

## Relative Strength of Common Fixed Point Results for Two Self Mapping in Fuzzy Metric Space

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### Article Info:

|              |              |              |             |
|--------------|--------------|--------------|-------------|
| Submitted:   | Revised:     | Accepted:    | Published:  |
| Nov 30, 2024 | Dec 15, 2024 | Dec 27, 2024 | Jan 3, 2025 |

### Abstract

The concept of fuzzy metric space, which was introduced by Kramosil and Michalek (1975), is used in this article. In this manuscript, we present and generalize some common fixed point theorems in fuzzy metric spaces, which is an extension of the well-known results given by shen, Y. et al. (2012) in the sense of Schweizer and Sklar(1983).

**Keywords:** Fuzzy metric space, Fixed Point, Common Fixed Point, Contractive Mapping

### INTRODUCTION

The fuzzy set theory was introduced by Zadeh (1965) there has been a great effort to obtain fuzzy analogues of classical theories. Many authors have work out on the theory of fuzzy sets and its applications and explored it successfully. After that, to use this concept in topology and analysis different authors have expansively developed the theory of fuzzy sets

and its applications. The theory of fuzzy topological spaces was introduced and developed by Chang, C. L. in 1968. In 1975, Kramosil and Michalek, introduced the fuzzy metric spaces. Some fixed point results for set valued mappings on fuzzy metric spaces were found in [M. Grabiec(1988), Sadeghi (1960), Hadzic and Pap, E. (2002), and Kiani and Harad (2011)] in the sense of Kramosil and Michalek. Especially, the work of Deng (1982), Erceg (1979), Kaleva and Seikkala(1984), Kaleva(1985), Subrahmanyam(1993), and Upadhyay and Choudhary(2002) are of great significance. Manadhar et al. (2014) extended the results of Singh and Singh (2007) and Pant, V. (2009) in fuzzy metric space for the concept of compatible mapping of type (E). In 2002, Sharma, S., obtained the common fixed point results for three self mappings in fuzzy metric spaces. Many researchers have been obtained fixed p point results in fuzzy metric space by using the concept of compatible mp, implicit maps, control function(see for instances Das and Saha(2014), Dey and Saha(2014), Dinarvand, M.(2017), Dosenovic, T. et al.(2014), Patel and Bhardwaj (2013), Shukla and Abaas (2014), Warirojjana et al. (2015), Ahmad, M.A. (2014), Gupta et al. (2015), Georgr and Sapena (2002), Imdad and Ali (2006), Cho et al. (2009),and Manadhar and Jha (2015)).A study of fuzzy fixed point theorem using the Handorff distance function between the  $\alpha$  –level sets corresponding to fuzzy sets was introduced and presented by Fadhel in 1968. In 2012, Fadel and Majeed (2012) obtained the completeness and the fixed point theorem of a certain fuzzy metric space, which is the family of all fuzzy sets in which any intersection or union of fuzzy sets are fuzzy numbers or fuzzy point. In the same year in 2012, Shen, Y. et al, established fixed point theorems for a new class of self maps in complete fuzzy metric space and wairojjana et al. (2015), generalized the results of shen, Y.et al.(2012).Patir et al. (2018) derived some fixed point theorem in fuzzy metric spaces for self mappings with different types of contractive conditions. In 2019, Karayilan and Telci, proved and extended of generalized czarists fixed point theorem ,which prove by Bollenbacher and Hicks for p- arbitrary complete fuzzy metric space. Recently, in 2021, Huang, H. et al. introduced new type of contraction mapping, which are called f- contraction and obtained some fixed point theorems in fuzzy metric spaces for such type contraction. Eidi, J.H. et al. (2022), investigated two fixed point theorems related to two different contractive mappings in addition some theoretical results concerning fuzzy metric space. In the same year, Gupta et al. (2022) derived common fixed point results for weakly compatible self mappings satisfying E.A. properties in fuzzy metric spaces.

In this article, we generalize and improve the results of Shen Y. et al.(2012) in the sense of Schweizer and Sklar(1983) and obtain common fixed point theorem in fuzzy metric space for contractive condition.

## PRELIMINARY NOTES

In this section, we define some important definition and results which are used in sequel.

**Definition 1 ([Zadeh, L.A.(1965)]):** Let  $X$  be any set . A fuzzy set  $A$  in  $X$  is a function with domain  $X$  and value in  $[0, 1]$ .

**Definition 1 ([Schweizer, A. and Sklar, (1960, 1983)]):** A binary operation  $*$  :  $[0,1] \times [0,1] \rightarrow [0,1]$  is called a continuous t-norm if it satisfies the following conditions:

- (1)  $*$  is commutative and associative
- (2)  $*$  is continuous
- (3)  $1 * a = a$  for all  $a \in [0, 1]$
- (4)  $a * b \leq c * d$ , Whenever  $a \leq c$  and  $b \leq d$ , for each  $a, b, c, d \in [0, 1]$ .

**Definition 2 ([Kramosil and Michalek (1975)]):**A fuzzy metric space is an ordered triple  $(X, M, *)$  such that  $X$  is a non- empty set,  $*$  is continuous t-norm and  $M$  is a fuzzy set on  $X \times X \times (0, \infty)$  satisfying the following conditions, for all  $x, y, z \in X, s, t > 0$

- (i)  $M(x, y, 0) > 0$ ;
- (ii)  $M(x, y, t) = 1$  iff  $x = y$ , for all  $t \geq 0$ ;
- (iii)  $M(x, y, t) = M(y, x, t)$
- (iv)  $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s), \forall x, y, z \in X$  and  $t, s > 0$ ;
- (v)  $M(x, y, \cdot) : (0, \infty) \rightarrow [0,1]$  is left continuous; and
- (vi)  $\lim_{t \rightarrow \infty} M(x, y, t) = 1, \forall x, y \in X$ .

**Definition 3 ([George A., and Veermani, P. (1994)]):** Let  $(X, M, *)$  be a fuzzy metric space,  $x \in X$  and  $\{x_n\}$  be a sequence in  $X$ . Then,

- (1)  $\{x_n\}$  is said to converge to  $x$  if  $M(x_n, x, t) \rightarrow 1$  as  $n \rightarrow \infty$ , for all  $t \geq 0$ .
- (2)  $\{x_n\}$  is said to Cauchy sequence if  $M(x_{n+p}, x_n, t) \rightarrow 1$ , as  $n \rightarrow \infty$  for every  $t > 0$  and each positive integer  $p$ .
- (3)  $(X, M, *)$  is said to be complete fuzzy metric space if every Cauchy sequence is

convergent in  $X$ .

**Definition 4:** A map  $f: X \rightarrow X$  is said to be continuous on  $x \in X$ , if  $f(x_n) \rightarrow f(x)$ , whenever  $x_n \rightarrow x$ .

Thus it clearly follows that a sequence can converge to only one point of  $X$ .

## MAIN\_RESULTAS

In this section we have extend and generalize the results of hen Y. et al.(2012) in the sense of Schweizer and Sklar (1983) and obtain common fixed point result in fuzzy cone metric space. The main results are as follows:

**Theorem 5.** Let  $(X, M, *)$  be a complete fuzzy metric space and  $S, T : X \rightarrow X$  be any two continuous self mapping, satisfying the following

$$2M(Sx, Ty, t) \geq M\left(x Sx, \frac{t_1}{a}\right) + M\left(y, Ty, \frac{t_2}{b}\right) \dots (5.1)$$

Where  $t = t_1 + t_2$ ;  $t_1, t_2 \geq 0$ ;  $t > 0$ ;  $a, b > 0$ , with  $b > a$ ;  $0 < a + b < 1$  and  $x, y \in X$ .

Then  $S$  and  $T$  have a unique common fixed point.

**Proof :** Let  $x_0 \in X$  and  $\{x_{2n}\}$  and  $\{x_{2n+1}\}$  be any two square in  $X$ , such that

$$x_{2n+1} = Sx_{2n} = S^{2n} x_0$$

and

$$x_{2n+2} = Tx_{2n+1} = T^{2n+1} x_0$$

Putting  $x = x_{2n}$  and  $y = x_{2n+1}$  with  $t_1 = (1 - b)t$  and  $t_2 = bt$  in (5.1), we get

$$2M(x_{2n+1}, x_{2n+2}, t) \geq M(x_{2n}, x_{2n+1}, (1 - b)t/a) + M(x_{2n+1}, x_{2n+2}, t)$$

Implies that  $M(x_{2n+1}, x_{2n+2}, t) \geq M(x_{2n}, x_{2n+1}, (1 - b)t/a)$ , for  $n = 0, 1, 2, \dots$

Where  $0 < c = \frac{a}{1-b} < 1$ , since  $a + b < 1$ . So,

$$M(x_{2n+1}, x_{2n+2}, t) = 0. \text{ Must we have } M(x_{2n+1}, x_{2n+2}, t/c) = 0.$$

If  $M(x_{2n+1}, x_{2n+2}, t) \neq 0$ , then

$$M(x_{2n+1}, x_{2n+2}, t) \geq M(x_{2n}, x_{2n+1}, t/c), \text{ where } 0 < c < 1.$$

For any positive integer  $p$ , we must have

$$\begin{aligned} M(x_{2n+1}, x_{2n+p+1}, t) &\geq M\left(x_{2n+1}, x_{2n+2}, \frac{t}{p}\right) \star \dots \dots p \text{ times } \dots \dots \\ &\star M(x_{2n+p}, x_{2n+p+2}, t/p). \end{aligned}$$

$$M(x_{2n+1}, x_{2n+p+1}, t) \geq M\left(x_0, x_1, \frac{t}{pc^{2n+1}}\right) \star \dots p \text{ times } \dots \\ \star M\left(x_0, x_1, \frac{t}{pc^{2n+p+1}}\right).$$

Since  $0 < c < 1$ , therefore, by definition 1, implies that  $M(x_{2n+1}, x_{2n+p+1}, t) \rightarrow 1$  as  $n \rightarrow \infty$ . Therefore, by definition 3(2),  $\{x_{2n}\}$  is a Cauchy sequence and hence is convergent in the fuzzy metric space  $X$ . Let  $\{x_{2n}\} \rightarrow z$ , as  $n \rightarrow \infty$ . Then

$$Sx_{2n} = x_{2n+1} \rightarrow z \text{ as } n \rightarrow \infty.$$

Similarly,  $Tx_{2n+1} = x_{2n+2} \rightarrow z$  as  $n \rightarrow \infty$ . Since  $S$  and  $T$  are continuous, so, by above equation

$$Tz = Sz = z$$

Now, we next prove the uniqueness. If possible, let  $u$  and  $v$  be any two common fixed point s of  $S$  and  $T$  with  $u \neq v$ , now, putting  $t_1 = (1-b)t$ ,  $t_2 = bt$  in equation 5.1., we have

$$M(u, v, t) \geq M(u, v, \frac{t}{c}), \text{ where } 0 < c < 1.$$

And subsequently, must we have, for all  $t > 0$

$$M(u, v, t) \geq M\left(u, v, \frac{t}{c^2}\right) \\ \geq \dots \geq \\ \geq M(u, v, \frac{t}{c^n}) \rightarrow 1 \text{ as } n \rightarrow \infty.$$

This shows, by definition 1 and Definition 2, that,  $u = v$  which proves the uniqueness.

Hence the proof is complete.

**Theorem 6.** Let  $(X, M, \star)$  be a complete fuzzy metric space and  $S, T : X \rightarrow X$  be any two continuous self mapping, satisfying the following

$$[2M(Sx, Ty, t)]^2 \geq [M\left(x, Sx, \frac{t_1}{e}\right) + M(y, Ty, \frac{t_2}{a})] \\ \star [M\left(y, Sx, \frac{t_2}{f}\right) + M(Sx, Ty, \frac{t_2}{b})] \dots \dots \dots (6.1)$$

Where  $t = t_1 + t_2$ ;  $t_1, t_2 \geq 0$ ;  $t > 0$ ;  $a, b, e, f > 0$ , with  $0 < a + b < 1$ ;  $0 < e < a$  and  $x, y \in X$ . Then  $S$  and  $T$  have a unique common fixed point.

**Proof :** Let  $x_0 \in X$  and  $\{x_{2n}\}$  and  $\{x_{2n+1}\}$  be any two square in  $X$ , such that

$$x_{2n+1} = Sx_{2n} = S^{2n} x_0$$

and

$$x_{2n+2} = Tx_{2n+1} = T^{2n+1} x_0$$

Putting  $x = x_{2n}$  and  $y = x_{2n+1}$  with  $t_1 = at$  and  $t_2 = bt$  in (6.1), we get

$$\begin{aligned} [2M(x_{2n+1}, x_{2n+2}, t)]^2 &\geq [M(x_{2n}, x_{2n+1}, \frac{at}{e}) + M(x_{2n+1}, x_{2n+1}, t)] \\ &\quad \star [M(x_{2n+1}, x_{2n+1}, \frac{bt}{f}) + M(x_{2n+1}, x_{2n+2}, t)] \end{aligned}$$

$$\begin{aligned} \Rightarrow [2M(x_{2n+1}, x_{2n+2}, t)]^2 &\geq [M(x_{2n}, x_{2n+1}, \frac{t}{k}) + M(x_{2n+1}, x_{2n+2}, t)] \\ &\quad \star [1 + M(x_{2n+1}, x_{2n+2}, t)], \text{ for all } t > 0, \text{ and } n = 0, 1, 2, \dots, \end{aligned} \quad (6.2)$$

When  $M(x_{2n+1}, x_{2n+2}, t) = 0$ , then we have  $M(x_{2n}, x_{2n+1}, \frac{t}{k}) = 0$ . If

$M(x_{2n+1}, x_{2n+2}, t) \neq 0$ , Then from 6.2, it is obvious that

$$M(x_{2n+1}, x_{2n+2}, t) \geq M(x_{2n}, x_{2n+1}, \frac{t}{k}), \text{ where } 0 < k < 1. \quad (6.3)$$

For ant positive integer  $p$ , we must have

$$\begin{aligned} M(x_{2n+1}, x_{2n+p+1}, t) &\geq M(x_{2n+1}, x_{2n+2}, \frac{t}{p}) \star \dots \dots p \text{ times } \dots \dots \\ &\quad \star M(x_{2n+p+1}, x_{2n+p+2}, t/p). \\ M(x_{2n+1}, x_{2n+p+1}, t) &\geq M(x_0, x_1, \frac{t}{pk^{2n+1}}) \star \dots \dots p \text{ times } \dots \dots \\ &\quad \star M(x_0, x_1, \frac{t}{pk^{2n+p+1}}). \end{aligned}$$

Since  $0 < k < 1$ . Therefore, by definition 1, we get  $M(x_{2n+1}, x_{2n+p+1}, t) \rightarrow 1$  as  $n \rightarrow \infty$ .

Therefore, by definition 3(2),  $\{x_{2n}\}$  is a Cauchy sequence and hence is convergent in the fuzzy metric space  $X$ . Let  $\{x_{2n}\} \rightarrow z$ , as  $n \rightarrow \infty$ . Then

$$Sx_{2n} = x_{2n+1} \rightarrow z \text{ as } n \rightarrow \infty.$$

Similarly,  $Tx_{2n+1} = x_{2n+2} \rightarrow z$  as  $n \rightarrow \infty$ . Since  $S$  and  $T$  are continuous, so, by above equation

$$Tz = Sz = z.$$

Now, we next prove the uniqueness. If possible, let  $u$  and  $v$  be any two common fixed point s of  $S$  and  $T$  with  $u \neq v$ , now, putting  $t_1 = at$ ,  $t_2 = bt$  in equation 6.1., we get

$$[2M(u, v, t)]^2 \geq \left[ M\left(u, v, \frac{t}{k}\right) + M(u, v, t) \right] \star [1 + M(u, v, t)] \dots \quad (6.4)$$

Now, as show in 6.3 for all  $t > 0$ , in the similar manner, from 6.4, we must have

$$M(u, v, t) \geq M\left(u, v, \frac{t}{k}\right), \text{ where } 0 < k < 1 \quad (6.5)$$

And subsequently, from 6.5 for all  $t > 0$ , we have

$$\begin{aligned} M(u, v, t) &\geq M\left(u, v, \frac{t}{k^2}\right) \\ &\geq \dots \geq \\ &\geq M\left(u, v, \frac{t}{k^n}\right) \rightarrow 1 \text{ as } n \rightarrow \infty. \end{aligned}$$

This shows, by definition 1 and definition 2, that,  $u = v$  which proves the uniqueness.

This completes the proof of the theorem.

**Theorem 7.** Let  $(X, M, \star)$  be a complete fuzzy metric space and  $S, T : X \rightarrow X$  be any two continuous self mapping, satisfying the following

$$\begin{aligned} [2M(Sx, Ty, t)]^3 &\geq [M\left(x, y, \frac{t_1}{e}\right) + M(y, Sx, \frac{t_1}{a})] \star [M\left(y, Sx, \frac{t_2}{f}\right) + M(Sx, Ty, \frac{t_2}{b})] \\ &\star [M\left(Ty, Sx, \frac{t_3}{g}\right) + M(y, Ty, \frac{t_3}{c})] \dots \quad (7.1) \end{aligned}$$

Where  $t = t_1 + t_2 + t_3$ ;  $t_1, t_2 \geq 0$ ;  $t > 0$ ;  $a, b, c, e, f, g > 0$ , with  $0 < a + b + c < 1$ ;  $0 < e < a$  and  $x, y \in X$ . Then  $S$  and  $T$  have a unique common fixed point.

**Proof :** Let  $x_0 \in X$  and  $\{x_{2n}\}$  and  $\{x_{2n+1}\}$  be any two square in  $X$ , such that

$$\begin{aligned} x_{2n+1} &= Sx_{2n} = S^{2n} x_0 \\ &\text{and} \\ x_{2n+2} &= Tx_{2n+1} = T^{2n+1} x_0 \end{aligned}$$

Putting  $x = x_{2n}$  and  $y = x_{2n+1}$  with  $t_1 = at$ ,  $t_2 = bt$  and  $t_3 = ct$  in (7.1), we get

$$\begin{aligned} [2M(x_{2n+1}, x_{2n+2}, t)]^3 &\geq [M\left(x_{2n}, x_{2n+1}, \frac{at}{e}\right) + M(x_{2n+1}, x_{2n+2}, t)] \\ &\star [M\left(x_{2n+1}, x_{2n+1}, \frac{bt}{f}\right) + M(x_{2n+1}, x_{2n+2}, t)] \end{aligned}$$

$$\star \star [M(x_{2n+2}, x_{2n+2}, \frac{ct}{g}) + M(x_{2n+1}, x_{2n+2}, t)]$$

Let  $k = \frac{e}{a}$ , therefore,  $0 < k < 1$ , and hence we have

$$\begin{aligned} [2M(x_{2n+1}, x_{2n+2}, t)]^3 &\geq [M(x_{2n}, x_{2n+1}, \frac{t}{k}) + M(x_{2n+1}, x_{2n+2}, t)] \\ &\star [1 + M(x_{2n+1}, x_{2n+2}, t) +] \\ &\star [1 + M(x_{2n+1}, x_{2n+2}, t)], \end{aligned}$$

for all  $t > 0$  and  $n = 0, 1, 2, \dots$  (7.2)

Implies that when  $M(x_{2n+1}, x_{2n+2}, t) = 0$ , we must have

$$M(x_{2n}, x_{2n+1}, \frac{t}{k}) = 0.$$

If  $M(x_{2n+1}, x_{2n+2}, t) \neq 0$ , then from 7.2, it is obvious that

$$M(x_{2n+1}, x_{2n+2}, t) \geq M(x_{2n}, x_{2n+1}, \frac{t}{k}), \text{ where } 0 < k < 1 \quad (7.3)$$

For any positive integer  $p$ , we have

$$\begin{aligned} M(x_{2n+1}, x_{2n+p+1}, t) &\geq M(x_{2n+1}, x_{2n+2}, \frac{t}{p}) \star \dots \dots p \text{ times } \dots \dots \\ &\star M(x_{2n+p+1}, x_{2n+p+2}, t/p). \\ M(x_{2n+1}, x_{2n+p+1}, t) &\geq M(x_0, x_1, \frac{t}{pk^{2n+1}}) \star \dots \dots p \text{ times } \dots \dots \\ &\star M(x_0, x_1, \frac{t}{pk^{2n+p+1}}). \end{aligned}$$

Since  $0 < k < 1$ , therefore, by definition 7.2, we have

Therefore, by definition 3(2),  $\{x_{2n}\}$  is a Cauchy sequence and hence is convergent in the fuzzy metric space  $X$ . Let  $\{x_{2n}\} \rightarrow z$ , as  $n \rightarrow \infty$ . Then

$$Sx_{2n} = x_{2n+1} \rightarrow z \text{ as } n \rightarrow \infty.$$

Similarly,  $Tx_{2n+1} = x_{2n+2} \rightarrow z$  as  $n \rightarrow \infty$ . Since  $S$  and  $T$  are continuous, so, by above equation

$$Tz = Sz = z.$$

$$M(x_{2n+1}, x_{2n+p+1}, t) \rightarrow 1 \text{ as } n \rightarrow \infty.$$

Now, we next prove the uniqueness. If possible, let  $u$  and  $v$  be any two common fixed point s of  $S$  and  $T$  with  $u \neq v$ , now, putting  $t_1 = at$ ,  $t_2 = bt$  and  $t_3 = ct$  in equation 7.1., we get

$$[2M(u, v, t)]^3 \geq \left[ M\left(u, v, \frac{t}{k}\right) + M(u, v, t) \right] \star [1 + M(u, v, t)] \\ \star [1 + M(u, v, t)], \text{ where } 0 < k < 1. \quad (7.4)$$

Now, as show in 7.3 for all  $t > 0$ , in the similar manner, from 7.4, we must have

$$M(u, v, t) \geq M\left(u, v, \frac{t}{k}\right), \text{ where } 0 < k < 1 \quad (7.5)$$

And subsequently, from 7.5 for all  $t > 0$ , we have

$$M(u, v, t) \geq M\left(u, v, \frac{t}{k^2}\right) \\ \geq \dots\dots\dots \geq \\ \geq M\left(u, v, \frac{t}{k^n}\right) \rightarrow 1 \text{ as } n \rightarrow \infty.$$

This shows, by definition 1 and definition 2, that,  $u = v$  which proves the uniqueness.

This completes the proof of the theorem.

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