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Mathematical Modelling and Optimization of Inventory Control Through Linear Programming: A Case Study of Haske Modern Bakery in Bauchi State

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Abstract

This study investigates the application of linear programming optimization, complemented by primal and dual linear programming, to improve raw material allocation and maximize profitability for the Haske Modern Bakery in Bauchi, Nigeria. A mathematical model was developed to optimize raw material usage, increase production efficiency, and enhance company returns. Linear programming was employed to identify the most profitable production strategy. Additionally, the Economic Order Quantity (EOQ) model was utilized to effectively manage inventory. EOQ calculations determined the optimal order quantities for flour, sugar, butter, milk, and yeast to be approximately 59 bags, 22 bags, 11 cartons, 12 bags, and 56 cartons, respectively. The total costs of ordering flour, sugar, butter, milk, and yeast are N19,349, N11,171, N5,586, N6,119, and N27,928, respectively. Reorder points were established with stock levels triggering reorders in 20 bags of flour, 4 bags of sugar, 2 cartons of butter, 2 bags of milk, and 30 cartons of yeast, assuming a constant lead time of seven days. The results showed that the optimal production strategy involved producing 319.8294 units of small-medium loaf (x_5) and 533.0490 units of another product (y_3), with all other products (x_1, x_2, x_3, ..., x_{13} and y_1, y_2, y_3, ..., y_9) being zeros units. This strategy is projected to yield a maximum profit of N15991.47. This study underscores the significance of utilizing linear programming and EOQ models to enhance operational efficiency and profitability in the bakery industry.

Keywords: Linear Programming; Inventory Management; Economic Order Quantity; Optimization; Bakery Industry; Mathematical Modeling; Haske Modern Bakery

Introduction

Linear programming (LP) is a special technique employed in operational research to optimize linear functions subject to linear equality and inequality constraints. Linear programming determines how to achieve the best outcome, such as maximum profit or minimum cost, in a given mathematical model, and provides a list of requirements as a linear equation. Linear programming (LP) has been used in a wide range of applications. Including agriculture, Industry, transportation, economics, health systems, behavioral and social science, and the military, although many business organizations see linear programming as a “new science” or recently developed mathematical history. But there is nothing new about the maximization of profit in any business organization, i.e. in a production company or manufacturing company (Adebiyi et al., 2014).

Linear Programming, an optimization technique, is employed to address managerial decision-making challenges. This mathematical modeling method aids managers in planning and resource allocation. In today's complex social and business environment, decision makers grapple with multifaceted tasks. They strive to achieve multiple objectives while contending with conflicting interests, limited resources, incomplete information, and constrained analytical abilities. Managerial decision making is further complicated by factors such as technology costs, material expenses, competitive pressures, and the rapid evolution of knowledge and technology. The consequences of erroneous choices are significant, prompting decision makers to move beyond personal experiences and intuition. Understanding the relevance of quantitative methods to decision making becomes crucial. For instance, choosing wrong markets or producing unsuitable products can adversely affect organizations. Operational Research (OR) techniques such as linear programming must align with global trends. While OR sometimes falls short, often owing to implementation issues, linear programming offers a mathematical tool for optimizing resource allocation. This determines how a firm's limited resources can achieve optimal goals by minimizing or maximizing linear constraints. This method is vital in various fields such as Agriculture, Military, Production, Finance, Engineering, and Marketing (Oluwasey et al., 2020).

According to various studies, Small and Medium-Sized Enterprises (SMEs) in Ghana operate in an environment characterized by complexity, risks, and financial uncertainty, often relying on loans from financial institutions. This reliance on external financing makes it essential for these firms to implement effective inventory management practices to enhance performance and ensure sustainability. However, according to research, many SMEs struggle with implementing these practices, as they are often run by individuals with little or no formal education. According to reports, training manuals provided by the government of Ghana and other stakeholders to support the SME sector often go unused, gathering dust on shelves. The lack of effective inventory management practices leaves these firms vulnerable, with many either experiencing stock shortages or holding excessive stock. According to Chan et al. (2023), a shortage of finished goods can lead to customer dissatisfaction and potential loss of customers and sales. Conversely, excess inventory consumes physical space, creates financial strain, and increases the risk of damage, spoilage, and loss.

Haske Modern Bakery, located in Bauchi State, Nigeria, is a growing enterprise that faces challenges common to many businesses in the food production sector. The bakery's management must make crucial decisions daily, such as determining the optimal mix of products to bake, managing the supply of raw materials, and efficiently utilizing labor while striving to minimize costs and maximize profits. Given the competitive nature of the bakery industry and the economic constraints in the region, these decisions can significantly impact the bakery's sustainability and growth. In this study, we explore the application of linear programming optimization techniques to address the operational challenges faced by Haske Modern Bakery. The objective was to develop a mathematical model that can help the bakery make informed decisions regarding production scheduling, resource allocation, and cost management. By applying linear programming, the bakery can identify the optimal production levels for its various products, considering the constraints of raw material availability, production capacity, labor hours, and market demand. Recent studies have shown that the use of such optimization techniques can lead to substantial improvements in operational efficiency and cost reduction in the food industry (Alam et al., 2024).

The research explores the use of linear programming for profit maximization in the case of Glad Tidings (GT) in Benin. Secondary data was utilized for this research. The primary objective was to analyze how linear programming can enhance profit maximization for GT Food in Benin City, Edo State. The research employed the revised simplex method to address standard maximization problems, applying the echelon rule for this purpose. The findings indicate that among the various production areas, the chicken production sector offers the greatest potential for profit maximization for the company. Conversely, Jollof rice should be produced minimally, as it contributes only a small amount to the organization's overall growth. Additionally, the management should continuously develop new strategies to improve product quality and meet market demands. GT Food, Benin, produces three main products: Fried rice, Jollof rice, and Chicken. The study applied linear programming to determine the profit maximization level for October 2018, extending into November 2018. The analysis revealed that the maximum profit (Z) was 537.78, indicating that Crunches Fried Chicken could achieve a profit maximization of 97.27 for the specified period (Ozokeraha, 2020).

The research explores the utilization of linear programming in optimizing profit in Tehinnah Cakes and Crafts. Using the simplex algorithm in Mat-Lab software, the research recommends specific quantities for different products to achieve a maximum profit of N8,550.80k. The analysis emphasizes the prioritization of certain products, such as Scones, for profit maximization (Egharevba & Ojekudo, 2021).

The research explores the application of linear programming in the production of small Ayurvedic items. The focus is on optimizing production planning for manufacturing Ayurvedic mixtures or pastes that enhance the body's immunity and fulfill vitamin requirements, aiming to maximize profit (Jain *et al.*, 2021).

The research conducted at Shukura Bakery in Zaria, Kaduna State, Nigeria, used linear programming to determine the optimal profit. The study analyzed variables such as production costs, selling prices, quantities of raw materials, and raw material availability on a weekly basis for four bread types: small loaf, medium loaf, family loaf, and sliced family loaf. A linear programming model was developed and solved using the simplex method with specialized software. The findings indicated that increasing the production of family loaves would result in the highest profit. The optimal solution from the linear programming model was $x_1 = 0$, $x_2 = 0$, $x_3 = 5600$, and $x_4 = 0$, with a maximum profit (Z) of N336,000. This solution highlights that only the family loaf (x_3) significantly contributes to enhancing the objective function's value. Based on this result, it is recommended that Shukura Bakery focus on producing more family loaves, considering the instability of raw material prices and availability, provided there is demand. This strategy would result in the sale of 5600 units per week, generating an optimal profit of N336,000 weekly, given the current raw material costs (Zakariyya et al., 2022).

Baker et al. (2024), effective inventory management minimizes the cost associated with inventory by efficiently controlling stock levels to avoid excess inventory. According to Dey & Seok (2024), in today's intensely competitive market, businesses must maintain adequate inventory levels to meet customer demands consistently. The effectiveness of inventory management lies in balancing the costs associated with inventory against its benefits. According to existing literature, many organizations struggle with inventory management (Alnaim & Kouaib, 2023; Gołaś, 2020), often leading to profit losses. Proper inventory management allows organizations to provide goods and services in the right quantity, quality, and cost at the appropriate time.

According to Jonsson & Mattsson (2021), insufficient inventory can disrupt a company's operations, while excess inventory can lead to unnecessary holding costs, reducing profitability and overall performance. Supporting this view, According to various studies, Small and Medium-Sized Enterprises (SMEs) in Ghana operate in an environment characterized by complexity, risks, and financial uncertainty, often relying on loans from financial institutions. This reliance on external financing makes it essential for these firms to implement effective inventory management practices to enhance performance and ensure sustainability. However, according to research, many SMEs struggle with implementing these practices, as they are often run by individuals with little or no formal education. According to reports, training manuals provided by the government of Ghana and other stakeholders to support the SME sector often go unused, gathering dust on shelves. The lack of effective inventory management practices leaves these firms vulnerable, with many either experiencing stock shortages or holding excessive stock. According to Chan et al. (2023), a shortage of finished goods can lead to customer dissatisfaction and potential loss of customers and sales. Conversely, excess inventory consumes physical space, creates financial strain, and increases the risk of damage, spoilage, and loss.

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According to Anantadjaya (2023) and Acquah (2024), implementing an inventory control model can significantly enhance demand forecasting accuracy, optimize stock levels, and ensure efficient resource utilization in a bakery. This leads to improved profitability, reduced waste, and increased customer satisfaction through better product availability and freshness. The study analyses various inventory control models and evaluates their applicability to Haske Modern Bakery. It examines how these models can be tailored to fit the bakery's specific operational context, taking into account production schedules, product shelf life, and fluctuating customer demand. The ultimate objective is to identify an inventory control model that boosts the bakery's overall efficiency and fosters growth in a competitive market. Recent literature underscores the vital role of effective inventory management in maintaining a competitive advantage and facilitating growth in small businesses, particularly within the food industry.

According to Chaudhary and Giri (2024), determining optimal production and inventory policies is essential for food industry companies, particularly those handling multiple products. Achieving efficient operational solutions can significantly affect profits, resource allocation, and product quality. This study focuses on establishing optimal production and inventory strategies in a multiproduct bakery unit under two scenarios: (i) deterministic consumption without inventory control, and (ii) stochastic consumption with delayed inventory control. Each scenario is framed with a specific formulation. The model considers constraints related to workforce availability and incorporates a cost structure that includes four key components: (i) production costs, (ii) inventory costs, (iii) setup costs, and (iv) costs arising from the degradation of perceived quality. The problem is structured as a Mixed Integer Linear Programming challenge and solved using a branch-and-cut algorithm. The formulation is applied to a real bakery producing a variety of eight products, employing different demand levels and inventory thresholds to develop scenarios that test both models and assess the economic performance of the bakery unit.

The optimization of raw materials refers to the efficient use of raw materials to obtain the best value for every raw material invested in producing goods. Optimization of the raw material balance provides the best choice for maximizing the economic gain. In the bread baking industry. For instance, the optimization of raw materials makes the best use of the available inventory, taking into account the perishing ability of the products and the fact that bread has a different yield when used in different recipes or sold as such. It helps manage both the product portfolio and raw material balance, directly affecting profitability (Adebiyi et al., 2014).

According to Eze and Uchenu (2021), ABC analysis is an inventory control technique that provides a clear view and understanding of the entire assortment of products in the inventory, making it an efficient method to control inventory investment. The technique follows the Pareto Principle, which suggests that roughly 80% of the effects come from 20% of the causes. In a business context, this could mean that 20% of the products generate 80% of the income. In ABC analysis, a company categorizes its inventory into three groups: "A" items, "B" items, and "C" items. "A" items typically generate 70% of income, "B" items 25%, and "C" items 5%. This classification may vary across companies, but managers should identify the pattern that best suits their needs. According to Eze and Uchenu (2021), “A” items require closer attention and higher service levels to minimize the risk of stockouts, which would result in significant losses. Almrdof

& Attia (2021) also observed a positive connection between inventory management practices and productivity. Furthermore, Acquah (2024) concluded that effective inventory management positively affects organizational performance.

Methodology

Method of Data Collection

The data collection method employed in this research involves gathering secondary data obtained from records at Haske Modern Bakery.

Research Design

The following flowchart (Figure 3) summarizes the steps taken to achieve the research objectives.

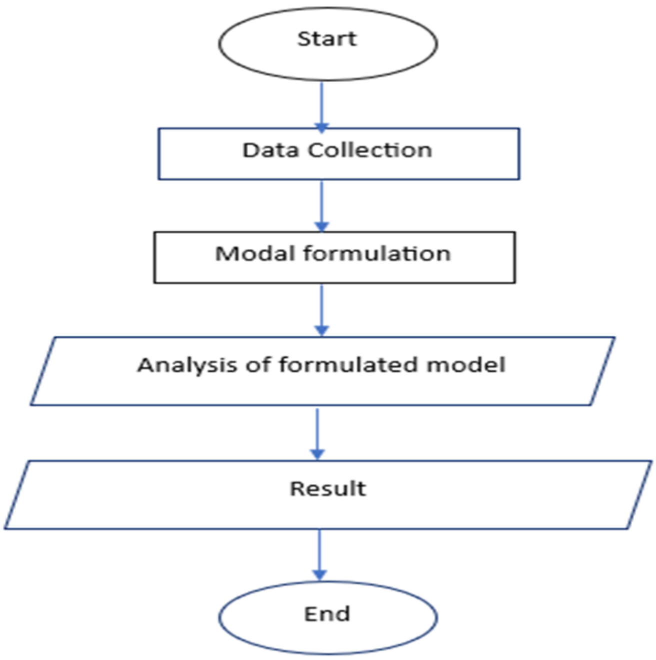

Figure 3.1: Research design flowchart

Mathematical Formulation of the Model

A Linear Programming Problem (LPP) has the general form maximize /minimize the objective function (Amit *et al.* (2020)):

$Z = c_1x_1 + c_2x_2 + c_3x_3 + \cdots + c_nx_n$ ....(3.1)

Subject to the constraints:

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n (\leq) b_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n (\leq) b_2$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3n}x_n (\leq) b_3$

… …. … … …

… …. … … …

$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n (\leq) b_m$

And the non-negative restrictions $x_j \geq 0$, $j = 1, 2, \ldots$, where a_{ij} , b_i , and c_i are constants and x_j are variables.

Primal Form

Maximize $Z = CX$

Subject to: $AX = B$

With: $X \geq 0$.

where X is the column vector of unknowns, including all slack, surplus, and artificial variables;

where C the row vector of the corresponding costs;

where A is the coefficient matrix of the constraint equations;

B is the column vector of the right-hand sides of the constraint equations.

Dual Form

Minimize $Z = BY$

Subject to: $A^T Y = C$

With: $Y \geq 0$.

where Y is the column vector of unknowns, including all slack, surplus, and artificial variables;

where B the row vector of the corresponding costs;

where A^T is the coefficient matrix of the constraint equations;

C is the column vector of the right-hand sides of the constraint equations.

Inventory Control Model

The EOQ technique, the variables consist of five inventory items which include flour, sugar butter milk and yeast which is judgmental in nature. For this research, the source of data is a secondary collected at Haske Modern Bakery through record, as information of the organization from the purchase department.

Economic Order Quantity Formula

EOQ inventory formula was written by Cargal. This formula is written below.

$Q = \sqrt{\frac{2SD}{H}}$ where Q is the EOQ order quantity

Total Cost (TC) = $\sqrt{2DSH}$

Expected number of orders $= \frac{D}{Q}$

ROP = $\left(\frac{A}{365}\right) * L$

Reorder point where lead time $= 7days$

ROP = (Annual demand/no. of working days in a year) * Lead time

D = Annual demand

H = Carrying Cost

S = Ordering cost

Results and Discussion

Table 4.1 represent the different types of bread produced by the bakery along with their respective raw material requirements, selling prices, and cost prices. Each bread type was denoted by x_i, where i ranged from 1 to 13. The raw materials considered were flour, sugar, butter, yeast, preservatives, milk, flavor, improvers, and salt. The quantities of these raw materials required to produce each type of bread per week are provided along with the total available quantities of these raw materials per week. The bread types and their production quantities are as follows: x_1=1350 (multi-colour), x_2=1200 (larger), x_3=1000 (big coconut), x_4=900 (big), x_5=500 (small medium), x_6=300 (Kalangu), x_7=250 (Nokia), x_8=270 (big Nokia), x_9=800 (Butter), x_{10}=700 (plane Boga), x_{11} = 300 (cheza), and

x_{13} =200 (small butter). The table shows the quantities of each raw material required to produce one unit of each bread type. For instance, producing one unit of multi-colour bread x_1 requires 6.6667 kg of flour, 1.3333 kg of sugar, 0.2 kg of butter, 0.1 kg of yeast, 0.1333 kg of preservative, 0.4 kg of milk, 0.1 kg of flavour, 0.1 L of improver, and 0.3333 kg of salt. The total available raw materials per week were 1000 kg flour, 200 kg sugar, 30 kg butter, 15 kg yeast, 20 kg preservative, 60 kg milk, 15 kg flavor, 15 L improver, and 50 kg salt. Each type of bread has an associated selling price and cost price per unit, indicating the revenue generated and cost incurred, respectively. For example, the selling price of multicolor bread x_1 is N1350, while its cost price is N1300.

TABLE 4.1 : DATA PRESENTATION

RAW MATERIALS	PRODUCT													TOTAL RAW MATERIAL PER WEEK
	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	
Flour	6.667	14.286	20	3.333	3.125	5	3.333	4	5	25	50	20	10	1000kg
Sugar	1.333	2.8571	4	0.667	0.625	1	0.667	0.8	1	5	10	4	2	200kg
Butter	0.2	0.4286	0.6	0.1	0.094	0.15	0.1	0.1	0.15	0.75	0.2	0.6	0.3	30kg
Yeast	0.1	0.2143	0.3	0.05	0.047	0.08	0.05	0.1	0.08	0.38	0.8	0.3	0.2	15kg
Preservative	0.133	0.2857	0.4	0.067	0.063	0.1	0.067	0.1	0.1	0.5	1	0.4	0.2	20kg
Milk	0.4	0.8571	1.2	0.2	0.188	0.3	0.2	0.2	0.3	1.5	3	1.2	0.6	60kg
Flavour	0.1	0.2143	0.3	0.05	0.047	0.08	0.05	0.1	0.08	0.38	0.8	0.3	0.2	15kg
Improver	0.1	0.2143	0.3	0.05	0.047	0.08	0.05	0.1	0.08	0.38	0.8	0.3	0.2	15L
Salt	0.333	0.7143	1	0.167	0.156	0.25	0.167	0.2	0.25	1.25	2.5	1	0.5	50kg
Selling price	1350	1200	1000	900	500	300	250	700	270	800	700	300	200	

RAW MATERIALS	PRODUCT													TOTAL RAW MATERIAL PER WEEK
Cost price	1300	1150	950	850	450	280	230	650	250	750	650	280	180	

Source: Haske Modern Bakery in Bauchi State through Record, 2024

Mathematical Model Formulated for Primal

Maximize $50x_1 + 50x_2 + 50x_3 + 50x_4 + 50x_5 + 20x_6 + 20x_7 + 50x_8 + 20x_9 + 50x_{10} + 50x_{11} + 20x_{12} + 20x_{13}$;

Subject to:

$$6.6667x_1 + 14.2857x_2 + 20x_3 + 3.3333x_4 + 3.125x_5 + 5x_6 + 3.3333x_7 + 4x_8 + 5x_9 + 25x_{10} + 50x_{11} + 20x_{12} + 10x_{13} \leq 1000$$

$$1.3333x_1 + 2.8571x_2 + 4x_3 + 0.6667x_4 + 0.625x_5 + x_6 + 0.6667x_7 + 0.8x_8 + x_9 + 5x_{10} + 10x_{11} + 4x_{12} + 2x_{13} \leq 200$$

$$0.2x_1 + 0.4286x_2 + 0.6x_3 + 0.1x_4 + 0.0938x_5 + 0.15x_6 + 0.1x_7 + 0.12x_8 + 0.15x_9 + 0.75x_{10} + 1.5x_{11} + 0.6x_{12} + 0.3x_{13} \leq 30$$

$$0.1x_1 + 0.2143x_2 + 0.3x_3 + 0.05x_4 + 0.0469x_5 + 0.075x_6 + 0.05x_7 + 0.06x_8 + 0.075x_9 + 0.375x_{10} + 0.75x_{11} + 0.3x_{12} + 0.15x_{13} \leq 15$$

$$0.1333x_1 + 0.2857x_2 + 0.4x_3 + 0.0667x_4 + 0.0625x_5 + 0.1x_6 + 0.0667x_7 + 0.08x_8 + 0.1x_9 + 0.5x_{10} + x_{11} + 0.4x_{12} + 0.2x_{13} \leq 20$$

$$0.4x_1 + 0.8571x_2 + 1.2x_3 + 0.2x_4 + 0.1875x_5 + 0.3x_6 + 0.2x_7 + 0.24x_8 + 0.3x_9 + 1.5x_{10} + 3x_{11} + 1.2x_{12} + 0.6x_{13} \leq 60$$

$$0.1x_1 + 0.2143x_2 + 0.3x_3 + 0.05x_4 + 0.0469x_5 + 0.075x_6 + 0.05x_7 + 0.06x_8 + 0.075x_9 + 0.375x_{10} + 0.75x_{11} + 0.3x_{12} + 0.15x_{13} \leq 15$$

$$0.1x_1 + 0.2143x_2 + 0.3x_3 + 0.05x_4 + 0.0469x_5 + 0.075x_6 + 0.05x_7 + 0.06x_8 + 0.075x_9 + 0.375x_{10} + 0.75x_{11} + 0.3x_{12} + 0.15x_{13} \leq 15$$

$$0.3333x_1 + 0.7143x_2 + x_3 + 0.1667x_4 + 0.1563x_5 + 0.25x_6 + 0.1667x_7 + 0.2x_8 + 0.25x_9 + 1.25x_{10} + 2.5x_{11} + x_{12} + 0.5x_{13} \leq 50$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13} \geq 0.$$

In equation 4.3, we solved a linear programming problem aimed at maximizing an objective function under a set of constraints. The objective function was defined as:

Maximize $50x_1 + 50x_2 + 50x_3 + 50x_4 + 50x_5 + 20x_6 + 20x_7 + 50x_8 + 20x_9 + 50x_{10} + 50x_{11} + 20x_{12} + 20x_{13}$. Subject to nine linear constraints, with non-negativity conditions on the decision variables x_1 through x_{13}. The optimal solution obtained was x_5=319.8294 with all other variables $x_1, x_2, x_3, \dots, x_{13}$ equal to zero. The maximum value of the objective function under these conditions was 15991.47. This result indicates that the optimal allocation of resources, as per the given constraints, is achieved by focusing entirely on the variable x_5, leading to the maximum possible outcome for the objective function. The solution satisfies all the imposed constraints, demonstrating both feasibility and optimality.

Mathematical Model Formulated for Dual

Minimize $1000y_1 + 200y_2 + 30y_3 + 15y_4 + 20y_5 + 60y_6 + 15y_7 + 15y_8 + 50y_9$

Subject to:

$6.6667y_1+1.3333y_2+0.2y_3+0.1y_4+0.1333y_5+0.4y_6+0.1y_7+0.1y_8+0.3333y_9 \geq 50$

$14.2857y_1+2.8571y_2+0.4286y_3+0.2143y_4+0.2857y_5+0.8571y_6+0.2143y_7+0.2143y_8+0.7143y_9 \geq 50$

$20y_1+4y_2+0.6y_3+0.3y_4+0.\ y_5+1.2y_6+0.3y_7+0.3y_8+y_9 \geq 50$

$3.3333y_1+0.6667y_2+0.1y_3+0.05y_4+0.0667y_5+0.2y_6+0.05y_7+0.05y_8+0.1667y_9 \geq 50$

$3.125y_1+0.625y_2+0.0938y_3+0.0469y_4+0.0625y_5+0.1875y_6+0.0469y_7+0.0469y_8+0.1563y_9 \geq 50$

$5y_1+y_2+0.15y_3+0.075y_4+0.1y_5+0.3y_6+0.075y_7+0.075y_8+0.25y_9 \geq 20$

$3.3333y_1+0.6667y_2+0.1y_3+0.05y_4+0.0667y_5+0.2y_6+0.05y_7+0.05y_8+0.1667y_9 \geq 20$

$4y_1+0.8y_2+0.12y_3+0.06y_4+0.08y_5+0.24y_6+0.06y_7+0.06y_8+0.2y_9 \geq 50$

$5y_1+y_2+0.15y_3+0.075y_4+0.1y_5+0.3y_6+0.075y_7+0.07y_8+0.25y_9 \geq 20$

$25y_1+5y_2+0.75y_3+0.375y_4+0.5y_5+1.5y_6+0.375y_7+0.37y_8+1.25y_9 \geq 50$

$50y_1+10y_2+1.5y_3+0.75y_4+y_5+3y_6+0.75y_7+0.75y_8+2.5y_9 \geq 50$

$20y_1+4y_2+0.6y_3+0.3y_4+0.4y_5+1.2y_6+0.3y_7+0.\ y_8+y_9 \geq 20$

$10y_1+2y_2+0.3y_3+0.15y_4+0.2y_5+0.6y_6+0.15y_7+0.15y_8+0.5y_9 \geq 20$

$y_1,y_2,y_3,y_4,y_5,y_6,y_7,y_8,y_9 \geq 0.$

In equation 4.4, we analyzed the dual formulation of a linear programming problem corresponding to a primal maximization problem. The dual objective function aimed to minimize the expression $1000y_1 + 200y_2 + 30y_3 + 15y_4 + 20y_5 + 60y_6 + 15y_7 + 15y_8 + 50y_9$, subject to a set of linear inequalities. The optimal solution for the dual problem was found with y_3=533.0490, while all other dual variables (y_1, y_2, y_3, ..., y_9) were zero. The objective value of the dual problem was 15991.47, which matches the optimal value of the primal problem, confirming strong duality. This result highlights that the third constraint in the primal problem is the binding constraint, driving the optimal value of the objective function. The other constraints do not influence the solution, as indicated by their corresponding dual variables being zero. The study demonstrates the importance of dual analysis in identifying key constraints and understanding resource valuation in optimization problems. In the context of the dual linear programming problem, y_3 = 533.0490 can be interpreted as the shadow price or dual price associated with the third constraint of the primal problem.

Interpretation of y_3:

i. **Marginal Value of the Resource:**

The value y_3=533.0490 indicates the rate of change in the objective value of the primal problem per unit increase in the right-hand side of the third constraint. Specifically, if the constraint associated with y_3 (the third constraint in the primal problem) were relaxed (i.e., the right-hand side value were increased by one unit), the objective value of the primal problem would increase by approximately 533.0490 unit

ii. **Binding Constraint:**

Since y_3 is non-zero, it implies that the third constraint in the primal problem is **binding** at the optimal solution. This means that this constraint is active and directly impacts the optimal solution. If this constraint were to be loosened, it would allow a better (higher) objective value in the primal problem.

iii. **Resource Scarcity:**

A high value of y_3 suggests that the resource corresponding to the third constraint is scarce and highly valuable in the context of the optimization problem. This scarcity drives its shadow price up, indicating that this resource is critical in determining the maximum possible value of the objective function.

Table 4.2 : Composition of Dough Makeup and Marker Prices Per 50kg Bag

S/N	ITEMS	WEIGHT (kg)	%	COST (N)
1.	Flour	1000kg	71.1	54500/50kg
2.	Sugar	200kg	14.2	77000/50kg
3.	Butter	30kg	2.1	25500/15kg
4.	Yeast	15kg	1.1	2500/500g
5.	Preservative	20kg	1.4	64000/20kg
6.	Milk	60kg	4.3	15000/25kg
7.	Flavour	15kg	1.1	72000/5L
8.	Improver	15L	1.1	88000/20L
9.	Salt	50kg	3.6	13500/50kg
	Total	1405	100	

Table 4.2 outlines the composition and market prices of ingredients used in dough preparation per 50 kg bag. Each item is listed with its corresponding weight in kilograms, percentage of total composition, and cost. Flour was the primary component, comprising 71.1% of the mix, with a weight of 1000 kg and a cost of N54,500 per 50 kg bag. Sugar

makes up 14.2% of the composition, weighing 200 kg and costing N77,000 per 50 kg. Butter accounted for 2.1% at 30 kg and priced at N25,500 per 15 kg Yeast contributes 1.1% with a weight of 15 kg and costs N2,500 per 500 g. The preservative, at 1.4%, weighs 20 kg, and is priced at N64,000 per 20 kg. Milk constitutes 4.3% of the mix, with a weight of 60 kg and a cost of N15,000 per 25 kg. Flavor, also at 1.1%, weighs 15 kg, and is priced at N72,000 per 5 L. The improver, another 1.1%, is 15 L in weight and costs N88,000 per 20 L. Lastly, salt comprises 3.6% of the mix, with a weight of 50kg and a cost of N13,500 per 50kg. The total weight of the ingredients was 1405 kg, which accounted for 100% of the dough composition.

Table 4.3 : Plant Capacity with Corresponding Weekly Raw Material Requirement

Ranking	Items	Bags/cartons	Qty (Kg)	Price/kg	Value	Percentage
1.	Flour	20	1000	1090	1090000	50.26342
2.	Sugar	4	200	1540	308000	14.20288
3.	Butter	2	30	1700	51000	2.351775
4.	Yeast	30	15	5	75	0.003458
5.	Preservative	1	20	3200	64000	2.951247
6.	Milk	2.4	60	6000	360000	16.60076
7.	Flavour	3	15	14400	216000	9.960458
8.	Improver	0.75	15	4400	66000	3.043473
9.	Salt	1	50	270	13500	0.622529

Table 4.3 details the plant capacity and the corresponding weekly raw material requirements. It ranks each item by quantity and provides the number of bags or cartons needed, the quantity in kilograms, price per kilogram, total value, and the percentage of the total cost. Flour tops the list with 20 bags (1000 kg), costing N1,090 per kg, totaling N1,090,000, and representing 50.26% of the total value. Sugar requires 4 bags (200 kg) at N1,540 per kg, totaling N308,000 (14.20%). Butter needs 2 cartons (30 kg) at N1,700 per kg, totaling N51,000 (2.35%). Yeast, with 30 cartons (15 kg) at N5 per kg, totals N75 (0.003%). Preservative needs 1 bag (20 kg) at N3,200 per kg, totaling N64,000 (2.95%).

Milk requires 2.4 bags (60 kg) at N6,000 per kg, totaling N360,000 (16.60%). Flavour needs 3 cartons (15 kg) at N14,400 per kg, totaling N216,000 (9.96%). Improver requires 0.75 cartons (15 kg) at N4,400 per kg, totaling N66,000 (3.04%). Lastly, salt needs 1 bag (50 kg) at N270 per kg, totaling N13,500 (0.62%).

Table 4.4 : Analysis of Inventory

[illegible] Annual demand (D)	[illegible]	EOQ (Q)	[illegible]
Carrying Cost (H)	[illegible]	Total Cost (TC)	[illegible]
Ordering cost (S)	[illegible]	Expected Number of order	[illegible]
		Re-order point	[illegible]

Table 4.4 provides an in-depth inventory of the key raw materials used in production: flour, sugar, butter, yeast, and milk. For each item, the table lists the annual demand (D), Economic Order Quantity (EOQ), carrying cost (H), total cost (TC), ordering cost (S), expected number of orders per year, and re-order point (ROP). Specifically, it includes flour with an annual demand of 1,040 units, an EOQ of 59 bags, a carrying cost of N300, and a total cost of N19,349. The ordering cost is N500, leading to 18 expected orders per year and a reorder point for 20 bags. Similarly, sugar has an annual demand of 208 units, an EOQ of 22 bags, a carrying cost of N500, a total cost of N11,171, and an ordering cost of N600, with nine expected orders per year and a reorder point of four bags. For butter, the annual demand is 104 units with an EOQ of 11 cartons, carrying cost of N500, total cost of N5,586, and ordering cost of N300, resulting in nine expected orders per year and a reorder point of two cartons. Yeast, with the highest annual demand of 1,560 units, has an EOQ of 56 cartons, a carrying cost of N500, a total cost of N27,928, an ordering cost of N500, leading to 28 expected orders per year, and a reorder point of 30 cartons. Lastly, milk has an annual demand of 125 units, an EOQ of 12 bags, a carrying cost of N500, a total cost of N6,119, and an ordering cost of N300, resulting in ten expected orders per year and a reorder point of two bags. This analysis aids in understanding the optimal inventory management strategies for these essential raw materials.

Table 4.5 : Summary of EOQ Model

	Flour	Sugar	Butter	Milk	Yeast
EOQ	59 bags	22 bags	11 Cartons	12 bags	56 Cartons
Total Cost	N19349	N11171	N5586	N6119	N27928
Expected Number of order per Year	18 orders	9 orders	9 **orders**	10 **orders**	28 **orders**
Re-order Point	20 bags	4 bags	2 cartons	2 bags	30 cartons

Table 4.5 presents a comprehensive summary of the Economic Order Quantity (EOQ) model applied to five key inventory items: flour, sugar, butter, milk, and yeast. The EOQ for each item is outlined, revealing the optimal order sizes of 59 bags for flour, 22 bags for sugar, 11 cartons for butter, 12 bags for milk, and 56 cartons for yeast. The total costs associated with these EOQs are detailed, with flour incurring N19,349, sugar N11,171, butter N5,586, milk N6,119, and yeast N27,928. Additionally, the table highlights the expected number of orders per year for each item, ranging from 9 orders for sugar and butter to 28 orders for yeast. The re-order points, indicating the inventory levels at which new orders should be placed to avoid stockouts, are specified as 20 bags for flour, 4 bags for sugar, 2 cartons for butter, 2 bags for milk, and 30 cartons for yeast. This summary provides crucial insights into the inventory management strategies for these essential items, ensuring efficient stock control and cost minimization.

Discussion of Result

Analysis of the bakery's production using a mathematical model revealed that the Primal result $x_5 = 319.8294, x_1, x_2, x_3, \dots, x_{13} = 0$, Objective Value = 15991.47

Dual result y_3=533.0490, y_1, y_2, ..., y_9=0, Objective Value = 15991.47

The primal problem maximizes the objective function ($50x_1 + 50x_2 + 50x_3 + 50x_4 + 50x_5 + 20x_6 + 20x_7 + 50x_8 + 20x_9 + 50x_{10} + 50x_{11} + 20x_{12} + 20x_{13}$) subject to resource constraints. The dual problem minimizes the objective function ($1000y_1 + 200y_2 + 30y_3 + 15y_4 + 20y_5 + 60y_6 + 15y_7 + 15y_8 + 50y_9$) subject to shadow price constraints. The optimal solution to the primal problem (x_5=319.8294, x_1, x_2, x_3, ..., x_{13}=0) corresponds to the optimal solution to the dual problem (y_3=533.0490, y_1, y_2, y_3, ..., y_9=0). The objective value of the primal problem (15991.47) equals the objective value of the dual problem (15991.47). The primal variables (x_1, x_2, x_3, ..., x_{13}) represent the deference sizes of bakery produce. The dual variables (y_1, y_2, y_3, ..., y_9) represent the raw material used. for Haske Modern Bakery in the Bauchi state.

To calculate the Economic Order Quantity (EOQ), it is essential to know the annual demand, holding cost, and ordering costs. The organization provided the annual demand, and other parameters were calculated. The EOQ for flour, sugar, butter, milk, and yeast were approximately 59 bags, 22 bags, 11 cartons, 12 bags, and 56 cartons, respectively. The reorder point assumes a constant lead time demand. Given a lead time of 7 days, the organization should be reordered when stock levels reach 20 bags of flour, four bags of sugar, two cartons of butter, two bags of milk, and 30 cartons of yeast. This aligns with the findings of Kehinde *et al.* (2020), who also noted that flour, sugar, and butter have the highest EOQs, in descending order, resulting in cost savings and optimized bakery capacity.

Conclusion

This research successfully demonstrated that by utilizing a mathematical model, Haske Modern Bakery can optimize its raw material usage and maximize profits. The optimal production mix was identified, which significantly enhanced profitability. Additionally, the implementation of the EOQ model provided a structured approach to inventory management, ensuring timely reordering and minimizing the costs associated with ordering and holding inventory. These findings highlight the importance of strategic resource allocation and efficient inventory management in achieving operational efficiency and financial performance in the bakery industry.

Recommendations

It is recommended that Haske Modern Bakery adopt the proposed mathematical model and an EOQ-based inventory management system to enhance its production efficiency and profitability. The bakery should regularly review and update the model parameters to reflect the changes in demand and costs, ensuring continued optimization. Furthermore, training staff on the implementation and benefits of these models could improve adherence and effectiveness. Additionally, conducting periodic case studies on other bakery products and incorporating more variables could refine the model and expand its applicability, ultimately driving greater operational success.

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