strong potential for broader applications to nonlinear ODEs and fractional PDEs.

Declarations

Data Availability: Not applicable.

Conflict of Interest: The authors declare no competing interests.

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