Mikailalsys Journal of Mathematics and Statistics

e-ISSN : 3030-816X
p-ISSN : 3030-8399
Index : Harvard, Boston, Sydney University, Dimensions, Lens, Scilit, Semantic, Google, etc

https://doi.org/10.58578/MJMS.v4i2.8764

Mathematical Modelling of Kidnapping Activities

A. R. Tasiu[1], Kabir O[2], Aminu Muhammad[3], M. S. Ayu[4], M. M. Ishaq[5]
[1,2]Usmanu Danfodiyo University, Sokoto, Nigeria; [3]Federal University Gusau, Nigeria;
[4]Abdullahi Fodiyo University of Science and Technology, Alieru, Nigeria;
[5]Kaduna State Polytechnic, Kaduna State, Nigeria
Corresponding email: tasiu.ahmad@uduok.edu.ng

Article Info:

Submitted:	Revised:	Accepted:	Published:
Feb 13, 2026	Mar 13, 2026	Mar 25, 2026	Mar 30, 2026

Abstract

Kidnapping has become one of the most severe security challenges in Nigeria, particularly in the northern regions, where it has evolved into a profitable criminal enterprise. This study develops a mathematical model to analyze the dynamics and control of kidnapping activities. The population is classified into five compartments: susceptible individuals, exposed individuals, informants, kidnappers, and repentant kidnappers. The model describes the transition of individuals from vulnerability to involvement as informants or kidnappers, as well as the possibility of repentance through rehabilitation. A basic reproduction number, (R_0), is derived to determine whether kidnapping activities will persist or decline. The analysis indicates that kidnapping can be eliminated when ($R_0 < 1$), whereas ($R_0 > 1$) implies its continued persistence. Numerical simulations further show that increasing the rehabilitation rate of kidnappers promotes repentance, while strengthening intelligence gathering through informants and reducing recruitment into kidnapping significantly suppress the expansion of this criminal activity. The study concludes that the proposed model provides useful quantitative insight into the mechanisms driving kidnapping and offers practical implications for policy interventions aimed at reducing kidnapping in Nigeria.

Keywords: Mathematical Modelling; Kidnapping Dynamics; Informants; Repentant Kidnappers; Crime Control

Introduction

Kidnapping is the unlawful abduction of a person without their consent, primarily for financial ransom, political motivations, or other malicious intent. Kidnapping is a crime at common law consisting of an unlawful restraint of a person's liberty by force. It is accompanied by bodily injury, sexual assault, or a demand for ransom (Iryang and Abraham 2013)

Kidnapping has evolved from a hidden crime into a significant enterprise. It thrives on fear, spreads rapidly, and finds support in underperforming systems. The landscape has transformed into a dangerous arena where criminals, their accomplices, and the surrounding communities are all complicit, with countries like Nigeria witnessing kidnapping as not merely a crime, but a multi-billionaire industry (Okrinya et al., 2022). Kidnapping has emerged as a significant threat and one of the key security issues in Nigeria. This illegal activity involves the forced abduction of individuals, often with the intent of extorting ransom from their families. The frequency of such incidents has led to Nigeria being ranked among the most dangerous places in the world, particularly between 2016 to 2023. Several factors contribute to this alarming trend, including the pursuit of illegal financial gains, the targeting of political rivals and the use of human sacrifices for ritualistic purposes. Kidnapping has become a widely discussed topic in both public and private discourse due to its prevalence in the country. We need to analyze the underlying causes and effects of kidnapping on Nigerian society, particularly concerning insecurity. A significant factor identified is the high level of unemployment in Nigeria, which poses a major risk to national security, particularly among the youth. Thus, we conclude that unemployment is a critical issue facing Nigeria, as the developing nation struggles with rising youth unemployment, which exacerbates criminal activities. Recommendations are made to address this crime wave, emphasizing the need for policies that attract local and foreign investment to create employment opportunities, especially for vulnerable youth (Abiodun 2021).

Today, kidnapping does not attract as much media coverage as other crimes, such as drug trafficking and terrorism. Nevertheless, its association with local criminal activities influences public perception.This issue extends beyond petty crimes or random acts of violence; it involves strategic organization. Kidnapping operations recruit members, engage in corrupt practices, and, most alarmingly, perpetuate their existence. While authorities attempt to counter these threats with force or negotiation, the culprits modify their tactics. Some engage in kidnapping for ransom, others adhere to ideological motives, and some simply seek to incite chaos. Victims are not just abducted; they are exploited as bargaining chips, used to exert influence, and serve as tools for psychological manipulation. Amidst this situation, it is evident that we must dig deeper into this issue beyond the headlines.

Mathematical modeling offers a valuable approach to simulate, forecast, and possibly manage the kidnapping epidemic. By regarding it as a spreading phenomenon similar to disease outbreaks, we can monitor how individuals transition from being innocent to becoming informants or even full-fledged kidnappers. This viewpoint aligns with the modeling concepts discussed by Brauer and Castillo-Chavez (2012), but is adapted to combat human predators rather than microbes.

Tasiu et al (2024) developed a mathematical model of kidnapping dynamics and control, their model categorizes kidnappers according to their activities, The model incorporates various parameters, including recruitment rates (Λ), death rates (δ), transmission rates between compartments, and probabilities of specific events such as informers becoming kidnappers or kidnappers becoming leaders, δ is the death due to kidnappers activities, death due to torture and life jail in rehabilitation, rate at which susceptible becomes informers, rate at which informers becomes kidnappers. μ natural death rates, ψ rate at which kidnappers become kidnapper's sponsors, t is the fraction at which kidnappers become military leaders, (1-t) is the probability that not all kidnappers become leaders, σ rate at which all individuals fall back to K(t) from R(t), β is a rate at which kidnappers sponsors moved to rehabilitation, $\propto$ movement from rehabilitation back to susceptible. τ is jail break due to kidnappers operations, e is recruitment from susceptible to kidnappers Kambai (2023) proposed a dynamic mathematical model aimed at analyzing the relationship between the proliferation of arms and the rise in kidnapping activities. The model introduces a nonlinear system of ordinary differential equations to investigate how arms usage influences the recruitment, behavior, and suppression of kidnapping within a population.Akpan and Thomas (2023) presented a compartmental deterministic model that captures the dynamics of kidnapping within human populations. The model introduces a structured framework that divides the population into Susceptible individuals K_s, Infected individual (Kidnapper) K_I, Kidnap victims K_v, Quarantined individuals K_d and Discovered individuals K_r, who have undergone reformation or rescue.

Therefore, by developing a mathematical model, it aims to offer a more solid perspective on the kidnapping environment, enabling us to make more informed and focused actions so as to provides resources for scholars, law enforcers and politicians to tackle the problems that kidnapping brings. This model captured rehabilitation of kidnappers as well as repentant kidnappers

1. **Mathematical Formulation**: The proposed Model is structured into five mutually exclusive sub-populations: The Susceptible individuals (S), the Exposed individuals (E), the Informants (I), the Kidnappers (K), and the Repentant kidnapper (R).

The Susceptible individuals (S) consist of individuals who have not yet been influenced by Kidnappers but are vulnerable to such influence. This class grows through recruitment or immigration at a constant rate Λ, and decrease due to exposure to banditry at rate λ as well as through natural death at rate μ.

The Exposed individuals (E) consist of the individuals who have had contact with Kidnappers or their influence but are not yet active as informants or Kidnappers. They are generated from the susceptible class at the rate λS and leave the exposed class at a transition rate ρ, either becoming informants or Kidnappers. The exposed class also diminishes through natural at a rate μ.

The Informants class (I) comprises individuals from the exposed group who do not acquire firearms and instead supply information to Kidnapper. This transition occurs at the rate $(1-\xi)\rho$. Informants may become repentant at the rate δ, may become Kidnappers at the rate σ, and are subject to natural death μ as well as vigilante or targeted killing at the rate d_1.

The Kidnappers (B) includes who acquire weapons and engage in Kidnapping activities. This group is generated from the exposed class at rate $\xi\rho$, and also through defection from the informant class at the rate σ. The bandit class diminishes through natural death at the rate μ, and additional removal by government authorities or killing at the rate d_2.

The repentant class (R) includes individuals who have abandoned their roles as Kidnappers or informants. This class is populated by transitions from the informant and Kidnappers classes at the rate δ. The repentant class decreases only through natural death at the rate μ.

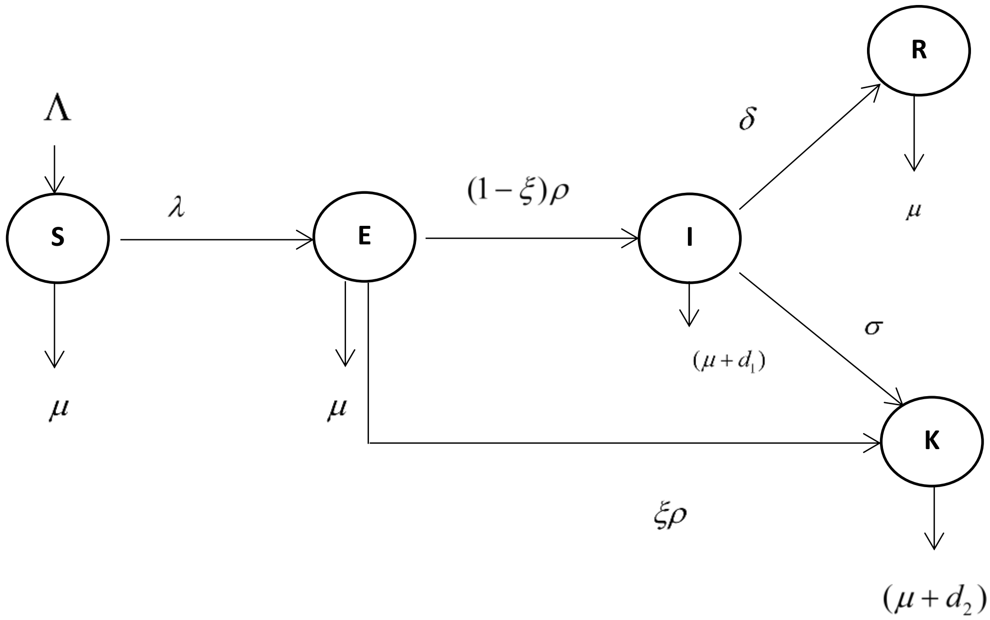


Figure 1: Schematic diagram of the Kidnapping Model

The Model Equations are derived as follows:

$$\frac{dS}{dt} = \Lambda - \lambda S - \mu S \tag{1}$$

$$\frac{dE}{dt} = \lambda S - (1-\xi)\rho E - \xi\rho E - \mu E \tag{2}$$

$$\frac{dI}{dt} = (1-\xi)\rho E - \sigma I - \delta I - (\mu + d_1) \tag{3}$$

$$\frac{dK}{dt} = \sigma I + \xi\rho E - (\mu + d_2)K \tag{4}$$

$$\frac{dR}{dt} = \delta I - \mu R \tag{5}$$

$$\text{let } \lambda = \frac{\beta I}{N} \tag{6}$$

Subject to the non-negative initial conditions

$$S(0)=E(0)=I(0)=R(0)=K(0)\geq 0$$

Where N(t) denotes total population,

$$N(t)=S(t) + E(t) + I(t) + K(t) + R(t)$$

The state variables parameters for the model are defined in the Table 1 and Table 2 respective.

Table 1. Description of the State Variable used in the Model

Variables	Description
S(t)	Susceptible individuals
E(t)	Exposed individuals
I(t)	Informers
K(t)	Kidnappers
R(t)	Repentant Kidnappers

Table 2. Description of the Parameter Used in the Model

Parameter	Description
Λ	Recruitment rate
λ	Force of becoming Kidnapper
ξ	Proportion of all Informers that acquire firearms
$1-\xi$	Remaining proportion that have no firearms
ρ	Progression rate to Informant and Kidnapper
δ	progression rate to Repentant kidnappers
σ	progression rate from informant to Bandit
d_1	Death of Informers by Military force
d_2	Death of Kidnappers by military
μ	Natural death

Basic Properties of the Model Equations

Invariant Region

The model system in (1)-(6) describe the dynamics of the armed banditry, therefore it is assumed that all variables and parameters of the model system are non-negative for all t≥0

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dE}{dt} + \frac{dI}{dt} + \frac{dK}{dt} + \frac{dR}{dt} \tag{7}$$

Thus

$$\frac{dN}{dt} = \Lambda - \mu N - d_1 I - d_2 K$$

In the absence of Kidnappers and Informant

$d_1 I = d_2 K = 0$, we obtained a differential equation:

$$\frac{dN}{dt} + \mu N = \Lambda \tag{8}$$

Theorem 1 The solution of the Model is bounded in the Invariant Region $\Omega_N \forall t \geq 0$

Proof: Let (S, E, I, R, B) R^5 be the solution of the system in (8) with non-negative initial condition using the method of Integrating Factor

$$1^{pdt} = 1^{\int \mu dt} = 1^{\mu t}$$

$$N(t)1^{\mu t} = \int \Lambda 1^{\mu} dt = \frac{\Lambda}{\mu} 1^{\mu t} + c$$

$$N(t) = \frac{\Lambda}{\mu} + 1^{-\mu t} \tag{9}$$

At t tends to zero in (9) we have:

$$N(0) = \frac{\Lambda}{\mu}$$

$$\Rightarrow c = \left(N(0) - \frac{\Lambda}{\mu} \right) \tag{10}$$

By substitution and simplification, we have

$$N(t) = \frac{\Lambda}{\mu}(1 - 1^{-\mu}) + N(0)1^{-\mu}$$

$$Hence,\ N(t) \to \frac{\Lambda}{\mu}\ as\ t \to \infty$$

$$\lim_{t \to 0} N(t) \le \frac{\Lambda}{\mu}$$

Then,

$$\Omega_N = \{S, E, I, R, K\} \in R_+^5 : \left(S(t) + E(t) + I(t) + R(t) + K(t) \le \frac{\Lambda}{\mu} \right)$$

Hence, it is positively invariant set under the flow induced by the Model (1) to (5), hence the model is sociologically well-posed in the domain. Thus, the result satisfies the existence, continuity and uniqueness of the system.

Positivity of Solutions

Here the following result guarantee by the Kidnappers governed in the equation is well-posed in a feasible region Ω

Lemma: All the solution of the systems (1) to (5) are positive for all time $t \ge 0$ provided that the initial conditions are positive. For the system (1) to (5) the region Ω is positively invariant and all the solutions stating in Ω approach or stay in the region.

Proof:

From equation (1)

$$\frac{dS}{dt} = \Lambda - (\lambda + \mu)S$$

$$\frac{dS}{dt} \ge -(\lambda + \mu)S$$

By separation of variables we have,

$$\frac{dS}{S} \geq -(\lambda + \mu)dt$$

Integrating both sides to have,

$$\ln(S) \geq -(\lambda + \mu)t + c$$

Take e of both sides,

$$S(t) \geq c_1 \ell^{-(\lambda+\mu)t}$$

At t=0

$$S(0) \geq c_1 \tag{10}$$

So, Also,

Similarly, From equation (2)

$$\frac{dE}{dt} = \lambda S - [(1-\xi)\rho - \xi\rho - \mu]$$

$$\ln(E) \geq -[(1-\xi)\rho - \xi\rho - \mu]t + c$$

Take e of both sides,

$$E(t) \geq c_2 \ell^{-(1-\xi)\rho - \xi\rho - \mu)t}$$

At t=0

$$E(0) \geq c_2 \tag{11}$$

Similarly it is evident that all variables S(t), E(t), I(t), R(t), K(t)>0.

Kidnapping-Free Equilibrium

At kidnaping-free equilibrium, there is absence of kidnapping activities. As a result, every infected class will be zero and everyone in the population will become susceptible

That is

$$[S^* \, E^* \, I^* \, R^* \, B^*] = \left\{ \frac{\Lambda}{\mu}, 0, 0, 0, 0 \right\}$$

Local Stability of Kidnapping-Free Equilibrium and Basic Reproduction Number

The concept of local stability is crucial for understanding the dynamics of kidnapping spread or population interacts. An endemic equilibrium represents a stable state where kidnapping persists at a constant level or a population maintains a stable size. To access the local stability of an endemic equilibrium, we typically use mathematical models and techniques such as Jacobian matrices. This method helps to determine whether small perturbations from the equilibrium point will lead to the system returning to that point (indicating stability) or moving away from it (indicating instability).

$$F = \begin{pmatrix} \frac{\partial F_1}{\partial E} & \frac{\partial F_1}{\partial I} & \frac{\partial F_1}{\partial B} \\ \frac{\partial F_2}{\partial E} & \frac{\partial F_2}{\partial I} & \frac{\partial F_2}{\partial B} \\ \frac{\partial F_3}{\partial E} & \frac{\partial F_3}{\partial I} & \frac{\partial F_3}{\partial B} \end{pmatrix}, F = \begin{pmatrix} \frac{\beta S}{N} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$V = \begin{pmatrix} \frac{\partial V_1}{\partial E} & \frac{\partial V_1}{\partial I} & \frac{\partial V_1}{\partial B} \\ \frac{\partial V_2}{\partial E} & \frac{\partial V_2}{\partial I} & \frac{\partial V_2}{\partial B} \\ \frac{\partial V_3}{\partial E} & \frac{\partial V_3}{\partial I} & \frac{\partial V_3}{\partial B} \end{pmatrix} \Rightarrow V = \begin{pmatrix} z_1 & 0 & 0 \\ z_2 & z_3 & 0 \\ \xi\rho & \sigma & z_4 \end{pmatrix}$$

Where,

$$z_1 = (1-\xi)\rho + \xi\rho + \mu, \quad z_2 = (1-\xi)\rho, z_3 = (\delta + \mu - d_1 - \sigma), z_4 = (\mu + d_2)$$

To get the inverse of matrix V

$$V^{-1}=\begin{pmatrix} \frac{1}{z_1} & 0 & 0 \\ \frac{-z_2}{z_1 z_3} & \frac{1}{z_3} & 0 \\ \frac{z_2\sigma - z_3\xi\rho}{z_1 z_3 z_4} & \frac{-\sigma}{z_3 z_4} & \frac{1}{z_4} \end{pmatrix} \tag{12}$$

We multiply F and V-1

$$FV^{1}=\begin{pmatrix} \frac{\beta S}{N} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{z_1} & 0 & 0 \\ \frac{-z_2}{z_1 z_3} & \frac{1}{z_3} & 0 \\ \frac{z_2\sigma - z_3\xi\rho}{z_1 z_3 z_4} & \frac{-\sigma}{z_3 z_4} & \frac{1}{z_4} \end{pmatrix} \tag{13}$$

$$FV^{-1}=\begin{pmatrix} \frac{\beta\Lambda}{\mu N z_1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

We obtained the Eigen values, and highest eigen value is the basic reproduction number as stated in Tasiu et al (2024)

Therefore,

$$R_0=\frac{\beta\Lambda}{\mu N z_1} \tag{14}$$

Which is the spectral Eigen Value

Existence of Local Stability of Kidnapping-Free Equilibrium

For non-linear differential equation systems, the asymptotic stability of equilibrium is established using the Routh-Hurwitz criteria. Asymptotic stability is determined by the

Routh-Hurwitz criteria, which give the necessary and sufficient conditions for all roots of the characteristic polynomial to contain negative portions. The equilibrium's local stability can be ascertained using the Jacobian matrix.

Theorem: the kidnapping-free equilibrium point is locally asymptotically stable if $R_0<1$ and unstable if $R_0 > 1$.

$$J(E_0)=\begin{pmatrix} \frac{\partial F_1}{\partial S} & \frac{\partial F_1}{\partial E} & \frac{\partial F_1}{\partial I} & \frac{\partial F_1}{\partial B} & \frac{\partial F_1}{\partial R} \\ \frac{\partial F_2}{\partial S} & \frac{\partial F_2}{\partial E} & \frac{\partial F_2}{\partial I} & \frac{\partial F_2}{\partial B} & \frac{\partial F_2}{\partial R} \\ \frac{\partial F_3}{\partial S} & \frac{\partial F_3}{\partial E} & \frac{\partial F_3}{\partial I} & \frac{\partial F_3}{\partial B} & \frac{\partial F_3}{\partial R} \\ \frac{\partial F_4}{\partial S} & \frac{\partial F_4}{\partial E} & \frac{\partial F_4}{\partial I} & \frac{\partial F_4}{\partial B} & \frac{\partial F_4}{\partial R} \\ \frac{\partial F_5}{\partial S} & \frac{\partial F_5}{\partial E} & \frac{\partial F_5}{\partial I} & \frac{\partial F_5}{\partial B} & \frac{\partial F_5}{\partial R} \end{pmatrix}$$

Reducing the above matrix to an upper triangular matrix give

$$J(E_0) = \begin{pmatrix} \mu & 0 & 0 & 0 & 0 \\ 0 & -z_1 & 0 & 0 & 0 \\ 0 & z_2 & -z_3 & 0 & 0 \\ 0 & \xi\rho & 0 & -z_4 & 0 \\ 0 & 0 & \delta & 0 & -\mu \end{pmatrix}$$

$$J(E_0 - \lambda) = \begin{pmatrix} \mu-\lambda & 0 & 0 & 0 & 0 \\ 0 & -z-\lambda_1 & 0 & 0 & 0 \\ 0 & z_2 & -z_3-\lambda & 0 & 0 \\ 0 & \xi\rho & 0 & -z_4-\lambda & 0 \\ 0 & 0 & 0 & 0 & -\mu-\lambda \end{pmatrix}$$

To find the determinant,

$$(\mu-\lambda)(-z_1-\lambda)(-z_3-\lambda)(-z_4-\lambda)(-\mu-\lambda)$$
$$\Rightarrow (\mu-\lambda)(-z_1-\lambda)$$
$$\left(-\mu z_1 - \mu\lambda + \lambda z_1 + \lambda^2\right)(-z_3-\lambda)$$
$$\Rightarrow \left(\mu z_1 z_3 + \mu\lambda z_1 + \mu\lambda z_3 + \mu\lambda^2 - \lambda z_1 z_3 - \lambda^2 z_1 - \lambda^2 z_3 - \lambda^3\right)(-z_4-\lambda)$$

$$= (-\mu z_1 z_3 z_4 - \mu z_1 z_3 \lambda - \mu\lambda z_1 z_4 - \mu\lambda^2 z_1 - \mu\lambda z_3 z_4 - \mu\lambda^2 z_3 - \mu\lambda^2 z_4 - \mu\lambda^3 + \lambda z_1 z_3 z_4 + \lambda^2 z_1 z_3 + \lambda^2 z_1 z_4$$
$$\lambda^3 z_1 + \lambda^2 z_3 z_4 + \lambda^3 z_3 + \lambda^3 z_4)(-\mu-\lambda)$$

$$= (\mu^2 z_1 z_3 z_4 + \mu\lambda z_1 z_3 z_4 + \mu^2\lambda z_1 z_3 + \mu\lambda^2 z_1 z_3 + \mu^2\lambda z_1 z_4 + \mu^2\lambda^2 z_1 + \mu\lambda^3 z_1 + \mu^2 z_3 z_4 + \mu\lambda^2 z_3 z_4 + \mu^2\lambda^2 z_3$$
$$+\mu\lambda^3 z_3 + \mu^2\lambda^2 z_4 + \mu\lambda^3 z_4 + \mu^2\lambda^3 + \mu\lambda^4 - \mu\lambda z_1 z_3 z_4 - \lambda^2 z_1 z_3 z_4 - \mu\lambda^2 z_1 z_3 - \lambda^3 z_1 z_3 - \mu\lambda^2 z_1 z_4 - \lambda^3 z_1 z_4 -$$
$$\mu\lambda^3 z_1 - \lambda^4 z_1 - \mu\lambda^2 z_3 z_4 - \lambda^3 z_3 z_4 - \mu\lambda^3 z_3 - \lambda^4 z_3 - \mu\lambda^3 z_4 - \lambda^4 z_4 - \mu\lambda^4 - \lambda^5)$$

By collecting the co-efficient of the Eigen value

$$\lambda^5 + q_4\lambda^4 + q_3\lambda^3 + q_2\lambda^2 + q_1\lambda + q_0 = 0$$

where,

$$q_5 = 1$$
$$q_4 = (\mu - z_1 - z_3 - z \ - \mu)$$
$$q_3 = (\mu z_1 + \mu z_3 + \mu z_4 + \mu^2 - z_1 z_3 - z_1 z_4 - \mu z_1 - z_3 z_4 - \mu z_3 - \mu z_4)$$
$$q_2 = (\mu z_1 z_3 + \mu z_1 z_4 + \mu^2 z_1 + \mu z_3 z_4 - \mu^2 z_3 + \mu^2 z_4 - z_1 z_3 z_4 - \mu z_1 z_3 - \mu z_1 z_4 - \mu z_3 z_4)$$
$$q_1 = (\mu z_1 z_3 z_4 + \mu^2 z_1 z_3 + \mu^2 z_1 z_4 - \mu z_1 z_3 z_4)$$
$$q_0 = (z_4 + \mu)\mu z_3 z_4$$

The Routh-Hurwitz criterion makes it evident that the eigenvalues have a negative real part and as a result, since there are no kidnappers, the kidnapping-free equilibrium is locally asymptomatically stable (LAS).

Results and Discussion

Variables Values

Choosing values for the variables that make up a mathematical model is the central task of every model. The process of figuring out and giving these variables values is crucial to our study of kidnapping. Each variable represents a unique aspect of the complex phenomenon of kidnapping, covering behavioral, regional and socioeconomic aspects. The selection and precise assignment of values to these variables are the bedrock of our modelling effort. They form the basis upon which we build a comprehensive understanding of the factors influencing the occurrence and the evolution of kidnapping within our study area.

We take a significant step in this part when we present and give values to the variables that we have a strong model that will hopefully offer comprehensive insights because these values are based on research, expert knowledge as well as data from simulations. The process we use here is about discovering the complex relationships between these variables, not just about numerical inputs. By doing this, we get a step closer to gaining a thorough understanding of kidnapping, which will enable us to create efficient plans of actions, laws, and other interventions to deal with this multifaceted kidnapping problem.

Table 3: Table for Variables

Variables	Values	Source
S	95000	Tasiu et al (2024b)
E	3000	Assumed
I	1000	Assumed
K	800	Tasiu et al(2024b)
R	400	Assumed
N	100400	Calculated

Parameter values

In mathematical modelling, parameter values are assigned by a process that includes expert consultation, data analysis and frequently validation to make sure the model accurately captures the phenomenon that occurs in real life. Determining these values requires data, expert opinion and a deep comprehension of the system being studied. The process may involve iterative adjustments and sensitivity analysis to gauge the impact of parameter variations on model outcomes ultimately leading to a more reliable and informative model.

PARAMETERS	VALUES	SOURCE
Λ	5000	Tasiu et al (2025)
μ	0.017	NBS, 2023
β	0.9	Imoukhedeme et al (2025)
ξ	0.25	Assumed
ρ	0.1	Assumed
d_1	0.02	Assumed
d_2	0.03	Assumed
δ,σ	0-1	Control

8. Validation of Analytical Results by Numerical Method

Numerical validation of analytical results is a crucial step in ensuring the accuracy and reliability of mathematical models used in scientific and engineering research. Analysis models offer a basic knowledge of real-life events, although they are frequently developed with simplifying assumptions. They might not be as applicable to complicated real-world systems though. Researchers use computational methods and numerical simulations to close this gap. This procedure involves numerically representing the analytical model and then carefully comparing the results. To find out if the two sets of findings agree, parameter, initial circumstances and mode setup must match. Validations measures are also established. When analytical solutions are unrealistic or inconceivable, the validation procedure is very important for demonstrating the reliability of analytical models.

In order to assure the validity of our model comprehending the dynamics of kidnapping, we use numerical approaches to validate analytical outcomes in this research. Our objective is to validate the accuracy and reliability of the model by running thorough numerical simulations using our analytical findings. This method allows us to confidently state that the predictions of our model are resilient, which is important for developing policy making decisions with the multifaceted social problem of kidnapping.

The Runge-Kutta formulas for each of the model equations were calculated using the Maple software. The values of the initial variables and parameters are taken from Table 2 and 3. The global stability of the endemic equilibrium was demonstrated by the numerical simulations we subsequently presented, which were designed to confirm our analytical results on the global stability of the kidnapping free-equilibrium and to track the dynamics of the model (1) to (5) for different values of the associated effective reproduction number.

Using the following initial conditions,

$S(0) = 95000$, $E(0) = 3000$, $L(0) = 1000$, $K(0) = 800$, $R(0) = 400$, $N(0) = 100200$

Figure 2: Graph of Susceptible (S) Individuals Under Varying Control Strategy

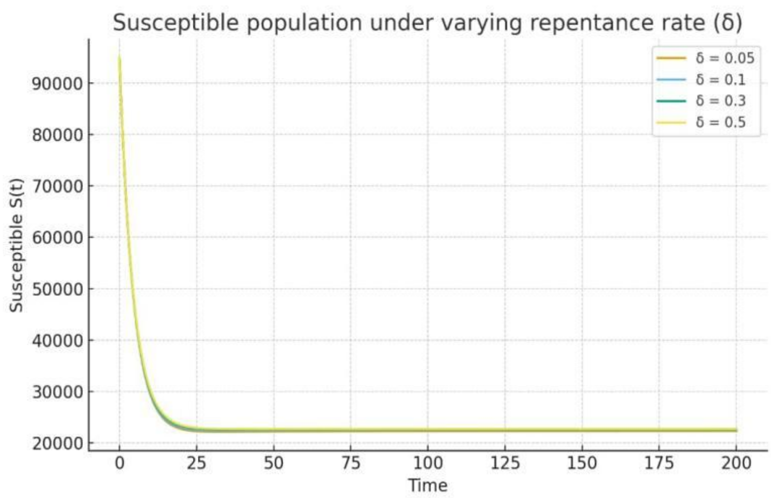


Figure 2: Susceptible individual under varying repentance strategy (δ =0.05, 0.1, 0.3, 0.5)

Clearly figure 2 illustrate that under high control strategy the population of susceptible rapidly decreases as individuals move into exposed, informant, or kidnapper classes through contact with active kidnappers and through direct recruitment (σ). After the initial transient each curve approaches a lower steady state. Higher repentance rates correspond to a slightly higher long-term susceptible level because more individuals leave criminal classes (I and K) and flow back into the repentant class rather than remain active, reducing ongoing transmission pressure on susceptible.

Figure 3: Graph of Exposed Population under Varying control Strategy

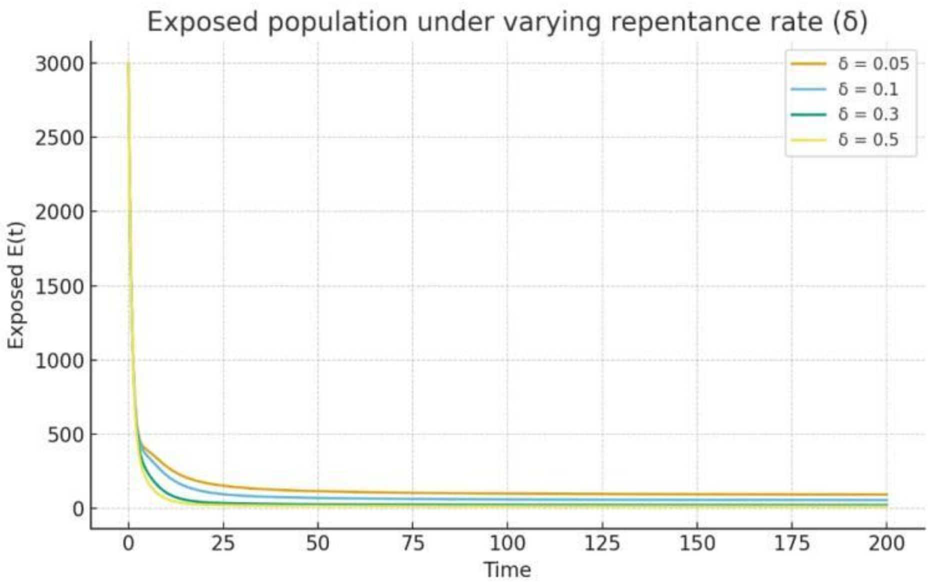


Figure 3: Expose individual under varying repentance strategy (δ =0.05, 0.1, 0.3, 0.5)

We observe that in figure 3 the exposed population increase immediately and then decrease to a low steady state for all δ scenarios. Importantly, increasing δ tends to lower the steady-state value of E(t) and shortens the tail of exposure: when repentance is high, informants and kidnappers are removed faster from the infectious chain, which reduces onward exposure of susceptible. Because the exposed compartment is fed primarily by contact with kidnappers.

Figure 4: Graph of Informant Population Under Varying Repentance Strategy

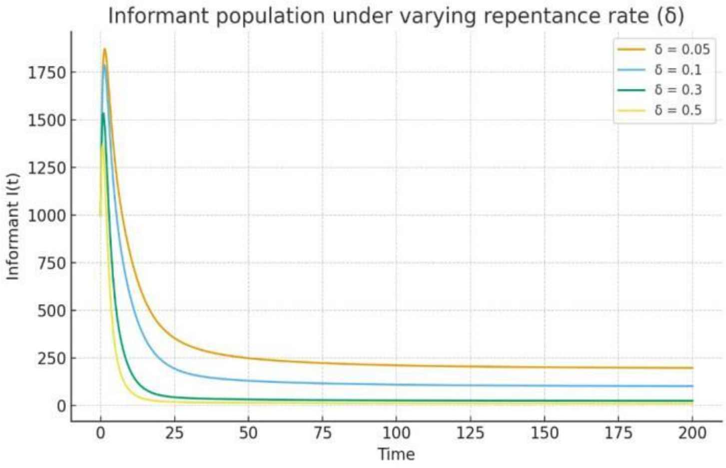


Figure 4: Informant population under varying repentance strategy (δ =0.05, 0.1, 0.3, 0.5)

From figure 4 we observe that the informant population changes over time. At first, the number of informants increases sharply and reaches a peak shortly after the start. After that, it begins to decrease until it becomes almost steady at a small value. When the repentance rate (δ) increases, both the peak and the final number of informants decrease. This happens because a higher repentance rate makes more informants leave the criminal group through surrender or rehabilitation. It also decreases the number of informants who later become kidnappers, since there are fewer active criminals influencing them. When the repentance rate is low, the decrease in informants is slower, and more people stay longer in the informant group.

Figure 5: Effect of Repentance rate on Kidnapper Population

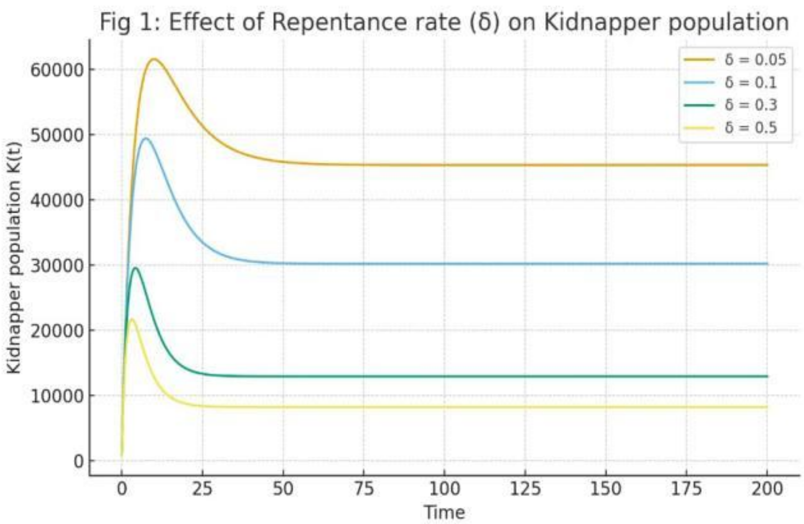


Figure 5: This shows how changes in the repentance rate (δ) affect the number of kidnappers over time. At first, the number of kidnappers increases slightly but later begins to decrease as repentance takes effect. When the repentance rate increases, the number of kidnappers decreases faster and settles at a much lower level. This means that encouraging more kidnappers to repent or surrender helps to reduce kidnapping activities. On the other hand, when the repentance rate is low, the number of kidnappers remains high and continues to grow slowly. This shows that an increase in repentance rate helps to control and decrease the spread of kidnapping within the population.

Figure 6: Effect of Recruitment rate on Kidnapper Population

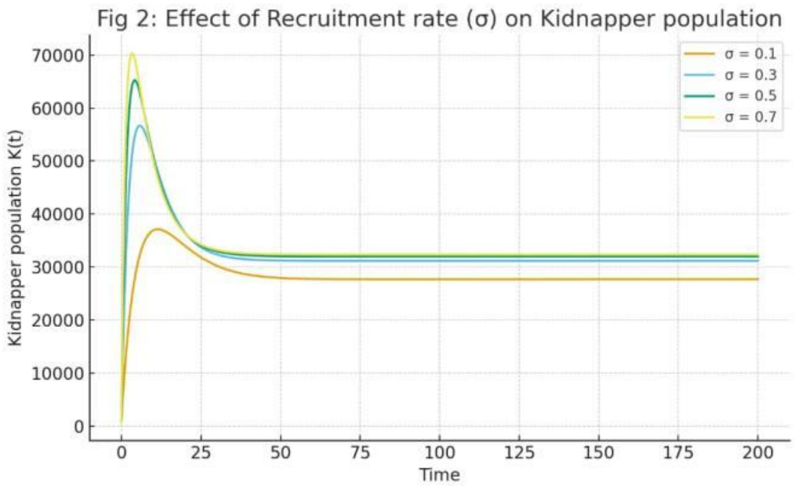
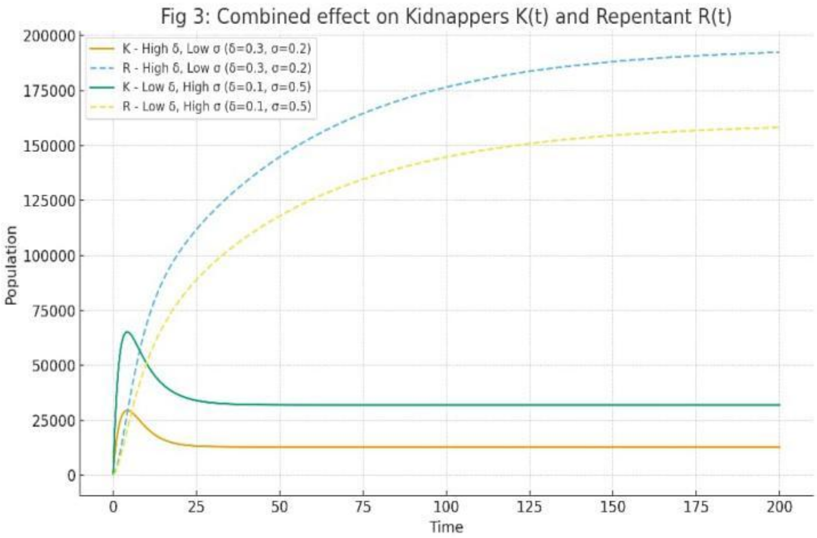


Figure 6: This shows the effect of different recruitment rates (σ) on the number of kidnappers. The graph shows that when the recruitment rate increases, the number of kidnappers also increases quickly. A higher recruitment rate means more people are joining the kidnapping group, which leads to a faster rise in criminal activities. In contrast, when the recruitment rate decreases, the number of kidnappers also decreases and the system becomes more stable over time. This result shows that controlling recruitment for example, by creating jobs or improving education can greatly reduce the number of kidnappers and the overall level of insecurity in society.

Figure 7: Combined effect on kidnappers and Repentant



Clearly figure 7 shows the combined effect of changing both the repentance rate (δ) and the recruitment rate (σ) at the same time. When the repentance rate increases and the recruitment rate decreases, the number of kidnappers drops quickly, while the number of repentant individuals increases. This shows a strong improvement in security and social stability. However, when the repentance rate is low and the recruitment rate is high, the number of kidnappers increases sharply and remains high for a longer period. The figure clearly shows that increasing repentance programs and decreasing recruitment together give the best result. In simple terms, reducing the inflow of new kidnappers and increasing the rate at which existing ones repent leads to a major decrease in kidnapping activities.

Conclusion

This presented a mathematical approach to understand and controlling kidnapping activities in society. By formulating a compartmental model that divides the population into Susceptible, Exposed, Informants, Kidnappers and Repentant classes. This analysis revealed that kidnapping behaves like epidemic. It increases when recruitment is high and repentance is low, but decreases when rehabilitation and preventive measures take place. This research show that the repentance rate plays a major role in reducing kidnapping. When more offenders repent or are integrated into the society, the number of active kidnappers decreases. Likewise reducing the recruitment of new members into criminal groups and strong security measures decreases the spread of kidnapping.

The model provides a framework for understanding the spread and persistence of kidnapping activities and for evaluating the effectiveness of various control strategies. However, it does not capture all possible realities of kidnapping dynamics, such as social influences, economic factors, and political motivations that may encourage or discourage the crime.

The numerical simulations from the model indicate that applying any of the control measures contributes to a reduction in the population of active kidnappers. Nonetheless, the most effective strategy is the detention or imprisonment of captured kidnappers, followed by the neutralization of active ones, and lastly, the prevention of susceptible individuals from being kidnapped. Therefore, the study concludes that an integrated control approach is the most reliable means of combating kidnapping and maintaining a kidnapping-free society.

Recommendation

Based on the outcomes of this mathematical modeling research on kidnapping dynamics with control, key recommendations emerge. The adoption of improved control measures should be a top priority for the government, with a special emphasis on the efficiency of imprisonment and detention, as well as focused interventions to curb kidnapping activities. Government attempts to prevent the abduction of susceptible individuals require special focus, and specific programs that address the root causes of kidnapping such as poverty, unemployment, and insecurity are necessary.

To keep up with the constantly changing nature of kidnapping operations, dynamic control strategy modifications and continuous monitoring are essential. Moreover, ongoing research and real-time data collection are needed to enhance the existing model and provide relevant insights for decision-making and long-term control of kidnapping activities.

The proposed model does not capture all the possible realities of the dynamics of kidnapping; therefore, future studies can incorporate additional variables such as repentant or rehabilitated kidnappers, who may offer useful knowledge about kidnapping operations and effective control measures. The results of this research can also be extended to other fields of study to analyze the patterns of kidnapping and other criminal activities in various geopolitical zones that pose significant threats to the public and national security.

References

Aguadze, M., Vivas, A., Pant, S., & Dagtoros, K. (2024). Spread of crime dynamics: A mathematical approach. *Turkish Journal of Mathematics and Computer Science, 16*(2), 481–489. https://doi.org/10.47000/tjmcs.1555972

Akpan, A. U., & Thomas, D. S. (2023). Mathematical modeling of the effect of kidnapping in Nigeria. *International Journal of Mathematical Analysis and Modelling, 6*(2), 305–320. https://tnsmb.org/journal/index.php/ijmam/article/view/102

Amin, R. (2017). A mathematical model on the role of social interactions in crime behavior. *International Journal of Mathematics Trends and Technology, 42*(2), 83–88. https://doi.org/10.14445/22315373/IJMTT-V42P513

Amin, R. (2019). Mathematical model on spatio-temporal variation of criminal activities. *International Journal of Mathematics Trends and Technology, 65*(7), 69–72. https://doi.org/10.14445/22315373/IJMTT-V65I7P511

Baba, I. A., Hincal, E., & Rihan, F. A. (2025). Kidnapping dynamics: A Lotka–Volterra approach with fractional order modeling. *The European Physical Journal Special Topics, 234*(8), 2097–2108. https://doi.org/10.1140/epjs/s11734-024-01208-0

Berestycki, H., Rodríguez, N., & Ryzhik, L. (2013). Traveling wave solutions in a reaction-diffusion model for criminal activity. *Multiscale Modeling & Simulation, 11*(4), 1097–1126. https://doi.org/10.1137/12089884X

Brauer, F., & Castillo-Chavez, C. (2012). *Mathematical models in population biology and epidemiology* (2nd ed.). Springer. https://doi.org/10.1007/978-1-4614-1686-9

Cartwright, E., Hernandez-Castro, J., & Cartwright, A. (2019). To pay or not: Game theoretic models of ransomware. *Journal of Cybersecurity, 5*(1), tyz009. https://doi.org/10.1093/cybsec/tyz009

Felson, M. (2010). What every mathematician should know about modelling crime. *European Journal of Applied Mathematics, 21*(4–5), 275–285. https://doi.org/10.1017/S0956792510000070

Imoukhedeme, P. K., Tasiu, A. R., & Jafar, A. (2025). Mathematical modeling of the effects of optimized drainage systems on mosquito dynamics in prevention of malaria. *Intermaths, 6*, 37–63. https://periodicos2.uesb.br/intermaths/article/download/16942/11138

Iqbal, A., Masson, V., & Abbott, D. (2017). Kidnapping model: An extension of Selten's game. *Royal Society Open Science, 4*(12), 171484. https://doi.org/10.1098/rsos.171484

Kazmierczak, A. (2021). Modeling and analysis of the interaction of police and criminal populations: A competing species model. *GPH-International Journal of Mathematics, 4*(10), 1–8. https://gphjournal.org/index.php/m/article/view/495

Kwofie, T., Dogbatsey, M., & Moore, S. E. (2023). Curtailing crime dynamics: A mathematical approach. *Frontiers in Applied Mathematics and Statistics, 8*, Article 1086745. https://doi.org/10.3389/fams.2022.1086745

Lacey, A. A., & Tsardakas, M. N. (2016). A mathematical model of serious and minor criminal activity. *European Journal of Applied Mathematics, 27*(3), 403–421. https://doi.org/10.1017/S0956792516000139

Okrinya, A. B., Timinibife, C. N., & Eli, I. C. (2024). Mathematical modelling on kidnapping with apprehended kidnappers. *The Transactions of the Nigerian Association of Mathematical Physics, 19*, 29–38. https://doi.org/10.60787/tnamp-19-29-38

Selten, R. (1988). A simple game model of kidnapping. In *Models of strategic rationality* (pp. 77–93). Springer. https://doi.org/10.1007/978-94-015-7774-8_4

Tasiu, A. R., Abdulrahman, S., Yabo, I. B., Dauran, N. S., & Abubakar, S. (2024). Dynamics of banditry activities: A mathematical modeling approach. *Bangmod International Journal of Mathematical and Computational Science, 10*, 77–96.

Tasiu, A. R., Abdulrahman, S., Idris, L., Pudza, E., & Bege, J. (2024). Mathematical modeling of cultism dynamics among youth in Southern Nigeria. *Revista O Universo Observável, 1*(8). https://ouniversoobservavel.com.br/mathematical-modeling-of-cultism-dynamics-among-youth-in-southern-nigeria-doi-10-69720-2966-0599-2024-00011/

Tasiu, R. A., Malan, S. S., Abdullahi, S., Hanafi, J., & Muhammad, A. (2024). Mathematical modeling of kidnapping: Dynamics and control. *UMYU Scientifica, 3*(3), 105–117. https://publications.umyu.edu.ng/scientifica/index.php/usci/article/view/247

Vannini, M., McCannon, B., & Detotto, C. (2012). *Understanding ransom kidnapping and its duration* (Working Paper CRENoS 2012/19). Centre for North South Economic Research, University of Cagliari and Sassari. https://crenos.unica.it/crenos/publications/understanding-ransom-kidnapping-and-its-duration