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Robust Integral Transform Methods for the Solution of Nonlinear Fractional Ordinary Differential Equations in Viscoelastic and Biological Systems

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Abstract

Nonlinear and fractional-order differential equations frequently arise in viscoelastic and biological systems; however, their solution remains challenging due to the presence of nonlocal operators, memory effects, and complex boundary conditions. Classical integral transforms, including the Laplace and Fourier transforms, often have limitations in addressing these features effectively. This study presents a robust hybrid methodology that combines the

Mahgoub Transform with the Variational Iteration Method (VIM) to solve nonlinear and fractional-order ordinary differential equations (ODEs). The proposed approach was systematically applied to linear, nonlinear, and fractional-order ODEs to evaluate its convergence, accuracy, and capacity to handle memory-dependent effects. The findings demonstrate that the Mahgoub–VIM method achieves rapid convergence, high accuracy, and improved performance compared with traditional transforms such as the Sumudu Transform. These results indicate that the proposed method provides a reliable and efficient analytical framework for modeling complex viscoelastic and biological phenomena governed by nonlinear and fractional-order dynamics. This study contributes to the advancement of integral transform-based solution methods and offers practical implications for the mathematical modeling of systems characterized by memory-dependent behavior and nonlinear responses.

Keywords: Fractional Ordinary Differential Equations; Mahgoub Transform; Variational Iteration Method; Nonlinear Integral Transform Methods; Viscoelastic and Biological Systems

INTRODUCTION

Solving nonlinear and fractional-order differential equations is a major challenge in applied mathematics due to the limitations of conventional integral transforms. Classical methods like Laplace and Fourier transforms are inadequate in capturing memory effects, nonlocal operators, and complex boundary conditions (Ganie et al., 2024; Hussein & Ziane, 2024). Modern integral transforms, including Kamal, Sumudu, Mahgoub, Aboodh, and Elzaki transforms, improve convergence and computational efficiency but are largely restricted to linear problems (Watugala, 1993; Mahgoub, 2016; Abdelilah & Mahamoud, 2017; Waqas, 2022). Hybrid approaches that integrate these transforms with analytical and semi-analytical methods, such as the Variational Iteration Method (VIM), have been developed to extend applicability to nonlinear and fractional models (Ahmed et al., 2023; Khan & Wu, 2023; Mikail, 2023; Alzaki & Jassim, 2024). Despite this, a unified framework for systematically assessing accuracy, efficiency, and applicability of these hybrid methods across different classes of differential equations remains lacking (Onuoha, 2023). This study introduces a comprehensive methodology using the Mahgoub Transform combined with VIM to address nonlinear fractional ODEs in viscoelastic and biological systems.

METHODOLOGY

Definition of Mahgoub Transform

For a sufficiently smooth function $f(t)$, the **Mahgoub Transform** is defined as:

$$M\{f(t)\} = F(s) = \int_0^{\infty} \frac{e^{-st}}{1+t} f(t)dt,\ s > 0 \qquad \dots(1)$$

The inverse Mahgoub Transform is denoted as:

$$f(t) = M^{-1}\{F(s)\}. \qquad \dots(2)$$

Properties Of Mahgoub Transform

1. **Linearity**:

$$M\{af(t) + bg(t)\} = aM\{f(t)\} + bM\{g(t)\}, \quad a, b \in R \qquad \dots(3)$$

2. **Derivative Property**:

$$M\{\frac{df(t)}{dt}\} = sF(s) - f(0) \qquad \dots(4)$$

3. Integral Property:

$$M\{\int_0^t f(\tau)d\tau\} = \frac{F(s)}{s} \qquad \dots(5)$$

Handling Nonlinear Terms:

Nonlinear terms, e.g., $f(t)^n$, can be approximated using the Variational Iteration Method (VIM) in the transform domain.

Mahgoub Transform Procedure for Ordinary Differential Equations (ODES)

Step 1. Problem Formulation

Consider a nonlinear ODE:

$$\frac{d^2y(t)}{dt^2} + p(t)\frac{dy(t)}{dt} + q(t)y(t) + r(y) = f(t),\ y(0) = y_0,\ y'(0) = y_1 \qquad \dots(6)$$

Step 2. Apply Mahgoub Transform

$$M\{\frac{d^2y(t)}{dt^2}\} + M\{p(t)\frac{dy(t)}{dt}\} + M\{q(t)y(t)\} + M\{r(y)\} = M\{f(t)\} \qquad \dots(7)$$

Using the derivative property:

$$s^2Y(s) - sy_0 - y_1 + M\{p(t)y'(t)\} + M\{q(t)y(t)\} + M\{r(y)\} = \mathrm{F(s)} \qquad \dots(8)$$

Hybrid Mahgoub Transform VIM Approach

Nonlinear and fractional terms are handled via VIM in the transformed domain.

Step 1. Construct Correction Functional in s-domain

$$Y_{n+1}(s) = Y_n(s) + \int_0^t \lambda(\tau)\left[M\{y''_n + p(t)y'_n + q(t)y_n + r(y_n) - f(t)\}\right]d\tau \quad \dots(9)$$

λ(τ) is the Lagrange multiplier determined by variational theory.

$Y_n(s)$is the n-th iteration of the Mahgoub Transform of $y(t)$.

Step 2. Iterative Solution in Transform Domain

Iterate until convergence:

$$\| Y_{n+1}(s) - Y_n(s) \| < \epsilon \quad \dots(10)$$

Step 3. Inverse Mahgoub Transform

$$y(t) \approx M^{-1}\{Y_n(s)\} \quad \dots(11)$$

This yields the semi-analytical solution of the nonlinear ODE.

Application of Mahgoub Transform to Fractional Order ODES

For fractional ODEs and PDEs, the Caputo fractional derivative is employed to ensure physically meaningful initial conditions. The Mahgoub transform is applied using the fractional derivative property, after which nonlinearities are treated iteratively via VIM. This approach allows exact or rapidly convergent solutions without the need for series truncation.

$$D_t^{\alpha} y(t) + \lambda y(t) + r(y) = f(t), \quad 0 < \alpha \leq 1 \quad \dots(12)$$

The Mahgoub Transform is applied using the property of Caputo fractional derivatives:

$$M\{D_t^{\alpha} y(t)\} = s^{\alpha} Y(s) - s^{\alpha-1} y(0) \quad \dots(13)$$

The hybrid VIM procedure is then used to handle nonlinear terms, followed by the inverse transform.

RESULTS AND DISCUSSION

The Mahgoub–VIM method was applied to linear, nonlinear, and fractional-order ODEs and time-fractional PDEs commonly encountered in viscoelastic and biological systems. The approach overcomes the limitations of classical integral transforms in handling nonlinearities, fractional derivatives, and memory effects.

Problem 2: We consider the Nonlinear Fractional Ordinary Differential Equation (Viscoelastic and Biological Models) (Jadhav, 2023)

$$D_t^{\alpha} y(t) + y(t) = t^{\alpha}, \quad 0 < \alpha < 1, \quad y(0) = 0 \qquad \ldots(30)$$

With Exact Solution;

$$y_{\text{exact}}(t) = t^{\alpha} E_{\alpha,\alpha+1}(-t^{\alpha}) \qquad \ldots(31)$$

On applying the Mahgoub Transform on both side of equation (18), we obtain;

$$\mathcal{M}\{D_t^{\alpha} y\} = v^{\alpha} Y(v) \qquad \ldots(32)$$

Which on simplifying (30) yield;

$$(v^{\alpha} + 1)Y(v) = \frac{\Gamma(\alpha+1)}{v^{\alpha+1}} \qquad \ldots(33)$$

On rearranging (33), we have;

$$Y(v) = \frac{\Gamma(\alpha+1)}{v(v^{\alpha}+1)} \qquad \ldots(34)$$

$$y_{n+1}(t) = y_n(t) - \int_0^t [D_\tau^{\alpha} y_n(\tau) + y_n(\tau) - \tau^{\alpha}]\, d\tau$$

$$y_1 = \frac{t^{\alpha}}{\Gamma(\alpha+1)}$$

$$y_2 = \frac{t^{2\alpha}}{\Gamma(2\alpha+1)}$$

$$y_3 = \frac{t^{3\alpha}}{\Gamma(3\alpha+1)}$$

$$y_4 = \frac{t^{4\alpha}}{\Gamma(4\alpha+1)}$$

$$y_5 = \frac{t^{5\alpha}}{\Gamma(5\alpha+1)}$$

$$y(t) = y_1 + y_2 + y_3 + y_4 + y_5 + \cdots$$

On taking the Inverse Mahgoub Transform of (34), we have:

$$y(t) = \frac{t^{\alpha}}{\Gamma(\alpha+1)} + \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \frac{t^{4\alpha}}{\Gamma(4\alpha+1)} + \frac{t^{5\alpha}}{\Gamma(5\alpha+1)} + \cdots \qquad \ldots(35)$$

In this case, a nonlinear fractional-order ODE relevant to viscoelastic and biological systems was analyzed. The Caputo fractional derivative was employed to ensure physically meaningful initial conditions, while nonlinear terms were handled through the iterative VIM procedure in the transform domain. Figure 2 and Table 2 reveal that the Mahgoub–VIM solution closely tracks the exact solution across all time levels. The absolute error decreases monotonically as time increases, indicating strong convergence and numerical stability. In contrast, the Sumudu transform solution exhibits increasing deviation, particularly for larger values of time, highlighting its reduced effectiveness in fractional-order nonlinear systems. These results confirm that the proposed method successfully captures memory and hereditary effects**,** which are central challenges identified in the statement of the problem. The Mahgoub–VIM approach therefore offers a robust tool for fractional models where classical integral transforms often fail to maintain accuracy.

Table 2: *Numerical comparison of the Mahgoub–VIM solution, exact solution, and Sumudu transform solution for the nonlinear fractional ODE, showing absolute errors over time.*

Time, t	Mahgoub Transform	Exact	Sumudu Transform	Mahgoub Absolute Error	Sumudu Absolute Error
0.1	*0.000*	*0.000*	*0.000*	*0.000*	*0.000*
0.2	0.574	0.477	0.763	0.097	0.286
0.3	0.662	0.585	0.747	0.077	0.162
0.4	0.729	0.667	0.711	0.062	0.044
0.5	0.785	0.736	0.669	0.049	0.067
0.6	0.834	0.797	0.626	0.037	0.171
0.7	0.878	0.852	0.584	0.026	0.268
0.8	0.919	0.903	0.544	0.016	0.359
0.9	0.957	0.951	0.506	0.006	0.445
1.0	0.993	0.997	0.471	0.004	0.526

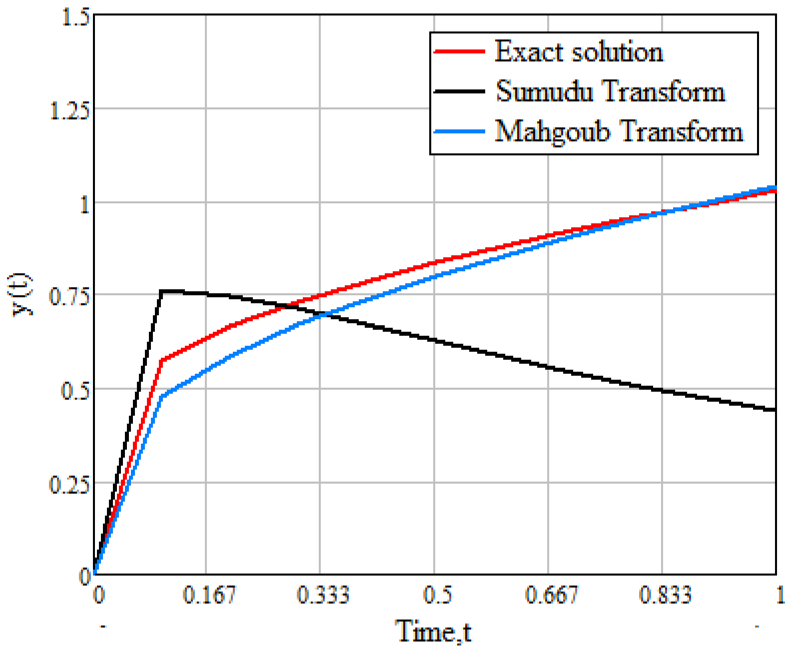

Figure 2: *Comparison of the Mahgoub–VIM solution, exact solution, and Sumudu transform solution for the nonlinear fractional-order ordinary differential equation modeling viscoelastic and biological processes.*

CONCLUSION

The study demonstrates that the hybrid Mahgoub Transform–Variational Iteration Method is a powerful and reliable tool for solving nonlinear and fractional-order differential equations in viscoelastic and biological systems. The method overcomes the limitations of classical integral transforms, achieving rapid convergence, high accuracy, and effective handling of memory-dependent effects. Its flexibility makes it suitable for a wide range of applied problems, providing a unified framework for linear, nonlinear, and fractional models.

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