Mikailalsys Journal of
Mathematics and Statistics

e-ISSN : 3030-816X
p-ISSN : 3030-8399
Index : Harvard, Boston, Sydney University, Dimensions, Lens, Scilit, Semantic, Google, etc

https://doi.org/10.58578/MJMS.v4i2.9210

Enhancing Volatility Forecasting in the Nigerian Stock Exchange: Evaluating GARCH-Type Models and Innovation Densities

Samuel Ohiorhenuan Oboh[1], Semiu Ayinla Alayande[2], Faith Oluwadamilola Olatunde[3]
[1]Federal University Wukari, Taraba State, Nigeria
[2,3]Redeemer's University Ede, Osun State, Nigeria
obohsamuel@fuwukari.edu.ng; alayandesa@run.edu.ng

Article Info:

Submitted:	Revised:	Accepted:	Published:
Mar 1, 2026	Apr 29, 2026	May 11, 2026	May 16, 2026

Abstract

Although volatility modeling in emerging stock markets has received increasing attention, limited research has jointly compared GARCH-type model structures under alternative symmetric and skewed innovation densities in the Nigerian capital market. This study aims to evaluate the forecasting performance of selected GARCH-type models under alternative innovation densities using daily returns of the Nigerian Stock Exchange All Share Index (NSE-ASI) from February 2012 to July 2023. A quantitative econometric time-series design was employed, involving 2,820 daily observations selected through purposive sampling based on data availability. Data were obtained from the official market database and analyzed using Maximum Likelihood Estimation, model selection criteria comprising Log-Likelihood (LL), Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC), and forecast accuracy measures including Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and Mean Absolute Error (MAE). The findings indicate that the APARCH(1,1)-GED model provides the best in-sample fit, whereas the APARCH(1,1)-SGED specification produces the most accurate out-of-sample forecasts. These results demonstrate the importance of innovation density selection in capturing asymmetry and fat-tailed behavior in stock return volatility. The study concludes that incorporating skewed heavy-tailed distributions enhances volatility forecasting accuracy in the Nigerian capital market. The findings contribute to the theoretical development of conditional heteroskedasticity modeling and offer practical implications for risk management, portfolio analysis, and regulatory forecasting in emerging markets. Future research may extend this work by examining advanced nonlinear and regime-switching volatility models across broader emerging market contexts.

Keywords: APARCH Model; GARCH-Type Models; Innovation Densities; Nigerian Stock Exchange; Volatility Forecasting

INTRODUCTION

In contemporary financial econometrics, volatility forecasting is gaining more attention than ever particularly in emerging markets where uncertainty is increased by structural and macroeconomic weaknesses. The Nigerian Stock Exchange (NSE), which is now the Nigerian Exchange Group (NGX), has seen increased volatility in Nigeria as a result of ongoing inflation, currency devaluation, political risk, inconsistent policies, and vulnerability to shocks from around the world. The returns on the Nigerian financial market show volatility clustering, persistence, and asymmetry, according to empirical data. This is consistent with stylized facts observed in emerging markets worldwide (Atoi, 2015; Ibrahim, 2017; Yaya et al., 2016; Adegboyo et al., 2025). Globally, GARCH-type models remain the cornerstone of volatility modeling (Bollerslev et al., 2020; Engle, 2021), yet their performance depends critically on both model specification and innovation density. These insights highlight the importance of adopting methodological frameworks capable of capturing the unique distributional and structural properties of emerging market data.

Despite substantial progress in volatility modeling, a significant limitation persists in the literature: studies rarely evaluate the combined effects of model choice and innovation distribution when forecasting NSE volatility. Most empirical work either focuses on comparing volatility models while relying on restrictive assumptions such as Gaussian or Student-t errors (Ibrahim, 2017), or investigates alternative error distributions without systematically assessing their interactions with different GARCH structures (Samson, 2020;

Ampadu et al., 2024). This fragmented research landscape limits the ability to identify the most accurate and reliable model–distribution pairing for the Nigerian market. Recent comparative studies involving machine learning approaches (Adams, 2024) also indirectly reveal that performance variability is driven in part by distributional misspecification, underscoring the methodological gap. Without integrated empirical analysis, volatility estimates may misrepresent tail risk, leverage effects, or extreme movements—outcomes with serious implications for risk managers, investors, and regulators.

Addressing this gap is essential because volatility forecasts shape decision making in portfolio allocation, risk management, derivative pricing, and regulatory planning. Emerging markets like Nigeria, characterized by liquidity constraints and heightened exposure to macroeconomic and geopolitical risks, require robust volatility frameworks that incorporate both model dynamics and realistic distributional assumptions. The absence of studies that systematically evaluate alternative GARCH specifications alongside multiple innovation densities represents a significant methodological deficiency, especially at a time when the NSE faces growing integration with global financial systems.

In response to these limitations, this study provides a comprehensive and rigorous assessment of volatility forecasting performance in the NSE by jointly evaluating three canonical GARCH-type models GARCH(1,1), EGARCH(1,1), and GJRGARCH(1,1) under three alternative innovation densities: the Normal distribution, Student-t distribution, and the Generalized Error Distribution (GED). Using recent NSE return data and maximum likelihood estimation procedures implemented in R, the study compares both in-sample estimates and out-of-sample forecasts to determine the optimal model distribution combination. By integrating model choice with distributional assumptions, this work offers a more holistic and empirically grounded framework than what currently exists in Nigerian volatility literature. The study contributes novel insights into the behavior of volatility in the NSE and provides practical implications for analysts, policymakers, and institutional investors navigating the complexities of the Nigerian capital market.

Literature Review

Volatility modelling in the Nigerian Stock Exchange (NSE) has been widely explored through ARCH–GARCH frameworks, with early studies consistently demonstrating the presence of volatility clustering, leverage effects, and heavy tailed return distributions. Emenike (2010) and later by Atoi (2015) established that asymmetric

GARCH variants such as EGARCH, TGARCH, and GJRGARCH offer improved explanatory power over symmetric models because they better capture the disproportionate impact of negative shocks on volatility. Subsequent contributions, including Ibrahim (2017), reaffirm these findings by showing that EGARCH models provide more accurate volatility forecasts than standard GARCH, particularly in the presence of persistent asymmetry in Nigerian financial returns. Collectively, these studies confirm the relevance of GARCH-type approaches for modelling volatility in emerging markets like Nigeria, where structural instability and market imperfections amplify uncertainty.

A parallel body of literature investigates the role of innovation density in improving the behaviour and performance of volatility models. Empirical evidence shows that Nigerian stock returns deviate substantially from normality, prompting researchers to consider heavy tailed alternatives. For instance, Atoi (2015) and Emenike (2010) demonstrate that the Student-t and Generalized Error Distribution (GED) better reflect the leptokurtic structure of NSE returns. More recent investigations expand this line of inquiry: Samson et al. (2020) compare multiple GARCH-type models estimated under five innovation distributions and conclude that non-Gaussian errors especially skewed specifications—yield superior model fit and forecasting performance. Similarly, Ampadu et al. (2024) employ simulation based evidence to show that innovation density assumptions critically influence forecast accuracy, with skewed GED performing consistently better than Normal or Student-t distributions. These studies underscore that choosing an appropriate density is as important as selecting the right GARCH variant.

However, despite these contributions, the empirical literature remains limited by several methodological shortcomings. First, many Nigerian studies evaluate either differences in model structure or differences in error distributions, but rarely both within a unified framework. As a result, findings on the optimal model–distribution pairing for the NSE remain fragmented and inconclusive. Second, several influential studies rely on pre-2015 or pre-COVID datasets (Ibrahim, 2017; Atoi, 2015), which do not capture the heightened volatility, macroeconomic dislocations, regulatory shocks, and global spillovers that have characterized the Nigerian market in recent years. Third, while some works employ non-Gaussian error distributions, they typically assess only a narrow subset often limited to the Student-t or GED thereby overlooking skewed alternatives shown to perform strongly in more recent cross market studies. Finally, comprehensive forecast evaluations remain scarce; many studies emphasize in-sample diagnostics while failing to incorporate rigorous out-of-sample testing across competing model–distribution combinations.

These gaps collectively point to the need for research that systematically evaluates multiple GARCH variants against a broad set of innovation densities using updated, post-COVID NSE data and robust predictive validation procedures. Although evidence from recent studies such as Adegboyo and Sarwar (2025) advances the literature by incorporating newer datasets and non-Gaussian errors, their evaluations still remain limited in scope—often focusing on only a few error distributions or omitting broader comparative analyses. Thus, despite notable progress, the literature continues to lack a comprehensive, empirically defensible, and replicable framework for identifying the most suitable volatility model for the Nigerian Stock Exchange.

This study addresses these shortcomings by jointly comparing the GARCH-type models GARCH(1,1), EGARCH(1,1), and GJRGARCH(1,1) across multiple innovation densities, including Normal, Student-t, and GED distributions, using recent high volatility, post-COVID Nigerian data. This study further responded directly to these gaps by jointly evaluating GARCH(1,1), EGARCH(1,1), and GJRGARCH(1,1) models under Normal, Student-t, and GED innovation densities, thereby offering the most extensive and methodologically coherent assessment of volatility forecasting currently available for the Nigerian Stock Exchange.

METHODOLOGY

This study adopts a quantitative time-series modeling framework in evaluating the performance of competing GARCH-type volatility models under alternative innovation densities. The methodological process consists of (i) dataset description, cleaning and stationarity check, (ii) model specification, (iii) distributional assumptions for innovations, and (iv) model selection and forecast accuracy evaluation, following standard practices in volatility modeling.

Dataset Description and Source

The Daily closing prices of the Nigerian Stock Exchange All Share Index (NSE-ASI) were obtained from http://Investing.com. The dataset was for the period 10years starting from 2 February 2012 to 12 July 2023, which yielded 2,934 observations., the sample was partitioned into two for in-sample estimation from (2 February 2012 – 30

December 2022; 2,703 observations) and an out-of-sample evaluation from (January – July 2023) to ensure robust forecast validation under post-COVID high-volatility conditions.

The ASI values represent broad market performance, making them suitable for global market volatility analysis. The time series plot and descriptive statistics were used to identify clustering, skewness, kurtosis, and volatility bursts typical of financial returns.

Return Computation and Stationarity Tests

Daily log-returns were computed from closing index values using:

$$r_t = \log\left(\frac{ASI_t}{ASI_{t-1}}\right) x\,100, \quad t = 2,\cdots,n \tag{1}$$

Where, ASI_t All Share Index at day t (present day)

ASI_{t-1} All Share Index at day t- 1(previous day)

n is the number of observation.

Stationarity of the return series were measured using both the Augmented Dickey–Fuller (ADF) and KPSS tests conducted in R, ensuring that the series satisfies the requirements for conditional heteroskedasticity of the model.

GARCH-Type Volatility Models

According to Bollerslev (1986), the GARCH model is key for modeling financial time series and a components for capturing and forecasting volatility. The primary structure includes a conditional mean equation, usually featuring Autoregressive (AR) and Moving Average (MA) terms, and conditional variance equation uniquely integrating autoregressive (ARCH) and moving average (GARCH) terms.

The GARCH (*p*, *q*) model considers the current conditional variance dependent on the p past conditional variances as well as the q past squared residual. These equations, expressed mathematically as:

$$y_t = \sigma_t\ \varepsilon_t \tag{2}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i\, y_{t-i}^2 + \sum_{j=1}^{q} \beta_j\, \sigma_{t-j}^2 \tag{3}$$

where $w > 0$, $a_i \geq 0$, $\beta_j \geq 0$, and (ε_t) is the white noise. In this model, σ_t^2 is the conditional variance of y_t. In addition to the non-negativity of the parameters here, there is parameter restriction to ensure the positivity of the conditional variance.

$$\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1 \tag{4}$$

(p,q) model consists of a conditional mean equation typically ARMA and a conditional variance equation: reflects asymmetric volatility responses to negative shocks.

Exponential GARCH (EGARCH) (p,q) Model

To remedy some weakness of symmetric GARCH model, Nelson, (1991) advanced the following model as: EGARCH (p,q) is given by:

$$In(\sigma_t^2) = \omega + \sum_{i=1}^{p} \alpha_i \frac{|u_{t-i}| + \gamma_i u_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{q} \beta_j \, In\left(\sigma_{t-j}^2\right) \tag{5}$$

The presence of parameter γ_i indicates an asymmetric effect of shocks on volatility and the value of u_{t-i} statistically different from zero or negative signing the asymmetry or the leverage effect. Nelson exploited the absolute value to react asymmetrically to positive and negative lagged values and utilized the logged conditional variance to relax the GARCH model's restriction.

APARCH Model

Ding, Granger, and Engle (1993) put the Asymmetric Power ARCH (APARCH) model. This APARCH (p,q) model can be expressed as stated below;

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \left(\left|\varepsilon_{t-i}^2\right| + \gamma_i \, \varepsilon_{t-i}^2\right)^{\delta} + \sum_{j=1}^{p} \beta_j \, \sigma_{t-j}^{\delta} \tag{6}$$

Where $\omega > 0,\ \delta \geq 0,\ \ \beta_j > 0,\ (j = 1, \cdots, p),\ \alpha_t \geq 0, and\ -1 < \gamma_i < 1\ \ (i, \cdots q)$

This model couples the flexibility of a varying exponent with the asymmetry coefficient to take care of the "leverage effect".

Conditional Innovation Densities

To evaluate the sensitivity of model performance to error-term assumptions, six standardized innovation distributions were considered: Normal, Skew-Normal, Student-t, Skew-t, GED, and Skew-GED. Their functional forms follow standard parametrizations

(Eqs. 7–13). These distributions allow for skewness, kurtosis, and fat-tailed behavior commonly observed in financial returns.

Standardized Normal Distribution

The Standard Normal Distribution is given as:

$$f(z) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{z^2}{2}\right), \; E|z| = \sqrt{2/\pi}\,, \gamma_2 = 0 \tag{7}$$

Standardized Skew Normal Distribution

The Skewed Normal Distribution:

$$f(z_t) = \frac{1}{\sigma\pi} e^{-\frac{(z_t-\varepsilon)^2}{2\sigma^2}} \int_{-\infty}^{\alpha} \frac{z_t - \varepsilon}{\sigma} e^{-\frac{t^2}{2}} dt, \quad -\infty < z_t < \alpha \tag{8}$$

where ε is the location, σ is the scale and α denotes the shape parameter

Standardized Student-t Distribution

The Student -t Distribution (STD) is given by:

$$f(x) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{(v\pi)}\Gamma\left(\frac{v}{2}\right)} \left(1 + \frac{x^2}{v}\right)^{-\left(\frac{v+1}{2}\right)} \qquad ; -\infty < x < \infty \tag{9}$$

Where the degree of freedom $v >$ as $v \leftrightarrow \infty$

Standardized Skew Student-t Distribution

The Skew Student -t Distribution (SSTD) is given by

$$f(x,\mu,\sigma,v,\lambda)=\begin{cases} bc\left(1+\dfrac{1}{v-2}\left(\dfrac{b\left(\dfrac{x-\mu}{\sigma}\right)+a}{1-\lambda}\right)^{2}\right)^{\frac{v+1}{2}}, & if\ x<-\dfrac{a}{b} \\ bc\left(1+\dfrac{1}{v-2}\left(\dfrac{b\left(\dfrac{x-\mu}{\sigma}\right)+a}{1+\lambda}\right)^{2}\right)^{\frac{v+1}{2}} & if\ x\geq-\dfrac{a}{b} \end{cases} \qquad (10)$$

Where v is a shape parameter with $2 < v < \infty$ and λ is a skewness parameter with. $-1 < \lambda < 1$. The constant a, b and c are given as:

$$a=4\lambda c\left(\frac{v-2}{v-1}\right),\quad b=1+3(\lambda)^{2}-a^{2},\quad c=\frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi(v-2)\Gamma\left(\frac{v}{2}\right)}} \qquad (11)$$

Where μ and σ^2 are mean and variance of the skewed student-t distribution respectively.

Standardized Generalized Error Distribution (GED)

The Generalized Error Distribution (GED) is given as:

$$f(x,\mu,\sigma,v)=\frac{\sigma^{-1}v\exp\left(-\frac{1}{2}\left|\frac{\frac{x-\mu}{\sigma}}{\lambda}\right|^{v}\right)}{\lambda 2^{\left(1+\left(\frac{1}{v}\right)\right)}\Gamma^{\left(\frac{1}{v}\right)}}\quad 1<x<\infty \qquad (12)$$

The parameter for tail thickness or degree of freedom is V > 0. $= \sqrt{2^{(-2/v)}r^{(3/v)}}$

The Generalized Error Distribution (GED) is normal distribution if the degree of freedom of shape parameter v = 2, and fat tail if v < 2.

Standardized Skew Generalized Error Distribution (SGED)

$$f(\varepsilon) = \frac{v}{2\lambda\Gamma\left(\frac{1}{v}\right)} \exp\left(-\left|\frac{\varepsilon-\mu}{\lambda}\right|^{v}\right) \tag{13}$$

Where:

$v > 0$ is shape parameter

ε is random error term

λ is Scale parameter

μ is location parameter

γ is skewness parameter usually constrained by $-1 < \gamma < 1$

Model Estimation, Selection, and Forecast Evaluation

All models were estimated via maximum likelihood under their respective distributions. Competing specifications were compared using: Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Log-Likelihood (LL) and Out-of-sample forecast performance was evaluated using the Root Mean Squared Error (RMSE) MSE and MAE

$$MSE = \frac{1}{n}\sum_{t=1}^{n}\left(y_t - \hat{y}_t\right)^2 \tag{14}$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(y_t - \hat{y}_t\right)^2} \tag{15}$$

$$MAE = \frac{1}{n}\sum_{t=1}^{n}\left|y_t - \hat{y}_t\right|^2 \tag{16}$$

By standard, the Lowest RMSE, MAE and MSE values indicate superior predictive accuracy. This integrated methodological framework enables a rigorous comparative assessment of GARCH-type models and innovation densities in forecasting volatility of the Nigerian stock market.

RESULTS AND DISCUSSION

Descriptive Statistics

Table 1 displays the descriptive statistics for the Nigeria All Share Index, which includes 2703 data points, from 02/02/2012 to 30/12/2022 of the return series.

Table 1: Descriptive statistics for Daily return series ASI between 02/02/2012 and 30/12/2022

	Mean	Median	Min	Max	Std. Dev.	Skewness	Kutosis	JB(p-value)
Return series	0.03335	-0.0010	-5.033	7.985	0.9694	0.3026	5.631	3618(<0.0001)

The summary statistics indicate that the daily return on ASI for Nigeria stocks computed with minimum and maximum daily returns of -5.033% and 7.985% respectively. The high degree of variability and dispersion from mean of the returns series throughout the study are shown by the positive standard deviation. The skewness of 0.3026 and kurtosis of 5.631 were obtained. The skewness was 0.31 while their kurtosis was far above 3 (5.631) shows fat-tails in the series. This is an indication that the returns are not normally distribution. This Jarque-Bera test of 3618 with a marginal p-value of 0.0001 also shows that the series is not normally distributed.

3.2 Graphical Examination of Daily Stock Prices and Returns

The initial step in time series analysis is to plot the original and return series; this helps ascertain the non-stationarity and price volatility of the data set. The graphical characteristics of the series were examined when these were plotted against time. Figures 1 and 2, respectively, display the plot.

Figure 1: Time Plot of the monthly All Share Index from 01/02/2012 to 30/12/2022

The daily stock prices presented in Figure 1 showed clearly that the series has mean and variance that changes with time and the presence of a trend indicating that the series is not covariance stationary.

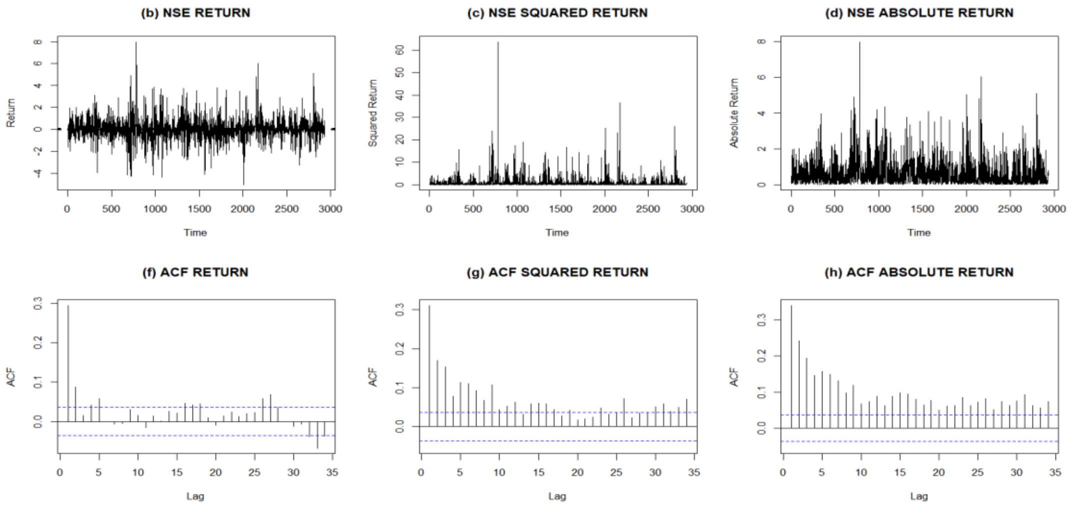

Figure 2: Daily All Share Index Returns, Squared Returns, Absolute Returns and their ACF's

These plots of daily return, squared returns and Absolute return series presented in Figure 2 suggests that the series has a constant mean and variance with absence of trend indicating that it is generated by a random walk and is thus weakly stationary. The plot in Figure 2 also indicates that some periods are more clustered than others as large changes in stock returns tend to be followed by large changes and small changes are followed by small changes. This phenomenon is described as volatility clustering. Also the ACF showed that the series is stationary and confirms volatility clustering.

Unit Root and Heteroskedasticity Tests Results

A stationarity test was conducted using the KPSS and ADF test. Based on the ASI of NSE index returns at all levels, it was determined that all return series were stationary.

Table 2:

	Stationarity test		
	Test Statistics	p-value	Comment
ADF	-12.829	0.01	Stationary at all lag
KPSS	0.15628	0.1	Stationary at all lag

Table 2 show that the ADF with p-value of 0.01 indicates that the return series is stationary at 5% level of significance while the KPSS with p-value of 0.1 which is a complete revise in the hypothesis also shows that the series is stationary.

ARCH effect tests for the daily returns

The Lagrange multiplier (LM) test statistic for the daily returns

H_0: There is no ARCH effect H_1: There is ARCH effect

Table 3

	Chi-squared	Df	p-value
ARCH LM-test	203	10	< 2.2e-16
	213	15	< 2.2e-16
	214	20	< 2.2e-16

The three (3) conditions for a GARCH process which are: stationarity of the data, Volatility clustering and the evidence of ARCH effect have being obtained and this allowed the usage of our GARCH or Volatility models. Consequently, we proceed to estimate the model parameters as show in the next table below

Parameter Estimates of the Volatility Models

The results of parameter estimations using the Student-t distribution, normal distrib ution, generalized error distribution, and suitably skewed versions for volatility models for Nigerian stocks are shown in Table 5. The findings show that in every model, the GARCH and ARCH terms were significant ($p < 0.05$). The importance of the ARCH and GARCH variables in these models suggests the existence of volatility persistence in the daily returns of the Nigerian stock market. The result shows that volatility has persisted quite a while, in dicating that there is more risk and uncertainty in the Nigerian stock market. The result also shows that the distribution parameters skewness and shape parameter were found to be sig nificant ($p<0.05$) in all volatility models.

Parameter of Volatility Models								Distributional parameters	
Models	**Distribution**	**μ (p-value)**	**ω (p-value)**	α_1 **(p-value)**	β_1 **(p-value)**	γ_1 **(p-value)**	δ **(p-value)**	**Skew (p-value)**	**Shape (p-value)**
GARCH (1,1)	**NORM**		0.132946 (0.024822) [0.0001]	0.251711 (0.0316) [0.0001]	0.612727 (0.0504) [0.0001]	-	-		
	STD		0.109868 (0.027278) [0.0001]	0.371839 (0.064957) [0.0001]	0.627161 (0.049798) [0.000]				3.184958 (0.214564) [0.0001]
	GED	-0.012694 (0.0043) [0.0031]	0.097427 (0.02396) [0.0001]	0.290295 (0.045615) [0.0001]	0.638605 (0.053823) [0.0001]				0.966752 (0.033975) [0.0001]
	SNORM		0.124674 (0.023215) [0.0001]	0.241308 (0.030100) [0.0001]	0.630152 (0.047723) [0.0001]	-	-	1.048782 (0.020833) [0.0001]	-
	SSTD		0.109613 (0.027276) [0.0001]	0.371054 (0.058527) [0.0001]	0.627946 (0.049892) [0.0001]	-	-	1.004470 (0.025085) [0.0001]	3.184658 (0.215687) [0.0001]
	SGED		0.096493 (0.022263) [0.0001]	0.285330 (0.043296) [0.0001]	0.642203 (0.050172) [0.0001]	-	-	1.014242 (0.015596) [0.0001]	0.969241 (0.034079) [0.0001]
EGARCH (1,1)	**NORM**	-	-	0.028267 (0.015322) [0.07]	0.850882 (0.02230) [0.0001]	0.398142 (0.033200) [0.0001]			3.073242 0.221867
	STD				0.899483 (0.020239) [0.0001]	0.512649 _(0.054505) [0,0001			3.073242 (0.221867) [0.0001]
	GED	-0.011178 (0.004341) [0.01]	-0.036228 (0.013005) [0.0053]		0.887193 (0.023241) [0.0001]	0.432031 (0.044638) [0.0001]			
	SNORM				0.858081 (0.016826) [0.0001]	0.394518 (0.032403) [0.0001]		1.047377 (0.164915) [0.0001]	
	SSTD				0.899513 (0.020653) [0.0001]	0.512444 (0.060856) [0.0001]		1.000510 (0.057355) [0.0001]	3.073729 (0.229446) [0.0001]
	SGED		-0.036049 (0.012732) [0.0046]		0.889114 (0.022479) [0.0001]	0.427138 (0.042947) [0.0001]		1.020253 (0.014770) [0.0001]	0.973074 (0.033923) [0.0001]
APARCH	**NORM**		0.149079 (0.025251) [0.0001]	0.244786 (0.025212) [0.0001]	0.656058 (0.042832) [0.0001]	-0.092136 (0.040462) [0.022781]	1.098289 (0.153125) [0.0001]		
	STD		0.104664 (0.022879) [0.0001]	0.317588 (0.042855) [0.0001]	0.698519 (0.038443) [0.0001]		0.962253 (0.150510) [0.0001]		3.071513 (0.219493) [0.0001]
	GED	-0.011859 (0.004070) [0.0035]	0.108209 (0.024029) [0.0001]	0.262621 (0.033598) [0.0001]	0.696487 (0.043520) [0.0001]		0.991241 (0.16862) [0.0001]		0.970916 (0.034030) [0.0001]
	SNORM		0.142223 (0.024572) [0.0001]	0.241044 (0.024743) [0.0001]	0.666042 (0.041771) [0.0001]		1.112489 (0.154398) [0.0001]	1.036112 (0.020731) [0.0001]	
	SSTD		0.105254 (0.023073) [0.0001]	0.319052 (0.043777) [0.0001]	0.697400 (0.038787) [0.0001]		0.958916 (0.151620) [0.0001]	0.992942 (0.030456) [0.0001]	3.065175 (0.221235) [0.0001]
	SGED	-0.008998 (0.003547) [0.0111]	0.107583 (0.023250) [0.0001]	0.261442 (0.032774) [0.0001]	0.697856 (0.042183) [0.0001]		0.992158 (0.167142) [0.0001]	1.005752 (0.017193) [0.0001]	0.971965 (0.034026) [0.0001]

Table 4: Parameter estimates of volatility models for Nigerian stock market

A variety of GARCH-type volatility models, such as GARCH(1,1), EGARCH(1,1), and APARCH(1,1), were used to investigate the parameter estimates of the volatility model s for the Nigerian stock market. Subsequently, the models were fitted to the Nigerian stock market data, taking into account various error distributions, including the Normal, Student' s t, Generalized Error Distribution (GED), and their skewed forms. For every model-distri bution combination, statistically significant estimates of the GARCH parameters (α and β) were discovered, suggesting volatility clustering and persistence in the data. In the EGARC H and APARCH models, the leverage parameter (γ) played a crucial role because it could ta ke into consideration the asymmetric impacts of both positive and negative shocks on volat ility. Notable form characteristics of the Student's t and GED distributions indicated that t here might be a wide tail to the data. In addition, the huge skew parameters for the skewed distributions imply that the data may not be balanced. When combined with the GED distr ibution, the Asymmetric Power Autoregressive Conditional Heteroscedasticity APARCH(1 ,1) model yielded the lowest log-likelihood value and the best trade-off between goodness-of-fit and model complexity. Additionally, it consistently offered the ideal fit. As a result, it was shown that the best specification for simulating the volatility dynamics in the Nigerian stock market was the APARCH(1,1) model with the GED distribution.

Model Selection Based on Fitness

Table 5 is the results of various symmetric and asymmetric GARCH models computed with various innovation densities.

Table 5: Model Selection

Model	Distribution	LL	AIC	BIC
GARCH(1,1)	NORM	-3415.891	2.5323	25411
	STD	-3208.87	2.3798	2.3907
	GED	**-3191.45**	**2.3669**	**2.3778**
	SNORM	-3413.013	2.5309	2.5418
	SSTD	-3208.854	2.3805	2.3936
	SGED	**-3191.419**	2.3676	2.3807
EGARCH(1,1)	NORM	-3408.751	2.5278	2.5387
	STD	-3198.789	2.3730	2.3861
	GED	**-3183.79**	**2.3619**	**2.3750**
	SNORM	-3406.353	2.5267	2.5398
	SSTD	-3198.789	2.3738	2.3891
	SGED	**-3183.603**	2.3625	2.3778
APARCH(1,1)	NORM	-3404.65	2.5255	2.5386
	STD	-3196.72	2.3722	2.3875
	GED	**-3182.333**	**2.3616**	**2.3769**

SNORM	-3403.181	2.5251	2.5404
SSTD	-3196.682	2.3730	2.3904
SGED	**-3182.329**	2.3623	2.3798

Bold faces denote the model selection criteria

A number of GARCH model specifications, including the Normal, Student's t, Generalized Error Distribution (GED), and their skewed counterparts, were fitted to the data in the empirical inquiry under various error distribution assumptions. In their model specifications, they included GARCH(1,1), EGARCH(1,1), and APARCH(1,1). The model selection criteria that consistently produced the best-performing combination—the APARCH(1,1) model with the GED distribution—were the log-likelihood, Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC). This model strikes the ideal balance between goodness-of-fit and model complexity, as evidenced by its lowest AIC and BIC values and greatest log-likelihood value, which shows a superior match to the data. As a result, it was shown that the APARCH(1,1) model was the best specification for simulating and predicting the volatility dynamics in the dataset being studied.

Forecast performance

Table 6 give details of the forecast performance measure

Model	**Innovation Dist.**	**MSE**	**RMSE**	**MAE**
GARCH(1,1)	NORM	0.868746	0.434373	0.34524
	STD	0.697623	0.348811	0.33627
	GED	**0.682917**	**0.341459**	**0.33156**
	SNORM	0.68747	0.343735	0.33387
	SSTD	0.697623	0.348811	0.33537
	SGED	0.687698	0.343849	0.33274
EGARCH(1,1)	NORM	0.74877	0.374385	0.31901
	STD	0.745114	0.372557	0.31679
	GED	**0.662542**	**0.331271**	**0.30015**
	SNORM	0.68077	0.340385	0.32873
	SSTD	0.679084	0.339542	0.32501
	SGED	0.670254	0.335127	0.30113
APARCH(1,1)	NORM	0.602683	0.301341	0.30016

STD	0.601558	0.300779	0.29857
GED	0.589488	0.294744	0.29361
SNORM	0.582683	0.291341	0.29024
SSTD	0.579356	0.289678	0.27656
SGED	**0.562848**	**0.281424**	**0.27362**

ASI return series is from 3 January, 2023 – 07 December, 2023 (232 daily returns). Bolded values indicate the conditional distribution with the least MSE, RMSE and MAE in each model.

Using 232 trading days out of sample forecast, the forecasting performance of these volatility models as well as the various error distributions were compared. The outcome sh ows that, for all six models, the innovation distribution is a critical factor in determining the forecast accuracy among the competing error distributions and their skewed versions. The GARCH(1,1) and EGARCH(1,1) models consistently perform best with the GED distribu tion. Since the APARCH (1, 1)-skewed Generalized Error Distribution produced the lowes t Root Mean Square Error of all the competing volatility models. Therefore, APARCH (1, 1) - SGED is recommended for forecasting daily returns of Nigerian stock market.

CONCLUSION

This study set out to provide a comprehensive and unified assessment of how model specification and innovation density jointly influence volatility forecasting in the Nigerian Stock Exchange, addressing a clear methodological gap in the literature. Strong volatility clustering, heavy tails, and asymmetric responses to market shocks are features of Nigerian stock returns that simple symmetric GARCH structures are unable to sufficiently capture, according to results from descriptive analysis, stationarity tests, parameter estimation, and thorough in-sample and out-of-sample evaluations. The APARCH(1,1) model consistently provided better fit across all model–distribution combinations examined, with the skewed GED distribution producing the most accurate forecasts out-of-sample and the GED performing best in-sample. These results demonstrate the importance of asymmetry and fat-tailed error distributions in simulating volatility in developing markets like Nigeria. The study offers significant insights for regulators, financial analysts, and portfolio managers that rely on accurate volatility estimates for well-informed decision-making by proving that the APARCH(1,1)–SGED model offers the most dependable and empirically supported forecasting framework. The findings highlight the benefits of combining sophisticated conditional variance structures with adaptable innovation densities and encourage the use of these models in the evaluation of market risk and the development of policies within the Nigerian financial system.

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