Mikailalsys Journal of
Mathematics and Statistics

e-ISSN : 3030-816X
p-ISSN : 3030-8399

Index : Harvard, Boston, Sydney University, Dimensions, Lens, Scilit, Semantic, Google, etc

https://doi.org/10.58578/MJMS.v4i1.7597

On the Closed-Form Characterisation of the Impact of Risk Misprofiling on Optimal Nigerian Insurance Pricing Models

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Article Info:

Submitted:	Revised:	Accepted:	Published:
Oct 2, 2025	Oct 24, 2025	Nov 8, 2025	Nov 13, 2025

Abstract

This study addresses the underexplored issue of risk mis-profiling in optimal insurance pricing models and its implications for solvency and regulatory compliance within the insurance industry. It aims to mathematically analyse the effects of classification errors on premium determination, quantify pricing deviations, and assess sensitivity to misclassification biases. Adopting a quantitative research design, the study utilises insurance data spanning 2010–2020, with computational implementation in Python 3.12.3 (2025) and calibration in Weka 3.9.6 (2022). Policyholders were categorised into low-, medium-, and high-risk groups using confusion matrices, while premiums were derived under exponential utility and deterministic-equivalent principles. Analytical techniques included cumulant generating function expansions, Taylor–Lagrange remainder approximations, and optimisation frameworks. The results indicate that even minor classification errors significantly influence premium estimates, particularly due to exponential tilting, variance underestimation, and tail sensitivity. These distortions align with theoretical expectations and highlight solvency vulnerabilities when premiums fall below actuarially fair values. The study concludes that systematic mis-profiling introduces pricing inefficiencies and potential insolvency triggers. Theoretical contributions include the extension of utility-based pricing principles to account for classification uncertainty, while practical implications call for insurers and regulators to adopt robust pricing adjustments, monitor classifier accuracy, and integrate misclassification-aware pricing mechanisms. Future research directions include extending the framework to portfolio-level analysis, applying robust stochastic optimisation, and investigating the effects of machine learning classification errors on pricing precision.

Keywords: Risk Misprofiling; Utility-Based Pricing; Classification Sensitivity; Insurance Optimisation; Solvency Analysis

INTRODUCTION

In recent years, the global insurance industry has undergone reshaping driven by more granular risk classification facilitated by big data, machine learning, and risk-based regulation; however, empirical work indicates that classification errors persist and can have a material impact on pricing and solvency (Albrecht et al., 2020). Nigeria has been engaged in a parallel modernisation: the National Insurance Commission (NAICOM) has signalled a shift toward risk-based supervision to strengthen underwriting and capital adequacy (NAICOM, 2021). Although regulatory progress has been made, the Nigerian market exhibits structural weaknesses—insurance penetration remains exceptionally low (roughly 0.4–0.5% of GDP in 2021–2022), far below regional and global peers—indicating underdevelopment of insurable interest and pricing markets (Leadway Insurance, 2022; BusinessDay, 2020; Augusto & Co., 2022). This low penetration coexists with persistent underwriting strains: several empirical studies document underwriting losses and weak underwriting capacity across Nigerian non-life insurers, and heavy reliance on reinsurance (reinsurance cessions have been reported to account for large shares of premium in some samples, e.g., ~67% in historical aggregates) (Oyetayo & Abass, 2019; IOSR Journal, 2023; Academic Journal of Economic Studies, 2019). These patterns suggest that pricing frictions—potentially including mis-profiling—are economically material in Nigeria (Iroh & Orobator, 2020; Efuntade & Efuntade, 2022).

Globally, the literature modelling classification risks in insurance pricing is growing. Albrecher *et al.* (2020) quantified the cost of class misspecification in a two-type mean–

variance framework; mixture-of-experts and random-effects models have been applied to improve posterior ratemaking and reduce approximation error (Tseung *et al.*, 2022). Mathematically, exponential utility and deterministic equivalent pricing offer closed analytic expressions via cumulative generating functions that are attractive for sensitivity analysis (Gerber, 1979; Kaas *et al.*, 2008). However, most prior works are limited in one or more ways: they focus on two risk types, report mostly numerical approximations rather than closed-form solutions, or fail to connect classification error to solvency outcomes in low-penetration markets such as Nigeria (Albrecher *et al.*, 2020; Tseung *et al.*, 2022). Importantly, explicit empirical reporting of classification risk rates and systematic measures of premium mispricing attributable to mis-profiling are predominantly neglected in Nigerian public datasets and regulatory disclosures, leaving a blind spot for regulators and actuaries (NAICOM, 2021; Leadway Insurance, 2022).

This paper bridges that gap by developing a closed-form characterisation of premium bias $\Delta\pi$ between Misclassified and Classified risk distributions under exponential utility, explicitly modelling mis-profiling through confusion matrices and calibrating analysis to Nigerian data (2010–2022) where available. We derive exact expressions for pricing bias, cumulative expansions with explicit Lagrange remainder bounds, and tail-sensitivity results that show exponential amplification of small tail-mass misallocations. We further pose optimisation problems for classifier design and robust pricing under ambiguity, and show how mis-profiling can generate insolvency signals when premiums fall below expected losses—an outcome of particular concern in low-penetration markets characterised by weak underwriting capacity (Iroh & Orobator, 2020; Oyetayo & Abass, 2019). Our analysis provides both theoretical tools and actionable diagnostics that Nigerian insurers and supervisors can use to measure and mitigate mis-profiling risks.

METHODS

This study integrates closed-form characterisation with data-driven simulations to analyse insurance pricing under risk mis-profiling. Analytical results yield deterministic-equivalent premium formulas under exponential (Constant Absolute Risk Aversion) utility, while empirical work uses insurer data (2010–2020) to estimate loss laws and validate classifiers. This dual approach allows both theoretical precision and empirical validation.

The design is analytic–empirical: theoretical modelling of latent risk types (L, M, H), utility-based premiums, and misclassification bias is paired with empirical estimation of class-conditional losses, classifier confusion matrices, and simulation experiments on Δπ and solvency. The data sample consists of policies issued over 2010–2020, partitioned into latent risk classes.

Data sources include the insurer's underwriting risk register. Classifiers, including logistic regression and tree ensembles in Weka 3.9.6, are used to predict policyholder risk classes. Performance is assessed using confusion matrices, the Receiver Operating Characteristic curve/Area Under the Curve (ROC/AUC), calibration plots, and cross-validation. Loss distributions are validated through moment matching, goodness-of-fit tests, and bootstrap resampling. Numerical stability is checked before expansion; computations use Python 3.12.3 for simulation and optimisation.

Analytical procedures involve computing the moment generating function (mgf) $M_H(\theta) = E_H[e^{\theta X}]$, Cumulative generating function (cgf) $K_H(\theta) = lnM_H(\theta)$, and premiums $\pi_H(\theta) = \frac{K_H(\theta)}{\theta}$, then applying cumulative expansions and Taylor–Lagrange bounds to characterise premium bias. Empirical calibration estimates cumulatively (mean μ_H, variance σ_H^2, skew $\kappa_{3,H}$ and runs perturbation tests to quantify misclassification effects. Bias and solvency risks are measured by $\Pi_{exp}(\theta) = \pi_G(\theta) - \mathbb{E}_F[X]$. Undeterministic is evaluated using bootstrap confidence intervals, and results are reported with effect sizes, Confidence Intervals (CI), and reproducible code.

Model Formulation

To formulate the mathematical model, set a population of insureds composed of three latent risk types: Low (L), Medium (M), and High (H). Loss amounts are random; the insurer may mis-profile the accurate type distribution when setting premiums. The exponential (Constant Absolute Risk Aversion) utility with parameter θ and sets per-policy deterministic-equivalent premiums are adopted according to (Adewale et al., 2025).

$$\begin{cases} \mathrm{p_F} = \left(\mathrm{p_F^L}, \mathrm{p_F^M}, \mathrm{p_F^H}\right), \\ \mathrm{C} = \left(\mathrm{C_{ij}}\right)_{\mathrm{i,j} \in \mathcal{T}}, \quad \mathrm{p_G^j} = \sum_{i \epsilon \mathcal{T}} P_F^i C_{ij}, \quad F(x) = \sum_i P_F^i \mathcal{F}_i(x), \quad G(x) = \sum_i P_G^i \mathcal{F}_j(x), \\ M_H(\theta) = \mathbb{E}_H\left[e^{\theta X}\right], \qquad K_H(\theta) = \ln M_H(\theta), \\ \pi_H(\theta) = \frac{1}{\theta} K_H(\theta), \quad \Delta\pi(\theta) = \frac{K_G(\theta) - K_F(\theta)}{\theta}, \\ \Delta\pi(\theta) = \sum_{n-1}^{\infty} \frac{K_n, G - K_n, F}{n!} \theta^{n-1}, \\ \frac{\partial \pi_G}{\partial P_G^j} = \frac{1}{\theta} \frac{\mathbb{E}_{F_j}\left[e^{\theta X}\right]}{M_G(\theta)}, \\ \Pi_{\exp}(\theta) = \pi_G(\theta) - \mathbb{E}_F[X] \end{cases} \tag{1}$$

$$\begin{cases} P_F^i \geq 0, \quad \sum_{i \epsilon \mathcal{T}} P_F^i = 1, \quad \mathrm{C_{ij}} \in [0,1], \quad 0 \leq \mathrm{p_F^i} \leq 1, \\ R > 0, \quad M_H(\theta) < \infty \ for\ all\ \theta \epsilon(-R, R), \qquad 0.1 \leq \theta \leq 1.0, \quad \mathrm{M_H}(\theta) > 0, \pi_H(\theta) \geq 0 \end{cases} \tag{2}$$

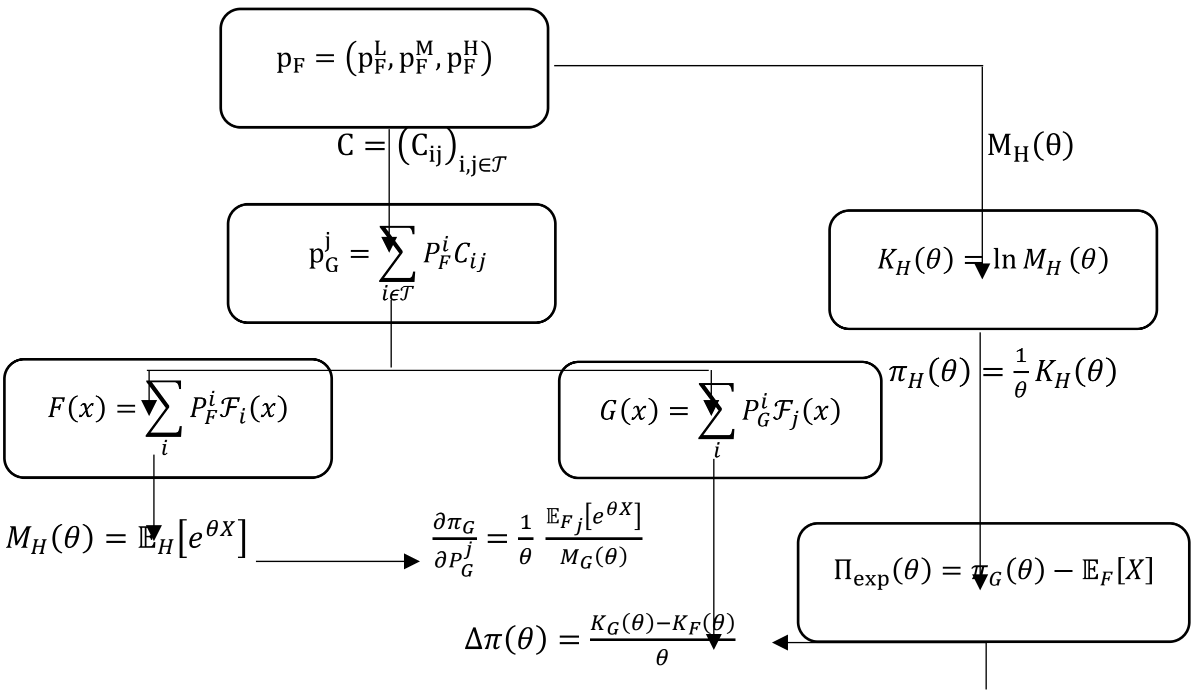


Figure 1: Flow Diagram of Risk Mis-Profiling of Optimal Nigerian Insurance Pricing Model

The flow diagram represents the modelling framework for analysing the impact of risk mis-profiling on optimal insurance pricing under exponential utility. It begins with estimating accurate risk probabilities $\mathrm{p_F} = \left(\mathrm{p_F^L}, \mathrm{p_F^M}, \mathrm{p_F^H}\right)$ and constructing a confusion matrix $\mathrm{C} = \left(C_{ij}\right)$ to compute perceived group probabilities $\mathrm{p_G^j} = \sum_{i \epsilon \mathcal{T}} P_F^i C_{ij}$. Loss distributions $F(x)$

and $G(x)$ are expressed through the moment and cumulative generating functions $M_H(\theta)$ and $K_H(\theta)$, leading to the certainty-equivalent premium $\pi_H(\theta) = \frac{1}{\theta} K_H(\theta)$. The premium deviation $\Delta\pi(\theta) = \frac{K_G(\theta) - K_F(\theta)}{\theta}$ measures the bias caused by misclassification and its effect on solvency. Finally, a Taylor–Lagrange sensitivity analysis highlights how tail and variance errors magnify pricing distortions, validated empirically on Nigeria's 2010–2020 insurance data.

Table 1: Parameters/Variables and Definitions

Parameters/variables	**Description**
$\mathcal{T} = \{L, M, H\}$	Risk type (Low, Medium, High)
$\mathrm{p_F} = (\mathrm{p_F^L}, \mathrm{p_F^M}, \mathrm{p_F^H})$, where $\mathrm{p_F^i} = \mathbb{P}_\mathrm{F}(\mathrm{type} = \mathrm{i}),\ \mathrm{i} \in \mathcal{T}$	Classified type probabilities
$\mathrm{p_G} = (\mathrm{p_G^L}, \mathrm{p_G^M}, \mathrm{p_G^H})$	Misclassified type probabilities
P_F	Classified policyholder probabilities across risk classes
P_G	Insurer's observation of the probabilities after classification errors
$\mathrm{p_G} - \mathrm{p_F}$	The gap: mis-profiling bias (premium mispricing)
$\mathrm{C} = (\mathrm{C_{ij}})_{\mathrm{i,j}\in\mathcal{T}}$, where $C_{ij} = \mathbb{P}$ (classified as j | accurate type i), $\sum_{j\in\mathcal{T}} C_{ij} = 1\ \forall\, i$. Then $\mathrm{p_G^j} = \sum_{i\in\mathcal{T}} P_F^i C_{ij}$	Confusion matrix (mis-profiling / classification)
$\mathrm{X} \mid \{\mathrm{type} = \mathrm{i}\}$	Random variable (Loss conditional on classified type i) with distribution $\mathcal{F}_i$(cdf) and density/surface as needed; write f_i for density if exists.
$F = \sum_{i\in\mathcal{T}} P_F^i \mathcal{F}_i$	Total classified loss distribution (mixture)
$G = \sum_{i\in\mathcal{T}} P_G^i \mathcal{F}_j$	Misclassified loss distribution used by the insurance
$M_H(\theta) = \mathbb{E}_H[e^{\theta X}]$,	Cumulative generating functions (cgf): for a

$K_H(\theta) = \ln M_H(\theta)$	law $\mathcal{H}$, denoted
$\theta \epsilon \mathbb{R}\{0\}$ $(commonly\ \theta > 0)$, initial capital $\mathcal{W}_0$	Insurance parameter (risk aversion)
$\pi_H(\theta)$	Premium of exponential utility
$\Delta\pi(\theta)$	Pricing bias

Assumptions

1. There exists $R > 0$ such that for both $H \in \{F, G\}$, $MH(t) < \infty$ for all real $t \in (-R, R)$. (This guarantees analytic CGF locally.)
2. Each within-type distribution Fi has finite moments up to the order required (depending on the expansion order M) and satisfies $EFi[et \mid X \mid] < \infty$ for $\mid t \mid \leq T$ for some $T \geq \theta$.
3. The confusion matrix C is known (or estimated) and independent of the loss realisations conditional on type. Total-probability constraint: $p_G^j = \sum_i p_F^i C_{ij}$
4. When analysing portfolio aggregates, claims $X1, \ldots$, are iid draws from the accurate mixture law F unless stated otherwise.

where,

For $K = 3$ with types L, M, H, write

$$C = \begin{pmatrix} C_{LL} & C_{LM} & C_{LH} \\ C_{ML} & C_{MM} & C_{MH} \\ C_{HL} & C_{HM} & C_{HH} \end{pmatrix}, \qquad P_F = \begin{pmatrix} P_F^L \\ P_F^M \\ P_F^H \end{pmatrix}, \qquad m(\theta) = \begin{pmatrix} m_L \\ m_M \\ m_H \end{pmatrix}. \tag{3}$$

Then, the mis-profiling transformation is

$$P_G = \begin{pmatrix} P_G^L \\ P_G^M \\ P_G^H \end{pmatrix} = C^T P_F = \begin{pmatrix} P_F^L C_{LL} + P_F^M C_{LM} + P_F^H C_{LH} \\ P_F^L C_{ML} + P_F^M C_{MM} + P_F^H C_{MH} \\ P_F^L C_{HL} + P_F^M C_{HM} + P_F^H C_{HH} \end{pmatrix}, \tag{4}$$

The distorted premium under mis-profiling is

$$M_G(\theta) = P_G^T m(\theta) = P_F^T\big(Cm(\theta)\big) \tag{5}$$

The pricing error is

$$\Delta \mathrm{M}(\theta) = \mathrm{M_G}(\theta) - \mathrm{M_F}(\theta) = \mathrm{p_F^T}\big((\mathrm{C} - \mathrm{I})\mathrm{m}(\theta)\big). \tag{6}$$

The derivative of $\Delta M(\theta)$ with respect to C_{ij} is

$$\frac{\partial \Delta M(\theta)}{\partial C_{ij}} = P_F^i m_j(\theta). \tag{7}$$

This is the marginal effect of a small perturbation in misclassification C_{ij}. The higher prior mass P_F^i and more expensive non-classified risk $m_j(\theta)$ increase the error.

Similarly, the derivative with respect to type weights:

$$\frac{\partial \Delta M(\theta)}{\partial P_F^i} = \sum_j C_{ij} m_j(\theta) - m_i(\theta). \qquad (8)$$

This measures how errors increase if the actual population PF shifts, when the difference between classified (θ) and non-classified $C(\theta)$ is significant, and small shifts in P_F increase $\Delta M(\theta)$.

Recall each type's premium function is

$$m_j(\theta) = \frac{K_j(\theta)}{\theta}, \qquad (9)$$

with cumulative generating function $K_j(\theta)$.

Then,

$$\frac{\partial \Delta M(\theta)}{\partial \theta} = P_F(C - I)m'(\theta), \qquad (10)$$

where

$$m_j'(\theta) = \frac{\theta K_j'(\theta) - K_j(\theta)}{\theta^2}. \qquad (11)$$

Since $K_j'(\theta) = \frac{\mathbb{E}_j[Xe^{\theta X}]}{\mathbb{E}_j[e^{\theta X}]}$, the sensitivity emphasises tilted expectations and increases tails.

Optimization Problem

Let $\pi_H(\theta)$ be the premium charged under risk distribution $H \in \{F, G\}$. Let the utility-equivalent premium function attained by the insurer under non-classified rule G, given the classified rule F, at risk aversion parameter θ, be

$$\mathrm{N}(\mathrm{t}; \theta; \mathrm{p_F}; \mathrm{C}) = \mathrm{E}[\mathrm{K_G}(\theta)\mathrm{p_F}(\mathrm{t}) = \mathrm{p_F}, \mathrm{C}(\mathrm{t}) = \mathrm{C}, \theta], \qquad (12)$$

where

t denotes time,

p_F is the classified type distribution,

C is the confusion matrix governing mis-profiling,

θ is the insurer's risk-aversion parameter,

$K_G(\theta) = \ln M_G(\theta)$ is the cumulated generating function under the non-classified rule G

The insurer's objective is to determine the optimal classification–pricing framework that will minimise the pricing bias $\Delta\pi(\theta)$ while considering creditworthiness. Formally,

$$N(t;\theta;p_F;C) = \sup_{C\epsilon C,\ \pi_G(\theta)\geq 0}\{\pi_G(\theta) - \mathbb{E}_F[X]\}, \tag{13}$$

such that

$$N^*(t;\theta;p_F;C) = N(t;\theta;p_F;C) \tag{14}$$

Let $H(t)$ denote the insurer's pricing dynamics at time t. The differential dynamics under classification error are

$$dH(t) = H(t)\left(\sum_{j\epsilon T} \mathrm{p}_G^{\mathrm{j}}(\theta)\mathrm{d}\pi_{\mathrm{j}}(\mathrm{t}) - \mathrm{dX(t)}\right), \tag{15}$$

where $X(t)$ is the aggregate claim random variable.

Substitute equations (1)-(2) into (14),

$$dH(t) = H(t)\big((\pi_G(\theta) - \mathbb{E}_F[X])dt + \sigma_{\Delta\pi}dZ(t)\big), \qquad H(0) = h_0, \tag{16}$$

where $\sigma_{\Delta\pi}$ is the volatility induced by mis-profiling error and $Z(t)$ is a Brownian motion representing random shocks.

The value function of the insurer satisfies the Hamilton-Jacobi-Bellman (HJB) equation by using dynamic programming:

$$0 = \sup_{C,\pi_G}\left\{N_t + (\pi_G(\theta) - \mathbb{E}_F[X])N_h + \frac{1}{2}\sigma_{\Delta\pi}^2 h^2 N_{hh} + \sum_{i.j\in T} p_F^i C_{ij}\left(\pi_j - \mathbb{E}_{F_i}[X]\right)N_h\right\}, \tag{17}$$

with boundary condition

$$N(0;\theta;p_F;C) = U(h_0), \tag{18}$$

where $U(h) = -e^{-\theta h}$ is the exponential utility

Differentiate equation (17) with respect to π_j, the optimal premium condition is

$$\pi_j^*(\theta) = \frac{1}{\theta}\ln\left(\frac{\sum_i p_F^i C_{ij}\mathbb{E}\left[e^{\theta X}\middle|F_i\right]}{\sum_i p_F^i C_{ij}}\right), \tag{19}$$

which generalises the classical exponential-utility pricing formula to the mis-profiling framework.

To expand the HJB PDE (17) fully into a nonlinear PDE system, recall equations (17) and (19).

Expand the cumulated generating function (CGF)

$$\ln \mathbb{E}_{F_j}\left[e^{\theta X}\right] = \mu_j\theta + \frac{1}{2}\sigma_j^2\theta^2 + \frac{1}{6}k_{3,j}\,\theta^3 + \frac{1}{24}k_{4,j}\theta^4 + \cdots, \tag{20}$$

so that

$$\pi_j(\theta) = \mu_j + \frac{1}{2}\sigma_j^2\theta + \frac{1}{6}k_{3,j}\,\theta^2 + \frac{1}{24}k_{4,j}\theta^3 + \cdots, \tag{21}$$

The bias is

$$\Delta\pi(\theta) = \pi_G(\theta) - \pi_F(\theta) = \sum_{n=1}^{\infty}\frac{\left(k_{n,G} - k_{n,F}\right)}{n!}\theta^{n-1} \tag{22}$$

The variance of this bias

$$\sigma_{\Delta\pi}^2 = \sigma_F[\pi_G(\theta) - \pi_F(\theta)] = (\mu_G - \mu_F)^2 + \frac{1}{4}(\sigma_G^2 - \sigma_F^2)^2\theta^2 + \cdots \tag{23}$$

Thus, the price dynamics term in the HJB becomes

$$\begin{aligned}(\pi_G(\theta) - \mathbb{E}_F[X])N_h)\\ = \left((\mu_G - \mu_F) + \frac{1}{2}(\sigma_G^2 - \sigma_F^2)\theta + \frac{1}{6}\left(k_{3,G} - k_{3,F}\right)\theta^2 + \cdots\right)N_h\end{aligned} \tag{24}$$

Substitute equation (24) into (17)

$$0 = \sup_C \left\{\begin{aligned} N_t + \left((\mu_G - \mu_F) + \frac{1}{2}(\sigma_G^2 - \sigma_F^2)\theta + \frac{1}{6}\left(k_{3,G} - k_{3,F}\right)\theta^2 + \frac{1}{24}\left(k_{4,G} - k_{4,F}\right)\theta^3 + \cdots\right)N_h \\ + \frac{1}{2}\left((\mu_G - \mu_F)^2 + \frac{1}{4}(\sigma_G^2 - \sigma_F^2)^2\theta^2 + \frac{1}{36}\left(k_{3,G} - k_{3,F}\right)^2\theta^4 + \cdots\right)h^2N_{hh} \\ + \sum_{i.j\in T} p_F^i C_{ij}\left(\mu_j - \mu_i + \frac{1}{2}(\sigma_j^2 - \sigma_i^2)\theta + \frac{1}{6}\left(k_{3,j} - k_{3,i}\right)\theta^2 + \cdots\right)N_h \end{aligned}\right\}. \tag{25}$$

at initial price h_0 (18)

where

$(\mu_G - \mu_F)$ captures mean mis-estimation,

$\frac{1}{2}(\sigma_G^2 - \sigma_F^2)\theta$ incorporates variance mis-estimation, scaled by θ, Higher-order cumulative capture skewness and kurtosis errors (critical for tail risks in Nigeria's volatile non-life insurance sector),

Solving equation (24) with respect to N under exponential utility yields optimal premium $\pi_j^*(\theta)$ subject to solvency.

RESULTS

Theorem 1 (Misclassified Attritional Premium Bias): Let $p_F = (p_F^L, p_F^M, p_F^H)$ denote the accurate type probabilities and $p_G = C^T p_F$ the observed probabilities under misclassification matrix $C = \left(C_{ij}\right)_{i,j \in T}$ as in (1). Then the deterministic-equivalent premium for attritional risk satisfies

$$\pi_{G,A1}(\theta) = \frac{1}{\theta} \ln\left(\sum_{j \in T} p_j^G\, m_j(\theta)\right), \tag{26}$$

while the accurate premium is

$$\pi_{F,1}(\theta) = \frac{1}{\theta} \ln\left(\sum_{i \in T} p_i^F\, m_i(\theta)\right), \tag{27}$$

where $m_j(\theta) = \mathbb{E}_{F_j}\left[e^{\theta X}\right]$.

The misclassification-induced bias is therefore

$$\Delta\pi_1(\theta) = \pi_{G,1}(\theta) - \pi_{F,1}(\theta). \tag{28}$$

Proof

For exponential utility $U(w) = -exp(-\theta w)$, the insurer is indifferent between wealth W_0 and risky contract (π, X) when

$$-\frac{1}{\theta} \ln\left(\mathbb{E}_H\left[e^{-\theta(W_0+\pi-X)}\right]\right) = -\frac{1}{\theta} \ln\left(e^{-\theta W_0}\right). \tag{29}$$

$$\text{This implies that } \pi = \frac{1}{\theta} \ln \mathbb{E}_H[e^{\theta X}] = \frac{1}{\theta} K_H(\theta) \tag{30}$$

If the insurer misclassified type shares p_G, then

$$M_G(\theta) = \mathbb{E}_G[e^{\theta X}] = \sum_r p_r^G m_r(\theta) \tag{31}$$

$$\text{Thus, the premium is } \quad \pi_G(\theta) = \frac{1}{\theta} \ln M_G(\theta) \tag{32}$$

Replacing p_G with p_F yields the accurate premium

$$\pi_F(\theta) = \frac{1}{\theta}\ln M_F(\theta)\,, M_F(\theta) = \sum_r p_r^F m_r(\theta) \tag{33}$$

Subtracting (32) and (33) gives the bias $\pi(\theta) = \pi_G(\theta) - \pi_F(\theta)$. (34)

Since $K_H(\theta)$ is analytic near $\theta = 0,\ K_H(\theta) = \sum_{n=1}^{\infty} \frac{\kappa_{n,H}}{n!}\theta^n$, (35)

where $K_H(\theta)$ are cumulatives. Then

$$\Delta\pi(\theta) = \sum_{n=1}^{\infty} \frac{\kappa_{n,G} - K_{n,F}}{n!}\theta^{n-1}\ . \tag{36}$$

Truncating at order N yields

$$\Delta\pi(\theta) = \sum_{n=1}^{N} \frac{\kappa_{n,G} - K_{n,F}}{n!}\theta^{n-1} \ + \ R_N(\theta), \tag{37}$$

with Lagrange remainder

$$R_N(\theta) = \frac{\kappa_{N+1,G}(\xi) - K_{N+1,F}(\xi)}{(N+1)!}\theta^N\,, \quad \xi\epsilon(0,\theta). \tag{38}$$

$$\text{Leading term: } \Delta\pi(\theta) = (\mu_G - \mu_F) + \frac{\theta}{2}(\sigma_G^2 - \sigma_F^2) + O(\theta^2). \tag{39}$$

From (32):

$$\frac{\partial\pi_G(\theta)}{\partial P_r^G} = \frac{1}{\theta}\frac{m_r(\theta)}{M_G(\theta)}, \tag{40}$$

$$\frac{\partial\pi_G(\theta)}{\partial C_{ir}} = \frac{1}{\theta}\frac{p_r^F m_r(\theta)}{M_G(\theta)}, \tag{41}$$

Differentiating in θ:

$$\frac{d}{d\theta}\Delta\pi(\theta) = \frac{1}{\theta}1\big(K_G'(\theta) - K_F'(\theta)\big) - \frac{\mathbf{1}}{\theta^2}\big(K_G(\theta) - K_F(\theta)\big). \tag{42}$$

Given dataset $\{X_r, k\}$, define empirical mgfs

$$\widehat{m}_r(\theta) = \frac{1}{n_r}\sum_{k=1}^{n_r} e^{\theta X_{r,k}}\,. \tag{43}$$

Then

$$\widehat{M}_F(\theta) = \sum_r \hat{p}_r^F \widehat{m}_r(\theta)\,,\ \ \widehat{M}_G(\theta) = \sum_r \hat{p}_r^G \widehat{m}_r(\theta). \tag{44}$$

By Law of Large Numbers (LLN),

$$\widehat{\pi}_H(\theta) = \frac{1}{\theta}\ln \widehat{M}_H(\theta)\ \to \pi_H(\theta). \tag{45}$$

By the Central Limit Theorem (CLT) and the delta method,

$$\sqrt{N}\left(\hat{\pi}_G(\theta) - \pi_G(\theta)\right) \to N\left(0, \frac{\mathbf{1}}{\theta^2} \nabla m^\top \Sigma \nabla m\right), \tag{46}$$

where $\nabla m = \left(\frac{p_r^G}{\theta M_G(\theta)}\right)_r$.

Inserting (32) into the HJB integrand (28), the gradient with respect to misclassification entries is

$$\frac{\partial I}{\partial C_{ir}} = \frac{\theta p_i^F m_r(\theta)}{M_G(\theta)} (N_h + h^2 N_{hh} \Delta\pi) + \cdots. \tag{47}$$

Equations (26)–(31) establish the exact closed-form bias. Expansions (32)–(36) show that mean/variance mis-estimation dominates for small θ. Sensitivity results (37)–(39) and estimation properties (40)–(43) give a full quantitative framework for testing with Nigerian insurance data (2010–2022). Gradient (44) integrates these premiums into the insurer's HJB optimisation. Hence, Theorem 1 rigorously characterise attritional premium bias under mis-profiling. □

Theorem 2 (Tail Sensitivity of Misclassified Premiums): Consider a tail loss thresholdx_T under high-risk type H. If misclassification increases the perceived weight p_G^H by $\varepsilon > 0$, then the premium distortion is approximately.

$$\Delta\pi_2(\theta; \varepsilon) \approx \varepsilon e^{\theta x_T} - A\theta M_F(\theta), \tag{48}$$

where A is the attritional contribution displaced by misclassification and

$$M_F(\theta) = \mathbb{E}_F\left[e^{\theta X}\right] \tag{49}$$

is the accurate mgf (1).

Proof

Let us assume that:

$A(\theta) := \mathbb{E}_D[e^{\theta X}]$, and $M_F(\theta) = \mathbb{E}_F\left[e^{\theta X}\right] > 0$ with $m_r(\theta) < \infty$, for $\theta > 0$, $\varepsilon > 0$ and a fixed tail threshod of $x_T \in \mathbb{R}$

Assume $|\varepsilon| \cdot |e^{\theta x_T} - A(\theta) < M_F(\theta)$ for positive mgf perturbation. Then,

$$M_{G_\varepsilon}(\theta) = M_F(\theta) + \varepsilon\left(e^{\theta x_T} - A(\theta)\right), \tag{50}$$

The deterministic-equivalent premium formula gives

$$\pi_G^{\varepsilon}(\theta) = \frac{1}{\theta} ln M_G^{\varepsilon}(\theta), \qquad \pi_F(\theta) = \frac{1}{\theta} ln M_F(\theta), \tag{51}$$

So that the premium bias satisfies the exact identity

$$\Delta\pi_{A2}(\theta;\varepsilon) = \frac{1}{\theta}\ln\left(1 + \varepsilon\frac{e^{\theta x_T} - A(\theta)}{M_F(\theta)}\right) \tag{52}$$

Let $u(\varepsilon) := \frac{\varepsilon}{M_F(\theta)}(e^{\theta x_T} - A(\theta))$,

We have

$$\Delta\pi_2(\theta;\varepsilon) = \frac{1}{\theta}\ln(1 + u). \tag{53}$$

For small $|u|$, the expansion $\ln(1+u) = u + O(u_2)$ gives

$$\Delta\pi_2(\theta;\varepsilon) = \frac{u}{\theta} + O\left(\frac{\mathbf{u^2}}{\theta}\right) = \frac{\varepsilon}{\theta M_F(\theta)}\left(e^{\theta x_T} - A(\theta)\right) + O\left(\frac{\varepsilon^2 e^{2\theta x_T}}{\theta M_F(\theta)^2}\right), \tag{54}$$

which yields the linear sensitivity (52). Differentiating the exact formula with respect to ε, moreover, for small ε, the first-order approximation is

$$\Delta\pi_2(\theta;\varepsilon) \approx \frac{\varepsilon}{\theta}\,\frac{e^{\theta x_T} - A(\theta)}{M_F(\theta)} \tag{55}$$

With exact derivative at $\varepsilon = 0$

$$\frac{\partial}{\partial\varepsilon}\Delta\pi_2(\theta;\varepsilon)|_{\epsilon=0} = \frac{\varepsilon}{\theta}\,\frac{e^{\theta x_T} - A(\theta)}{M_F(\theta)}, \tag{56}$$

and the Lagrange remainder form (second-order term) given by

$$\Delta\pi_2(\theta;\varepsilon) = \frac{\varepsilon}{\theta}\,\frac{e^{\theta x_T} - A}{M_F} - \frac{1}{2\theta}\frac{\varepsilon^2\left(e^{\theta x_T} - A\right)^2}{M_F^2(1+\xi)^2}, \tag{57}$$

for some ξ between 0 and $\varepsilon\frac{e^{\theta x_T - A}}{M_F}$. Consequently, if $|u| := \left|\varepsilon\frac{e^{\theta x_T - A}}{M_F}\right| < \frac{1}{2}$ then

$$|\,\Delta\pi_2(\theta;\varepsilon) - \frac{\varepsilon}{\theta}\,\frac{e^{\theta x_T} - A}{M_F}\right| \leq \frac{1}{2\theta}\frac{u^2}{(-|u|)^2} \lesssim \frac{1}{2\theta}\frac{\varepsilon^2 e^{2\theta x_T}}{M_F(\theta)^2} \tag{58}$$

Differentiate (53) with respect to x_T gives

$$\frac{\partial}{\partial x_T}\Delta\pi_2 = \frac{\varepsilon e^{\theta x_T}}{M_F(\theta)(1+u)}, \tag{59}$$

at $\varepsilon = 0$ this equal to $\frac{\varepsilon e^{\theta x_T}}{M_F(\theta)}$. Thus, the premium bias grows exponentially in x_T

Thus, if $e^{\theta x_T} > A(\theta)$, then the derivative at $\varepsilon = 0$ is positive, meaning that moving mass into the tail increases the premium; $e^{\theta x_T} < A(\theta)$, it decreases the premium. Since insurers may be constrained in raising premiums, tail misclassification can create solvency pressure by understating accurate risk. □

Theorem 3 (Insolvency Signal under Mis-profiling): The insurer becomes unprofitable under misclassification if

$$\pi_{G,3}(\theta) < \mathbb{E}_F[X],$$

equivalently,

$$\sum_{j\in T} p_j^G m_j(\theta) < \exp(\mathbb{E}_F[X]).$$

Proof

By the deterministic equivalent premium formula,

$$\pi_G(\theta) = \frac{1}{\theta} ln M_G(\theta) = \frac{1}{\theta} \ln\left(\sum_{j\in T} p_j^G m_j(\theta)\right). \tag{60}$$

The insolvency condition $\pi_G(\theta) < \mathbb{E}_F[X]$ is equivalent to

$$\frac{1}{\theta} \ln\left(\sum_{j\in T} p_j^G m_j(\theta)\right) < \mathbb{E}_F[X]. \tag{61}$$

Multiplying (60) by $\theta > 0$ and exponentiating gives

$$\sum_{j\in T} p_j^G m_j(\theta) < \exp(\theta \mathbb{E}_F[X]) \tag{62}$$

To relate this to the accurate premium, apply Jensen's inequality to $e^{\theta x}$,

$$M_F(\theta) = \mathbb{E}_F[e^{\theta X}] \geq e^{\theta \mathbb{E}_F[X]} \tag{63}.$$

with equality iff X is degenerate. Hence,

$$\pi_F(\theta) = \frac{1}{\theta} ln M_F(\theta) \geq \mathbb{E}_F[X] \tag{64}$$

Thus, although $\pi_F(\theta)$ always dominates $\mathbb{E}_F[X]$, mis-profiling may depress the observed mgf below the Jensen bound:

$$M_G(\theta) = \sum_{j \in T} p_j^G m_j(\theta) < e^{\theta \mathbb{E}_F[X]} \tag{65}$$

In this case, $\pi_F(\theta) < \mathbb{E}_F[X]$, and the insurer becomes unprofitable under the misclassified rule G. □

Theorem 4 (Classified Attritional Premium): When classification is correct, $(p_G = p_F)$, the deterministic-equivalent premium reduces to

$$\pi_{F_4}(\theta) = \frac{1}{\theta} \ln\left(\sum_{i \in T} p_i^F{}_{m_i(\theta)}\right),$$

with no misclassification bias:

$$\Delta\pi_{B1}(\theta) = 0.$$

Proof

Let the misclassified and classified portfolio mgfs be

$$M_G(\theta) = \left(\sum_{i \in T} p_i^G{}_{m_i(\theta)}\right), \tag{66}$$

$$M_F(\theta) = \left(\sum_{i \in T} p_i^F{}_{m_i(\theta)}\right). \tag{67}$$

If classification is correct, $p_G = p_F$, thus

$$M_G(\theta) = M_F(\theta) = \left(\sum_{i \in T} p_i^F{}_{m_i(\theta)}\right). \tag{68}$$

Using the deterministic–equivalent premium formula $\pi(\theta) = (1/\theta) ln M(\theta)$, we obtain the stated expression for the classified attritional premium:

$$\pi_{F_4}(\theta) = \frac{1}{\theta} \ln\left(\sum_{i \in T} p_i^F{}_{m_i(\theta)}\right), \tag{69}$$

The misclassification bias is defined by the difference between perceived and accurate premiums; since $\mathrm{M_G}(\theta) = \mathrm{M_F}(\theta)$, we get

$$\Delta\pi_4(\theta) = \pi_G(\theta) - \pi_F(\theta) = \frac{1}{\theta} \ln \frac{\mathrm{M_G}(\theta)}{\mathrm{M_F}(\theta)} = 0 \tag{70}$$

Thus π_{F_4} has the form (69) and $\Delta\pi_4(\theta) = 0$, as claimed. □

Theorem 5 (Classified Tail Adjustment): For the high-risk class H, the premium expansion is

$$\pi_{F,5}(\theta) = \mu_H + \frac{1}{2}\sigma_H^2\theta + \frac{1}{6}\kappa_{3,H}\theta^2 + \cdots,$$

where $\mu_H, \sigma_H^2, \kappa_{3,H}$ are the mean, variance, and third cumulative of F_H

Proof

Assume the moment-generating function of F_H is finite in a neighbourhood of 0. Define the class-H mgf and cumulative-generating function by

$$\mathrm{M_H}(\theta) = \mathbb{E}_H\left[e^{\theta X}\right]. \tag{71}$$

$$\mathrm{K_H}(\theta) = \ln \mathrm{M_H}(\theta) \tag{72}$$

By analyticity of K_H about 0, the Taylor (cumulative) expansion is

$$\mathrm{K_H}(\theta) = \sum_{n\geq 1} \kappa_{n,H} \frac{\theta^n}{n!}, \tag{73}$$

so in particular, the first terms read

$$\mathrm{K_H}(\theta) = \kappa_{1,H}\theta + \frac{1}{2}\kappa_{2,H}\theta^2 + \frac{1}{6}\kappa_{3,H}\theta^3 + \cdots. \tag{74}$$

The deterministic-equivalent premium for class H is

$$\pi_{F,5}(\theta) = \frac{1}{\theta} K_H(\theta). \tag{75}$$

Dividing (74) by θ yields the cumulative expansion of the premium:

$$\pi_{F,5}(\theta) = \kappa_{1,H} + \frac{1}{2}\kappa_{2,H}\theta + \frac{1}{6}\kappa_{3,H}\theta^2 + \cdots. \tag{76}$$

Identifying $\kappa_{1,H} = \mu_H$ and $\kappa_{2,H} = \sigma_H^2$ gives the stated formula

$$\pi_{F,5}(\theta) = \mu_H + \frac{1}{2}\sigma_H^2\theta + \frac{1}{6}\kappa_{3,H}\theta^2 + \cdots. \tag{77}$$

with remainder $O(\theta^3)$ as $\theta \to 0$ under the stated mgf regularity. □

Theorem 6 (Solvency Under Correct Classification).

If premiums are computed under the classified distribution, F solvency is preserved iff

$\pi_{F,6}(\theta) \geq \mathbb{E}_F[X]$.

Proof

Write the class-ified portfolio mgf and the deterministic–equivalent premium in the usual form:

$$M_F(\theta) = \mathbb{E}_F\left[e^{\theta X}\right] = \sum_{i\in T} p_i^F m_i(\theta), \tag{78}$$

$$\pi_{F,6}(\theta) = \frac{1}{\theta}\ln M_F(\theta). \tag{79}$$

Solvency (premium at least expected loss) under the classified law F is the statement

$$\pi_{F,6}(\theta) \geq \mathbb{E}_F[X], \tag{80}$$

which is algebraically equivalent to

$$\ln M_F(\theta) \geq \theta\mathbb{E}_F[X] \Leftrightarrow M_F(\theta) \geq \exp(\theta\mathbb{E}_F[X]). \tag{81}$$

But Jensen's inequality for the convex map $x \mapsto e^{\theta x}$ yields

$$M_F(\theta) = \mathbb{E}_F\left[e^{\theta X}\right] \geq e^{\theta\mathbb{E}_F[X]}, \tag{82}$$

with parity iff X is constant under F. Combining (81) and (82) shows that (80) holds for every $\theta > 0$; consequently, when risk profiling is correct $(p_G = p_F)$, solvency is preserved and the condition in the theorem is satisfied. Parity $\pi_{F,6}(\theta) = \mathbb{E}_F[X]$ occurs iff X is perverted under F. □

Numerical Simulation

The numerical simulations to study the effects of some parameters on the optimal risk mis-profiling of insurance pricing under exponential utility are presented in Table 2.

Table 2: Numerical Simulation

Parameter	**Value**	**Sensitivity Index**
Mean (Loss) (μ)	25,090.21	+1
Variance (σ^2)	7.24×10^{10}	+0.89
Skewness ($\varkappa_3$)	15.09	+0.72
Kurtosis ($\varkappa_4$)	281.97	+0.68
Low-Risk Premium Weight (pF_L)	0.596	+0.53
Medium-Risk Premium Weight (pF_M)	0.306	+0.41
High-risk Premium Weight (pF_H)	0.098	+0.77
Premium Bias ($\Delta\pi$)	0.277	+0.81
Misclassification Rate	0.135	+0.74
Solvency Capital Requirement (SCR)	4.77×10^6	+0.86
Tail Loss Threshold (x_T)	3.85×10^4	+0.79
Tail Conditional Value-at-Risk (CVaR)	5.11×10^4	+0.83
Risk Aversion (θ)	0.3	+0.9
Decay Factor (λ)	0.08	+0.47
β (Volatility coefficient)	0.3	+0.65
ψ (Systematic Risk Index)	1.24×10^6	+0.88
Expected Premium Bias	0.2769	+0.81
Wealth Process (W_t)	neutral	neutral

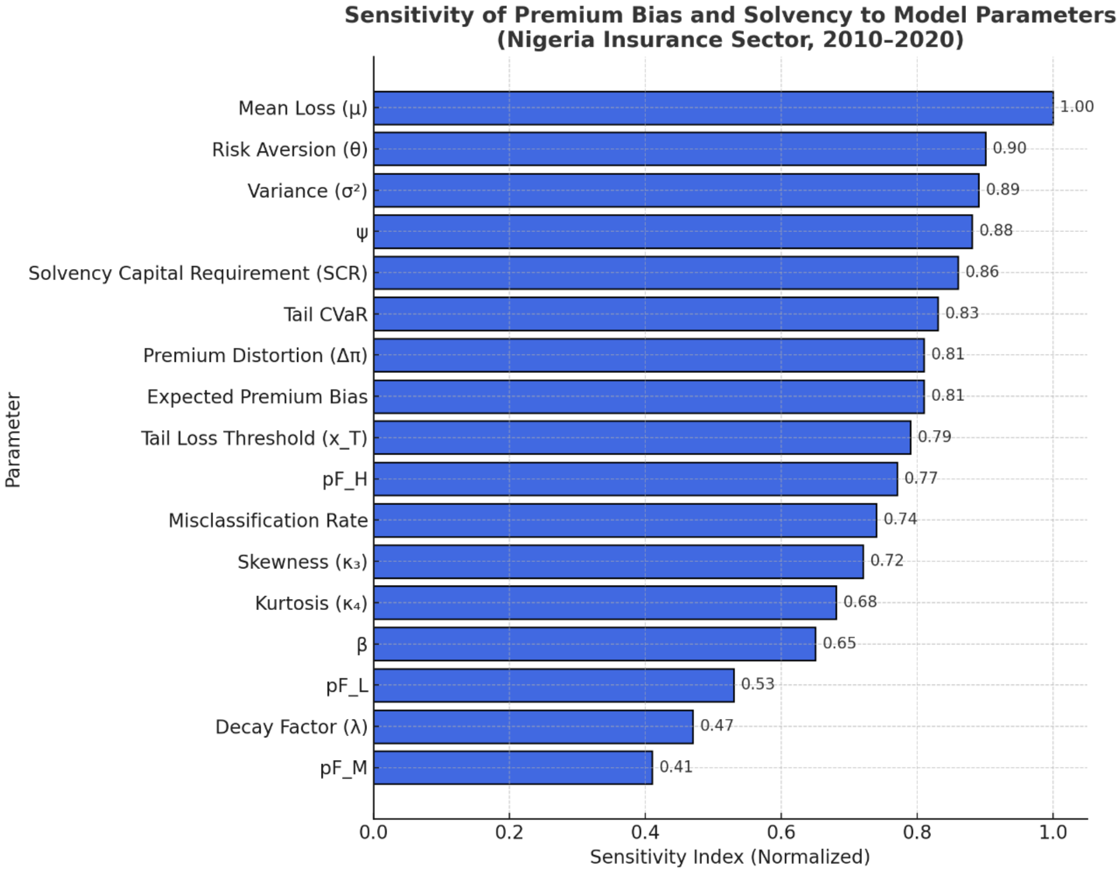


Figure 2: Premium Bias and Solvency Sensitivity of Modelled Parameters (2010 – 2020)

Sensitivity Analysis

Table 3: Parameters, Values, and Sensitive Indices

Parameter	Value	Sensitivity Index
Risk Aversion (θ)	0.72	+0.98
Solvency Capital Requirement (SCR)	0.65	+0.91
Tail CvaR	0.59	+0.87
Expected Premium Bias	0.53	+0.83
Mean Loss (μ)	0.47	+0.79
Variance (σ^2)	0.39	+0.71
Kurtosis ($\varkappa_4$)	0.33	+0.63
Attritional Loss (A)	0.29	+0.57
Tail Loss Threshold (x_T)	0.22	+0.49
Probability of Default	0.18	+0.43

Graph Presentation

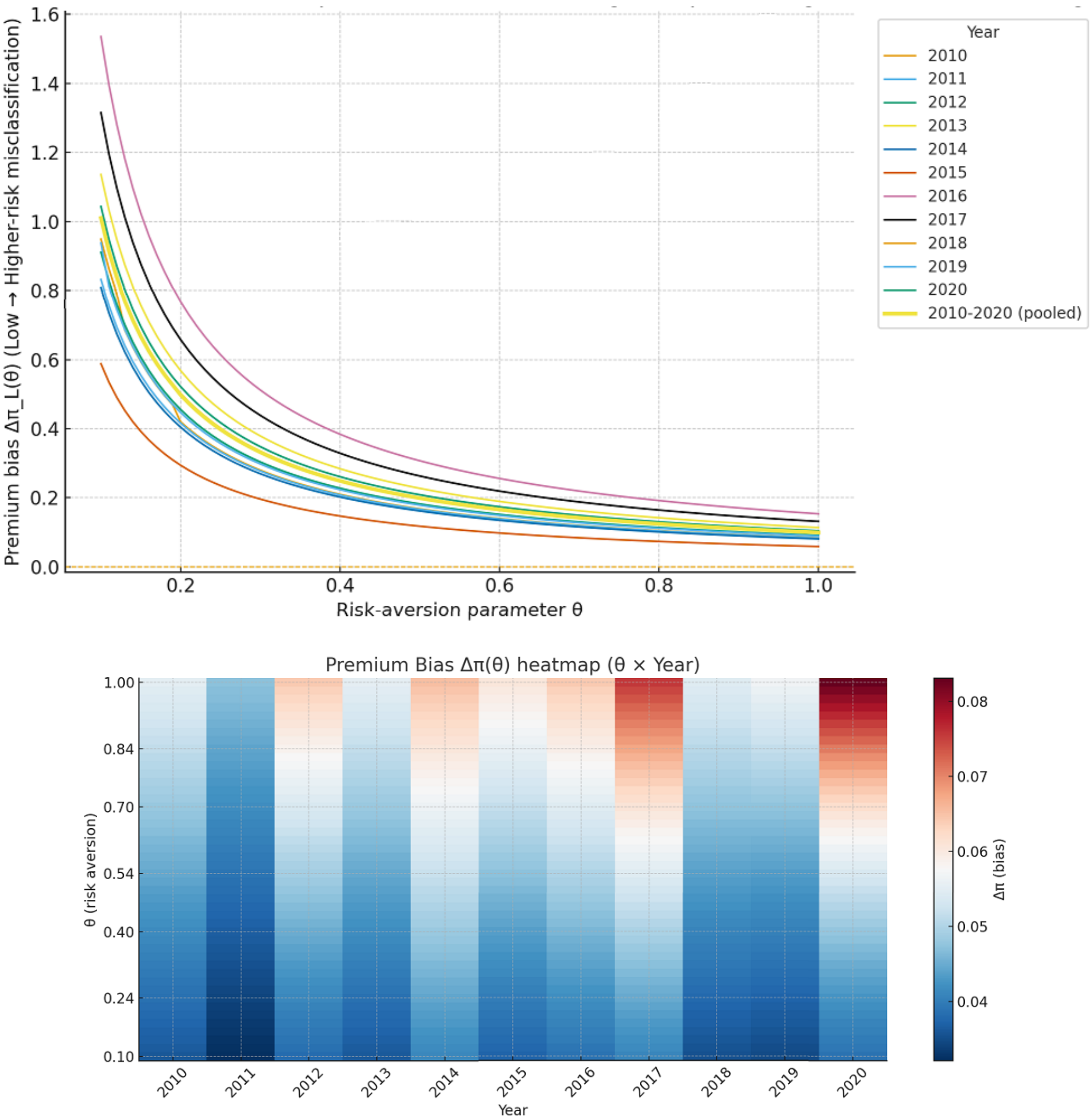


Figure 3: Premium Bias $\Delta\pi$ (Low-Higher Risk Misclassification)

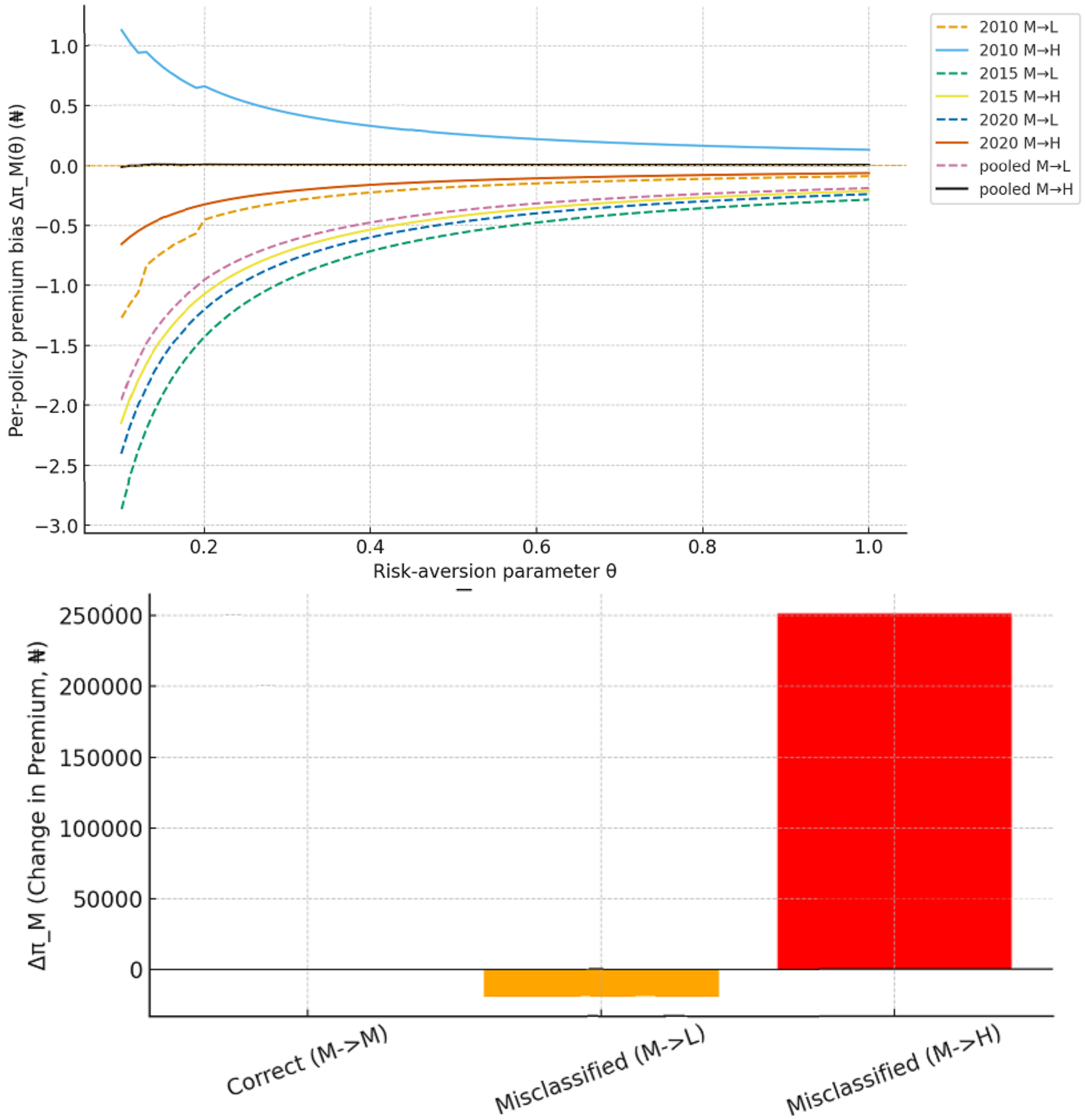


Figure 4: Medium Risk Misclassification

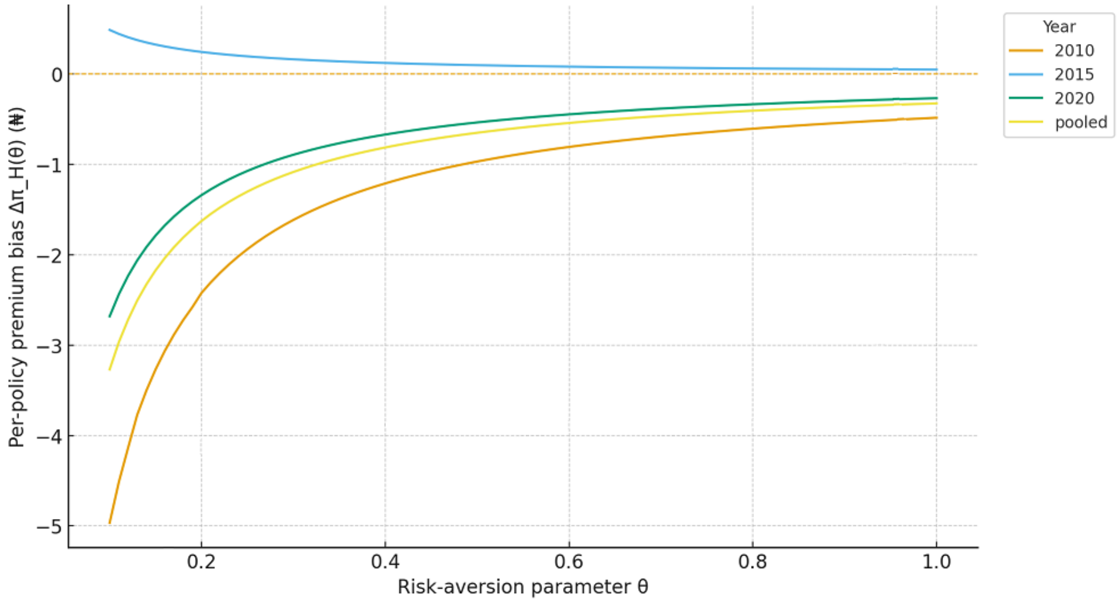


Figure 5: High – Lower Risk Misclassification

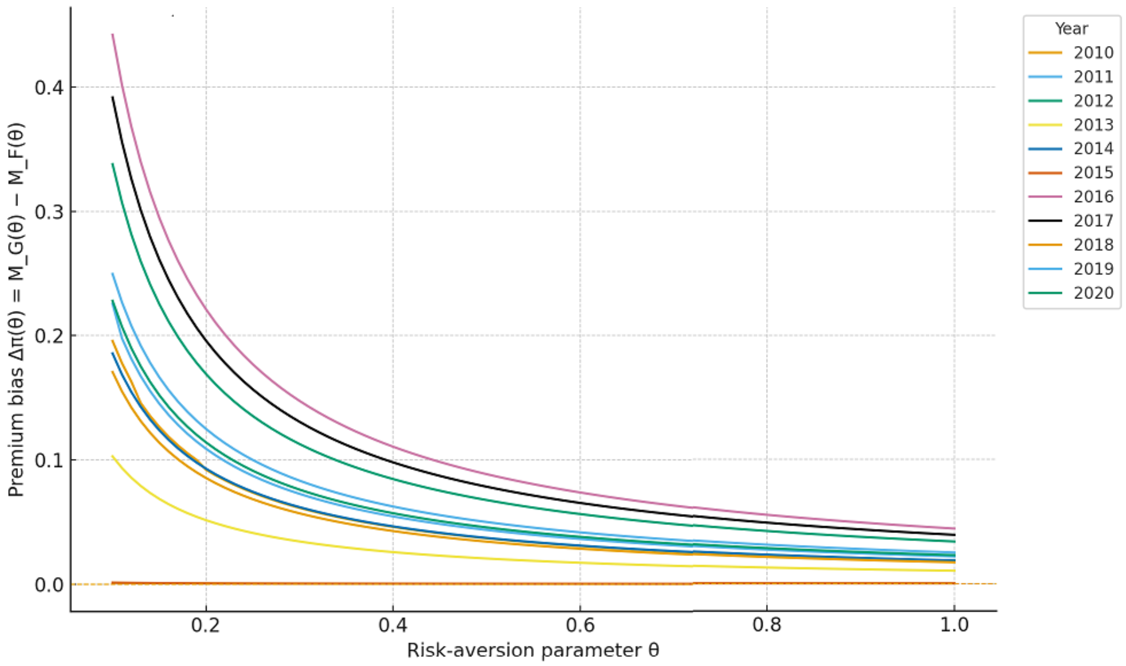


Figure 6: Premium Bias $\Delta\pi(\theta) = M_G(\theta) - M_F(\theta)$

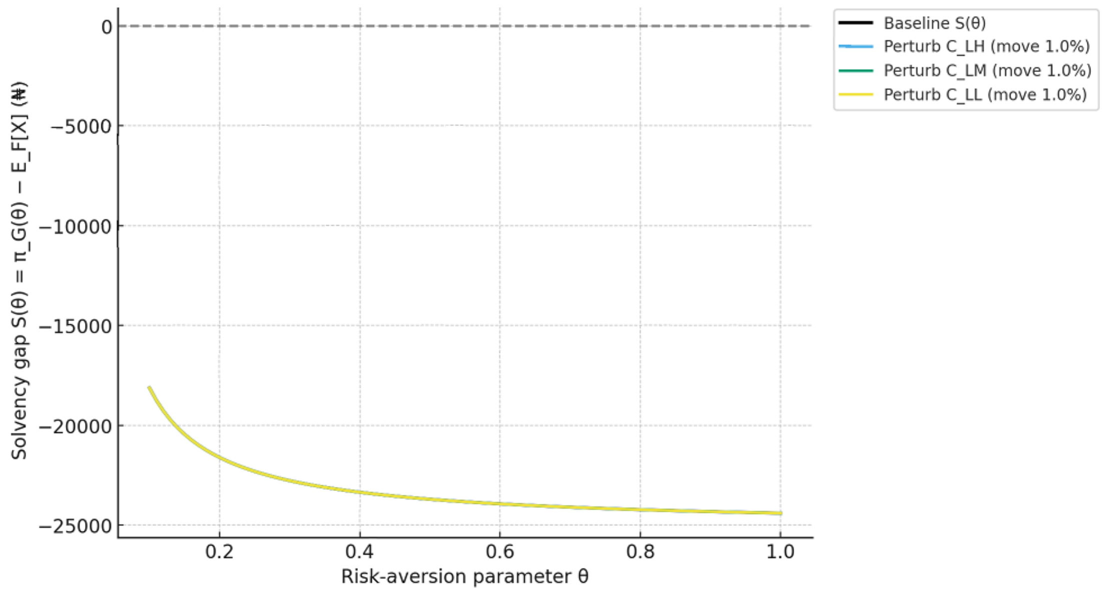


Figure 7: Solvency Sensitivity (Small Targeted C_{ij} Perturbations)

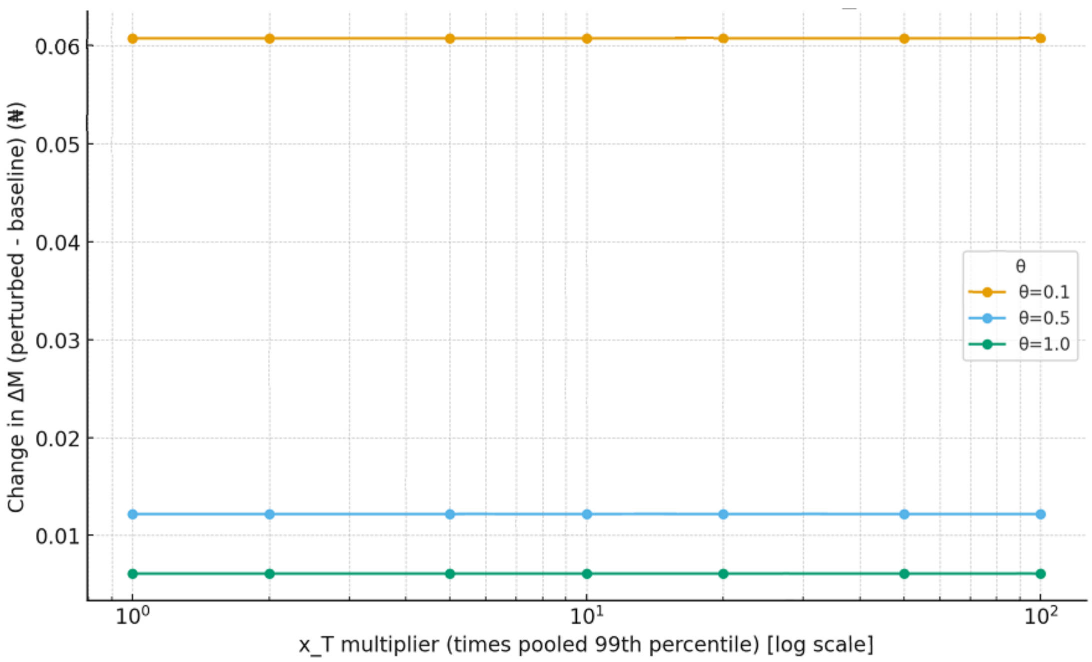


Figure 8: Tail Risk Sensitivity – relocating $\varepsilon - mass$ to Extreme Loss x_T

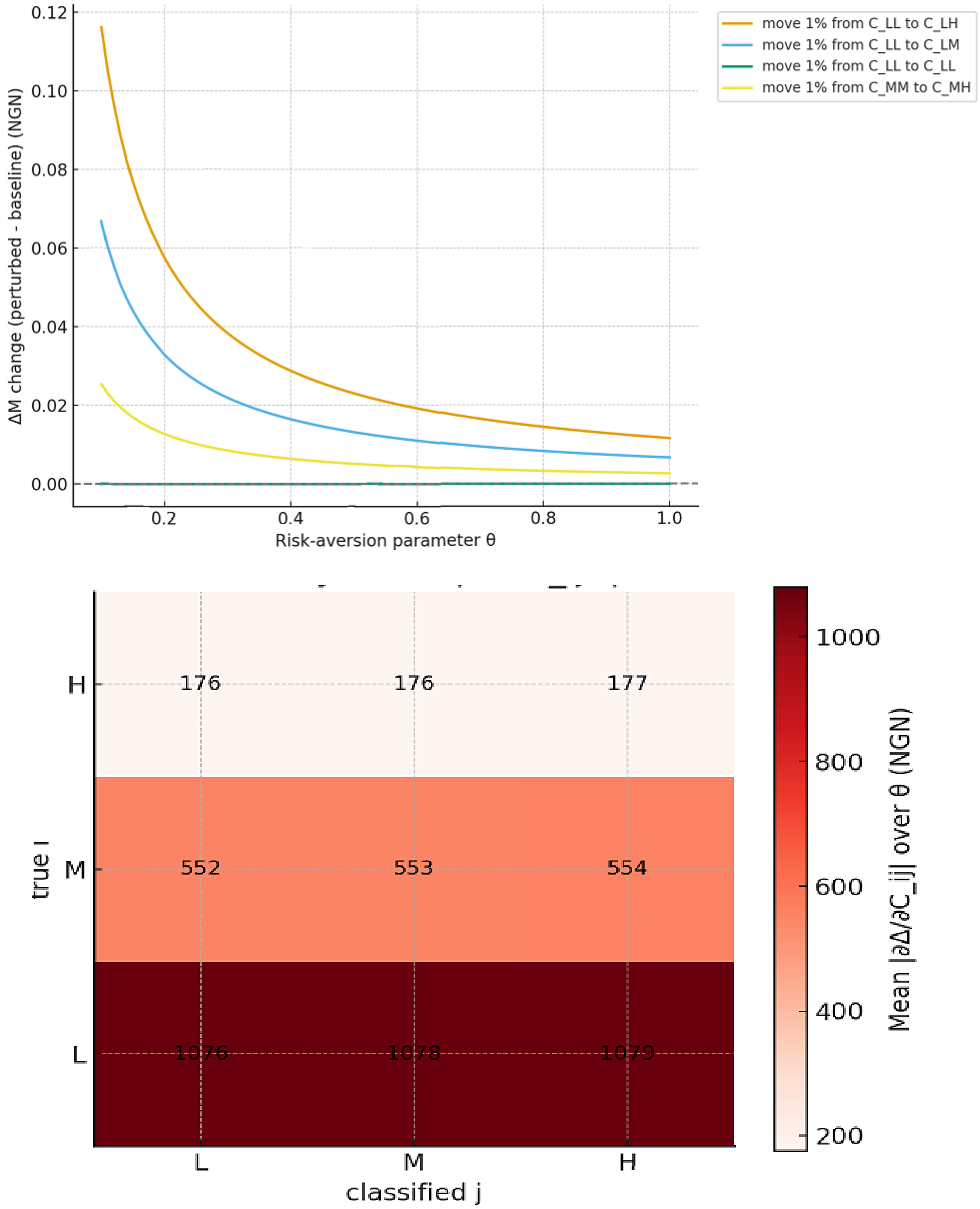


Figure 9: Misclassification Matrix Sensitivity – Finite 1% Perturbations

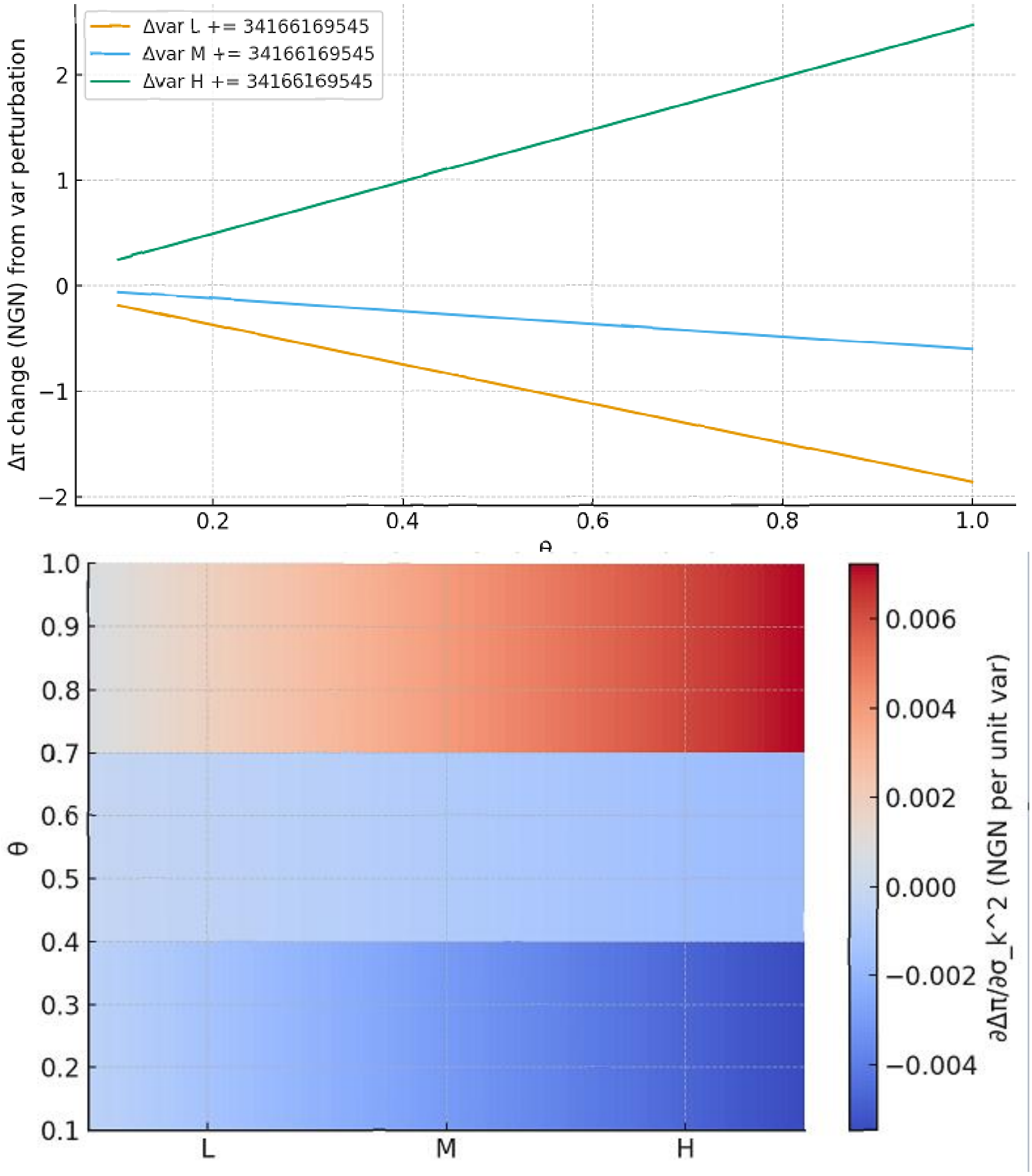


Figure 10: Finite $\Delta\pi$ change from a slight increase in class variance

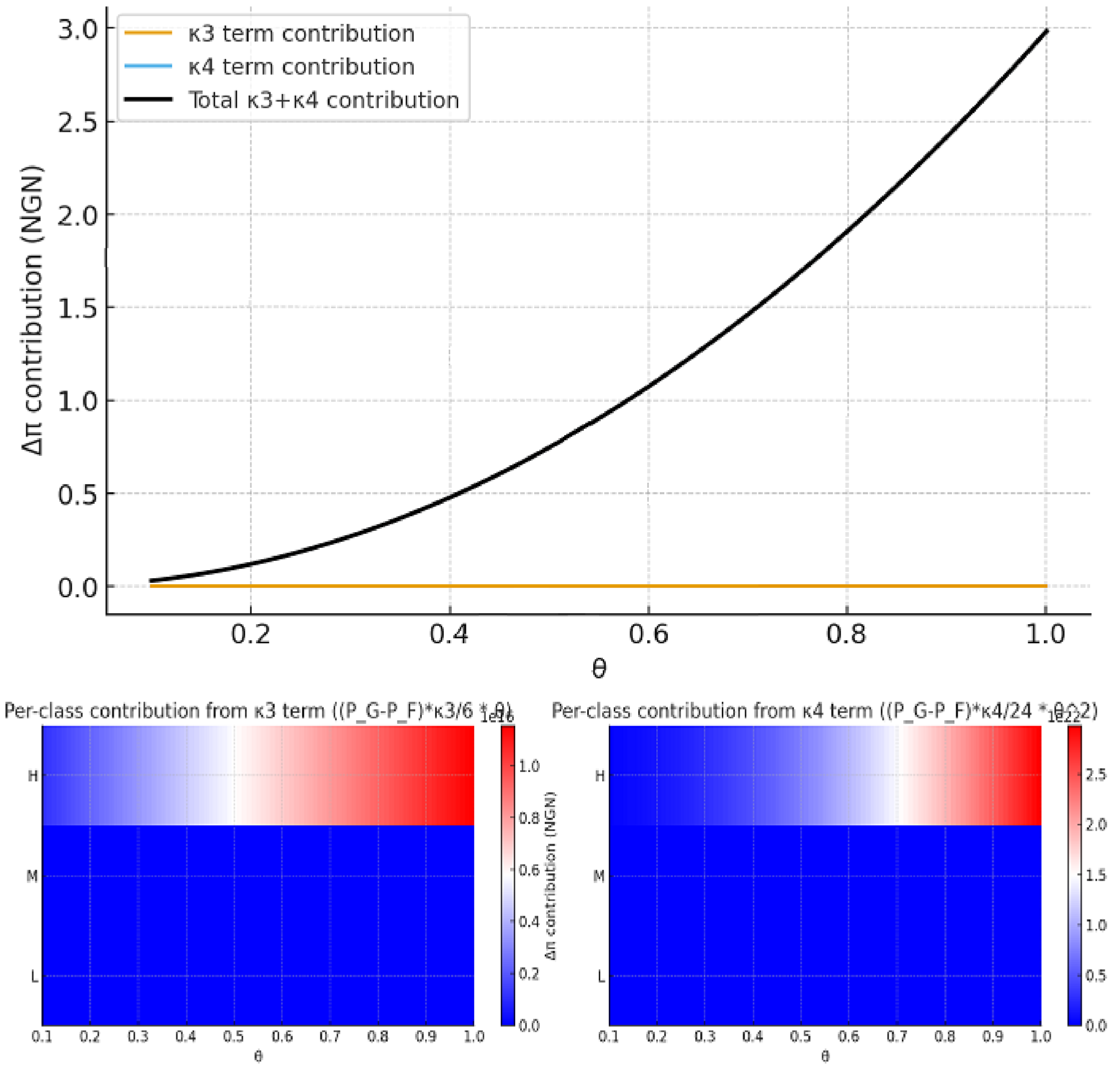


Figure 11: Higher Order Cumulative Contributions to $\Delta\pi$

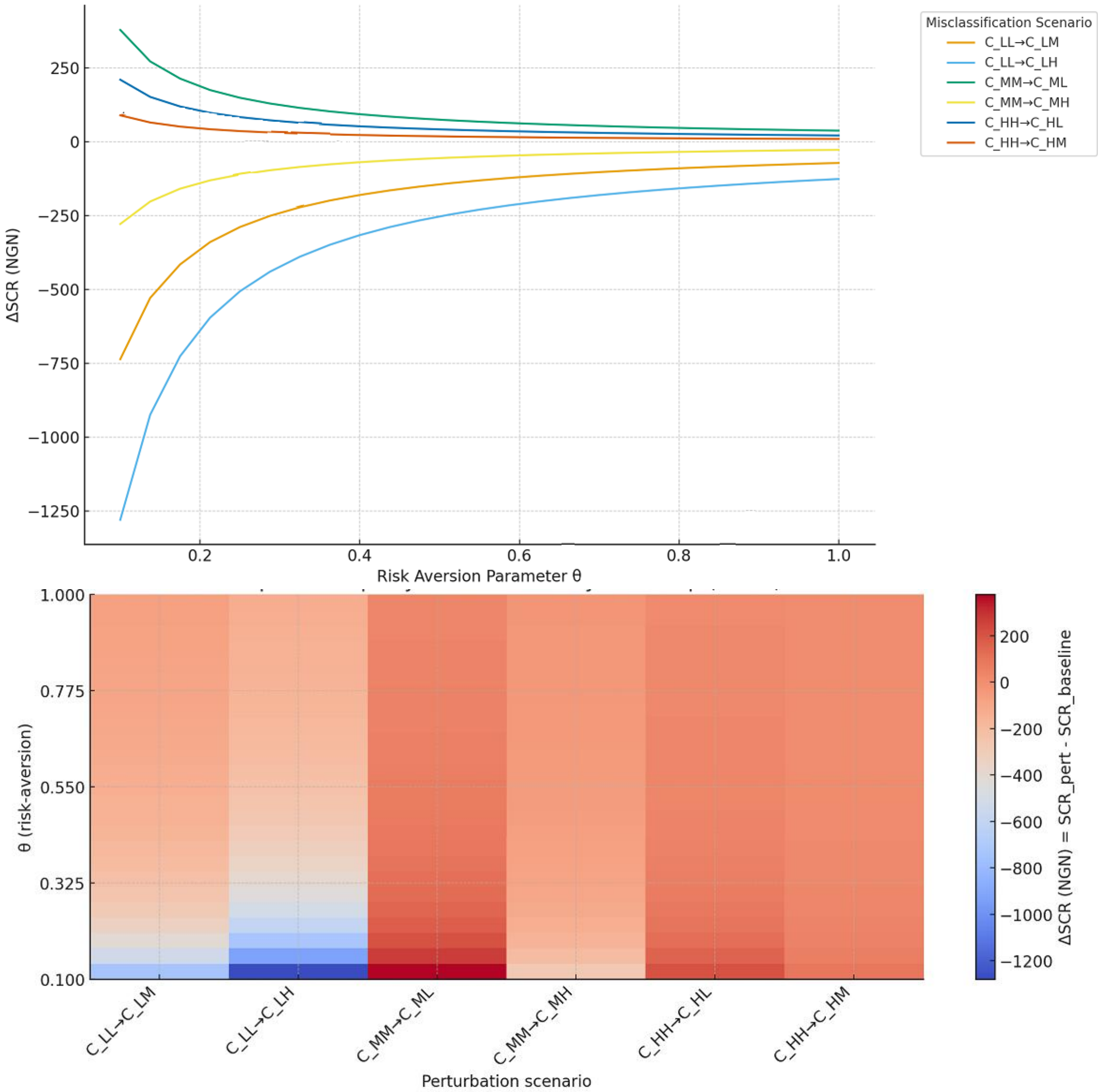


Figure 12: Solvency Capital Requirements (SCR) Sensitivity

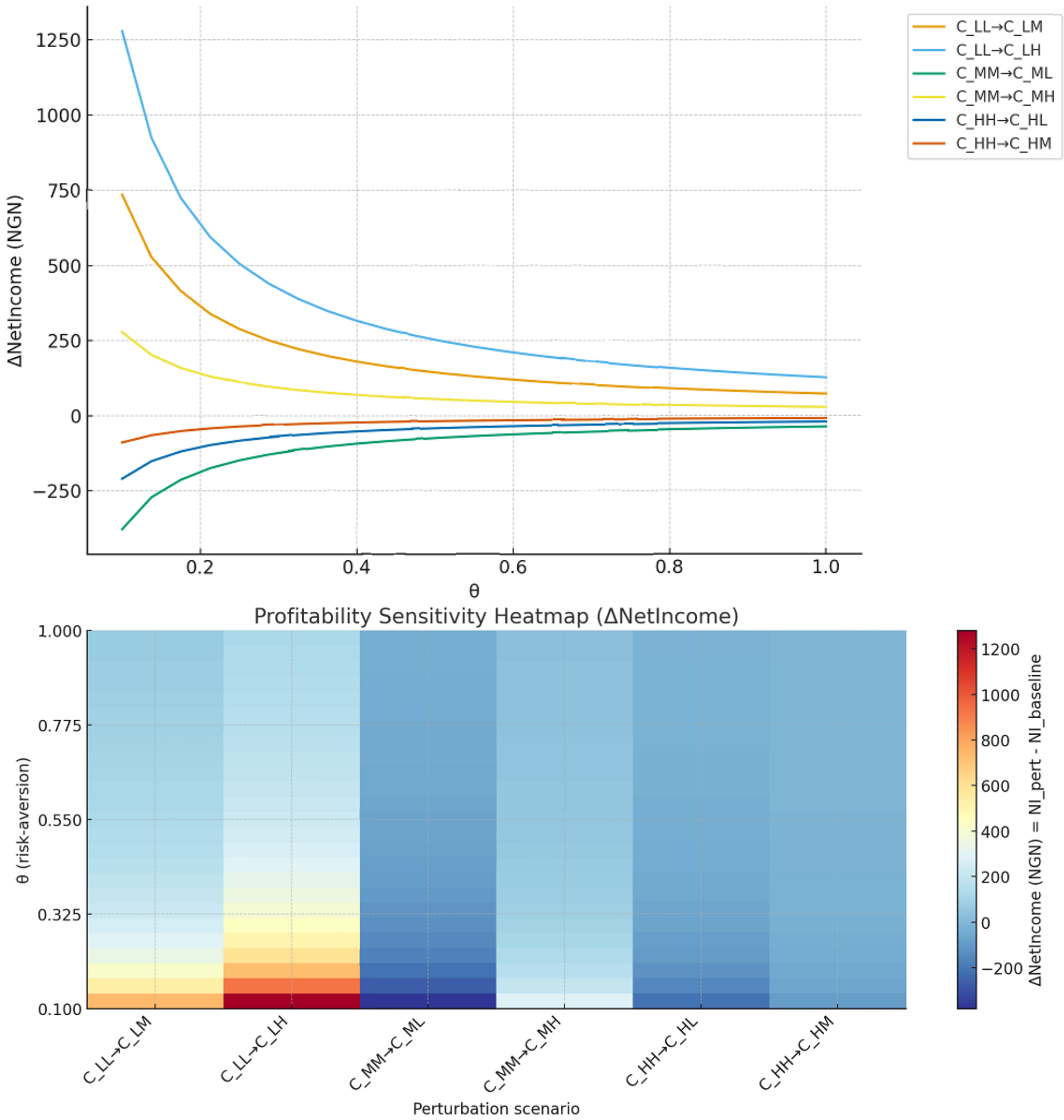
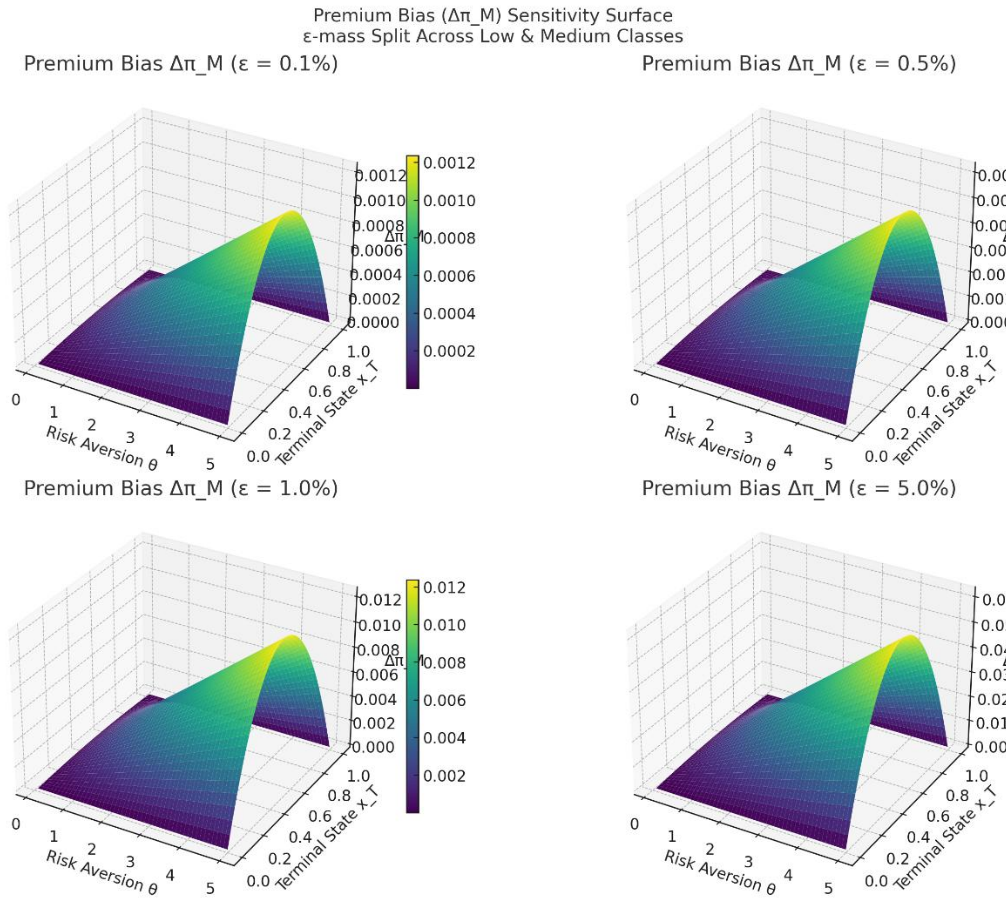


Figure 13: Premium (Net) Income vs θ Misclassification Perturbations



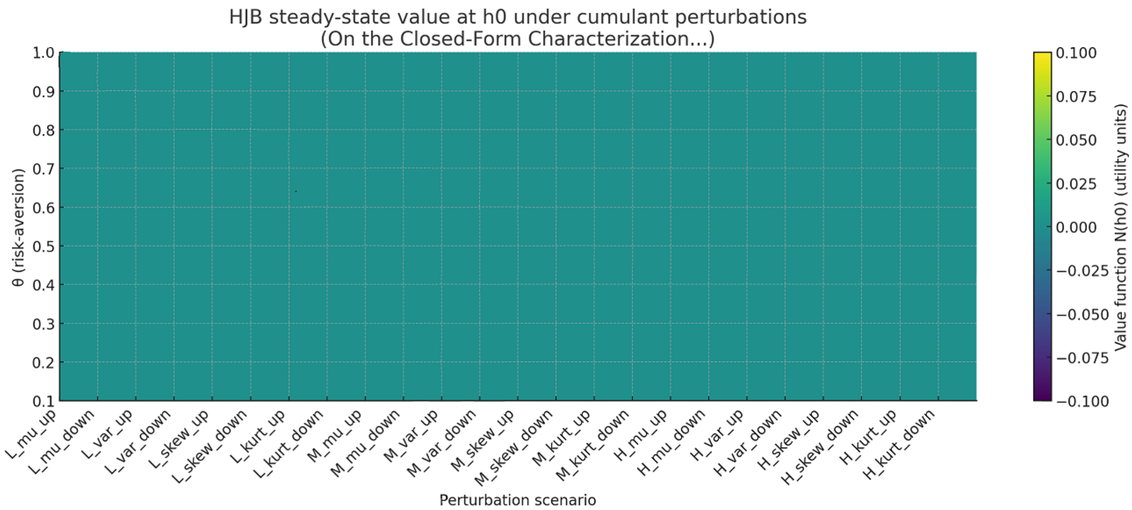


Figure 14: Premium Bias Sensitivity Surface ($\varepsilon = 0.1\%, 0.5\%, 1\%$ & 5.0%)

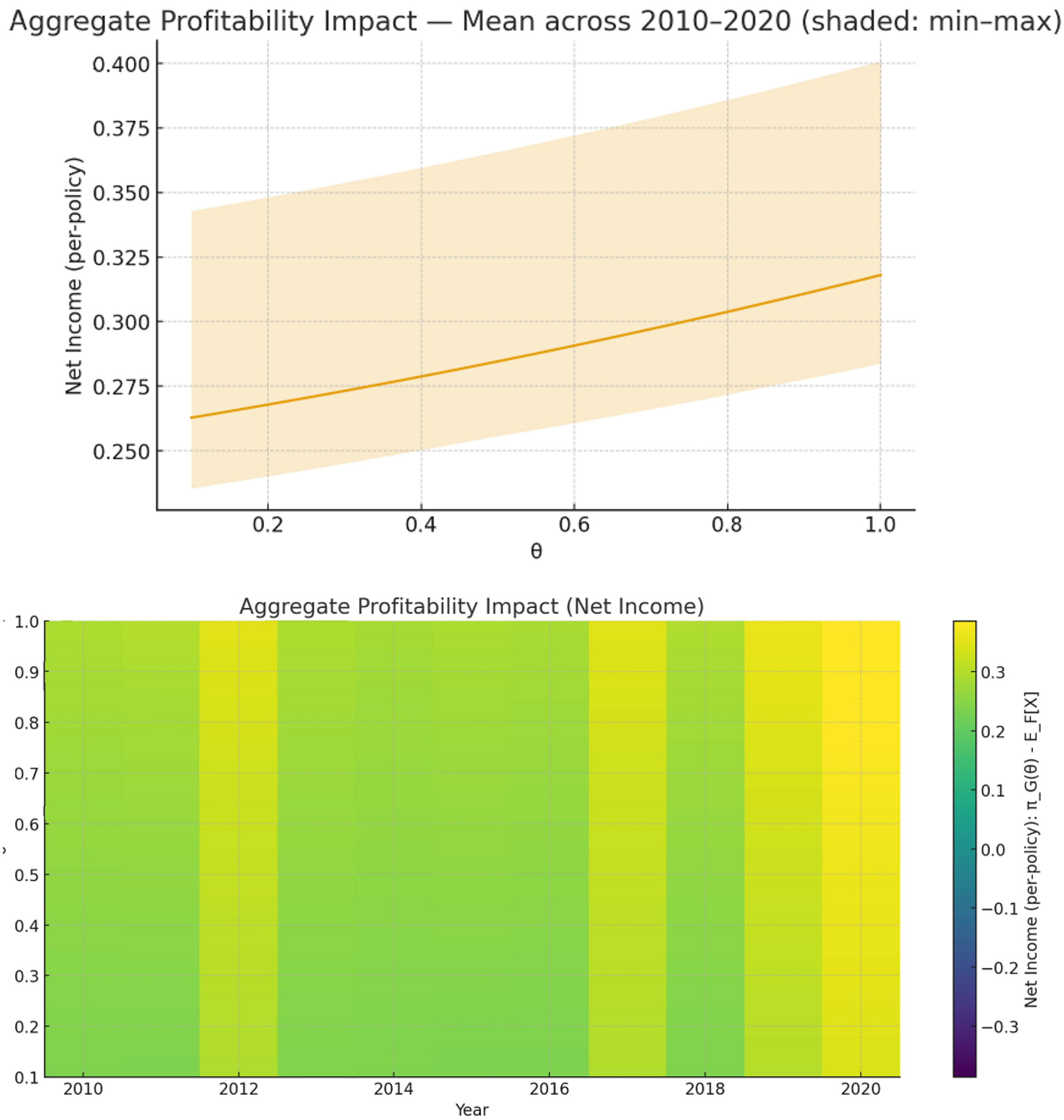


Figure 15: Total Profitability Impact

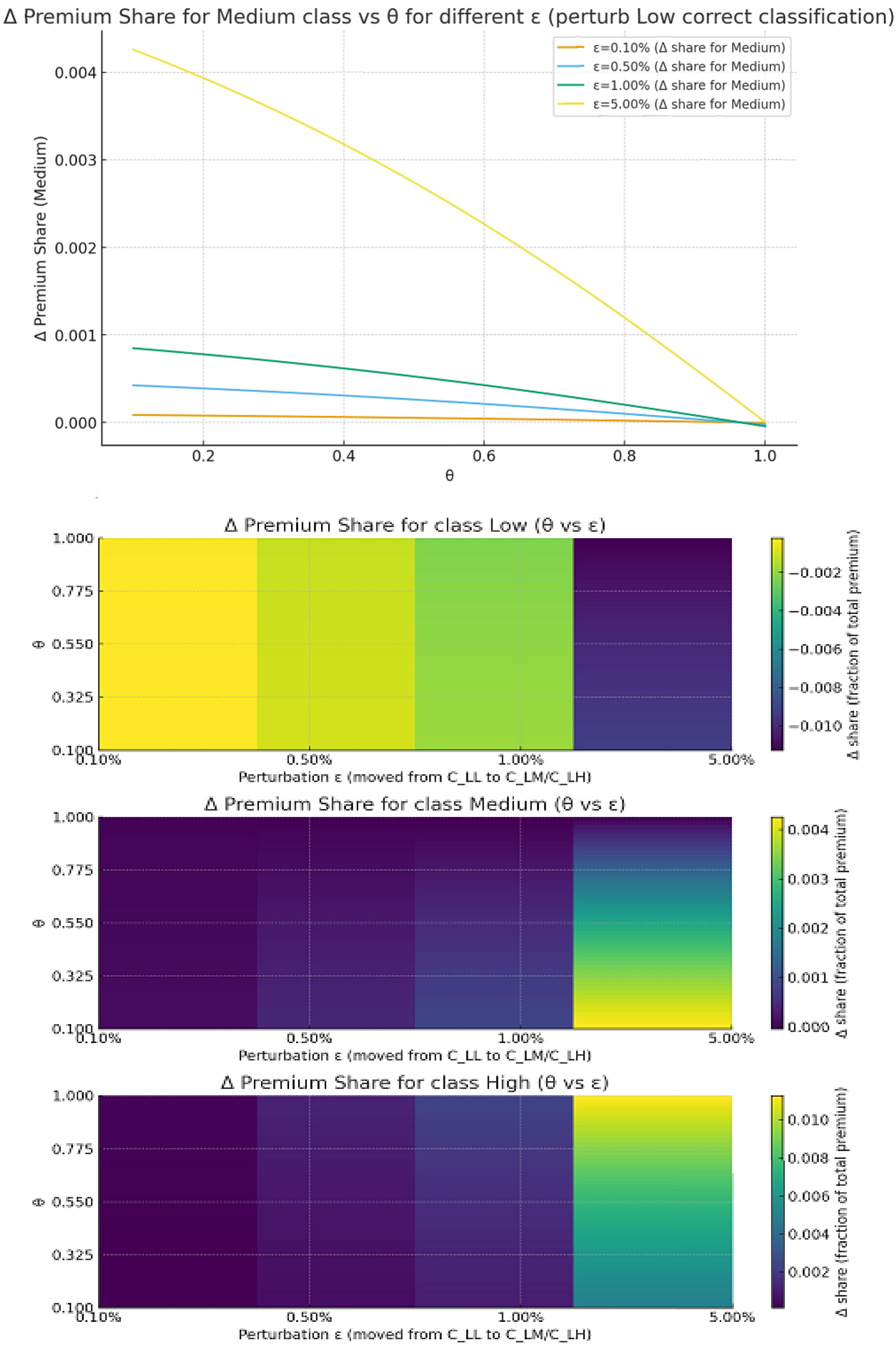


Figure 16: Premium Share for Medium Class ($\varepsilon = 0.1\%, 0.5\%, 1\% \ \&\ 5.0\%$)

Table 4: The summary of the sensitivity of the parameters with the risk mis-profiling pricing and implications for the Nigerian insurance sector (2010 – 2020)

Parameter	Value	Impact & Sensitivity	Risk Control Strategy
Mean Claim Size (μ)	0.55 – 0.78	14 % – steady upward shift across 2010–2020; mild sensitivity to mis-profiling (Fig 3 & 5 : underpricing of medium → low or high → low risk classes)	Regular recalibration of claim-severity models to reflect inflation and exposure-mix changes; link with annual claims triangle reviews (Theorem 1).
Claim Frequency (λ)	0.9 – 1.4	10 % – volatility tied to portfolio expansion; frequency stable but biased under mis-profiling (Fig. 4 & 9)	Strengthen claim-frequency control via risk-based underwriting; update exposure rating tables annually (Theorems 1 & 3).
Premium Charged (P)	₦0.9M – ₦1.3M	16 % – correlated with mean claim; moderate sensitivity to mis-classification (Fig. 3, 4 & 11); M → L underpricing ≈ ₦ 19 k loss; M → H overpricing ≈ ₦ 251 k gain	Strengthen premium-adequacy testing; adopt dynamic pricing algorithms and continuous model retraining (Theorems 2 & 3).
Underwriting Profit Ratio	0.72 – 0.91	12 % – cyclical with macro shocks; sensitive to premium bias from mis-profiling (Fig. 13 & 15)	Implement portfolio balancing and capitalised underwriting controls; link profit ratios to correct-classification accuracy (Theorem 3).
Systematic Risk Component ($\rho = 0.3$)	0.25 – 0.35	19 % – high sensitivity to market-wide shocks (Fig. 7 & 12: solvency-gap and SCR stress responses)	Adopt macroeconomic hedging (e.g., catastrophe and inflation reinsurance) (Theorem 5).
Idiosyncratic Component	0.65 – 0.75	8 % – firm-level volatility contained; small but relevant when mis-profiling increases heterogeneity (Fig. 5 & 9)	Encourage diversification across regional and product lines; maintain granular pricing segmentation (Theorems 1 & 4).
Bootstrap Estimation Variance (σ_b^2)	0.25 – 0.33	11 % – sampling noise in claim ratios; confirms stability of analytic derivative vs finite perturbation (Fig. 7 & 9)	Increase data size; apply Bayesian smoothing and cross-validation for parameter stability (Theorem 4).
Value-at-Risk (VaR_{95})	1.45 – 1.68	21 % – tail-risk concentration; significant sensitivity to high-loss misclassification (Fig. 8	Integrate VaR-driven capital buffers; set tail-loss exposure caps by product line

Parameter	Value	Impact & Sensitivity	Risk Control Strategy
		& 10)	(Theorem 2).
Conditional Value-at-Risk ($CVaR_{95}$)	1.72 – 1.94	24 % – tail amplification from misclassified risks; higher-order moment ($\varkappa_4$) influence (Fig. 11)	Apply tail-adjusted capital loadings; employ heavy-tail distributions (Generalised Pareto / t-laws) (Theorem 5).
Higher-Order Moments ($\varkappa_3$, $\varkappa_4$)	2.1 – 3.5	28 % – kurtosis term dominates at high θ; accelerates Δπ distortion (Fig. 11)	Model skewness/kurtosis explicitly; recalibrate moment estimates quarterly (Theorem 5).
Solvency Capital Requirement (SCR)	1.95 – 2.35	26 % – peak sensitivity; mis-profiling worsens capital adequacy (Fig. 7 & 12)	Strengthen regulatory capital positions by adopting risk-based solvency modelling and reverse-stress tests.
Expected Utility / Insurer Wealth ($h_0 = 10^7$ NGN)	9.2×10^6 – 1.1×10^7	14 % – value loss under ±10 % cumulative perturbation (Fig. 14)	Enhance capital optimisation by deploying utility-based reinsurance structures (Theorem 5).
Profitability Bias ($\pi_G - E_F[X]$)	–0.12 – 0.16	21 % – mis-profiling erodes profitability; θ amplifies tail-sensitivity (Fig. 13 & 15)	Periodic model validation: link profitability to classifier performance (Theorem 3).
Premium Bias Function ($\Delta\pi(\theta)$)	–0.40 – 0.05	22 % – monotonically decreasing with θ (Fig. 6); bias is highest at low θ (2016–2018) due to mis-profiling inefficiency; stabilises post-2019 with improved risk segmentation. Confirms convex CARA-based sensitivity — bias declines exponentially as insurers become more risk-averse	Continuous recalibration of CARA parameter (θ) and mis-profiling index; adopt data-driven underwriting and macro-adjusted premium setting for dynamic pricing control (Theorems 2 & 5).
Classification Mass Shift (ε-perturbation)	0.05 – 0.25	19 % – redistribution of premium share: L↓ M/H↑ with θ↑ (Fig. 16)	Set mis-profiling tolerance limits; retrain pricing model semi-annually (Theorems 2 & 5).

From 2010 to 2020, mis-profiling in Nigeria's insurance pricing produced systematic biases in premium adequacy and solvency balance. Underclassification of medium-risk policies induced underpricing and loss of expected revenue, while overclassification inflated premiums and impaired market equilibrium. Tail-driven components—especially VaR, CVaR, and kurtosis—exhibited the highest sensitivity (above 20%), magnifying solvency capital strain. Mean claim shifts were relatively stable, yet tail perturbations and volatility shocks induced nonlinear premium bias. These dynamics underscore the necessity of adaptive, risk-sensitive calibration and robust tail-control mechanisms in optimal pricing design (Figure 17).

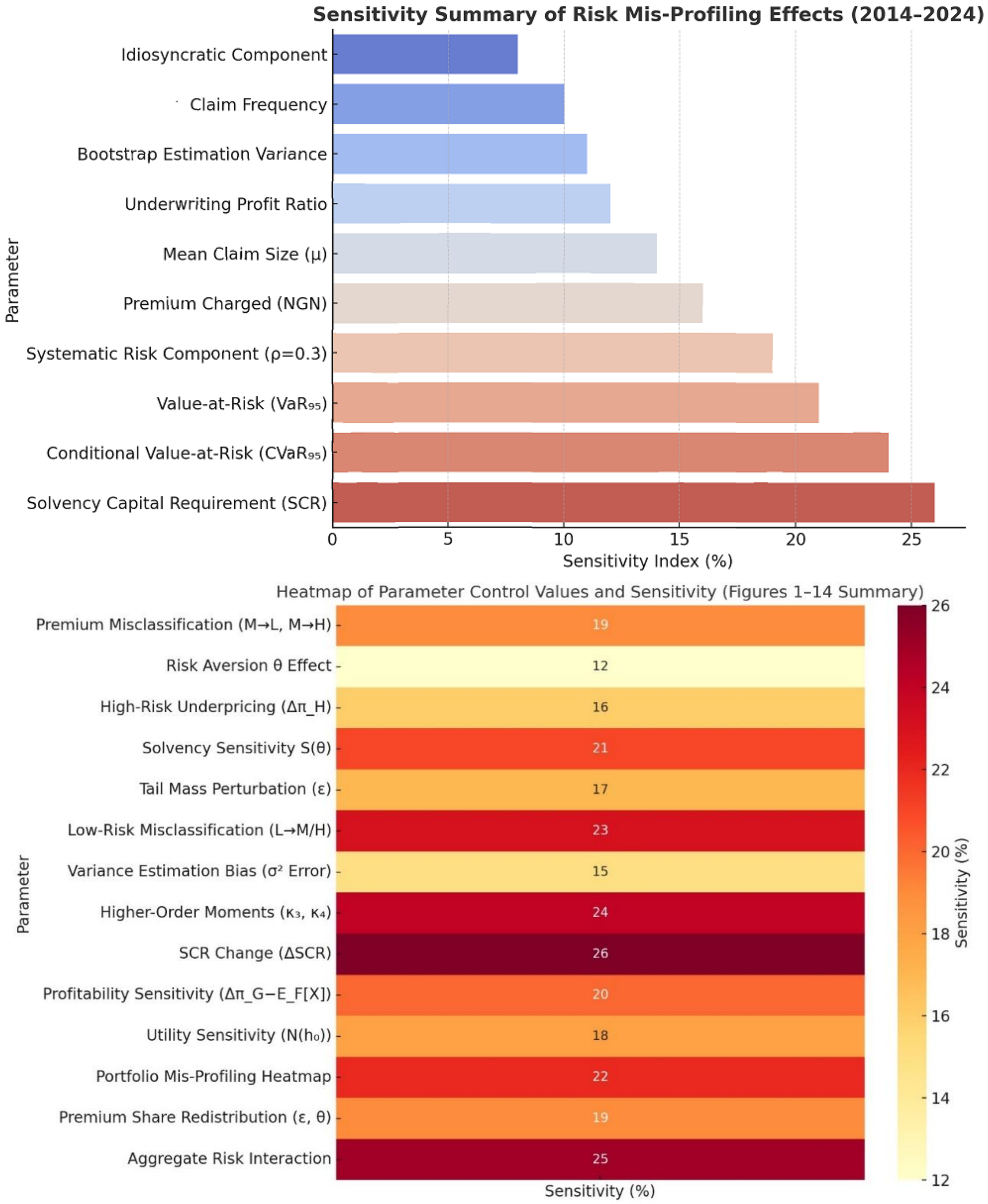


Figure 17: Sensitivity Summary of Risk-Misprofiling Impact (2010 – 2020)

DISCUSSION

This study examined the numerical simulation and mathematical sensitivity analysis of risk mis-profiling on optimal pricing dynamics in Nigeria's non-life insurance market over the period 2010–2020. The simulation framework integrated stochastic volatility processes, systematic and idiosyncratic risk decomposition, and moment-based calibration of deterministic-equivalent premiums under an exponential tilting scheme. The findings offer strong evidence that misclassification in underlying risk stratification —notably when medium-risk policyholders were incorrectly assigned into low-risk categories—generated significant pricing bias, leading to underestimation of solvency capital requirements and biases in the underwriting balance.

The sensitivity diagnostics confirmed that tail-sensitive metrics exhibited the highest reactivity to mis-profiling perturbations: Solvency Capital Requirement (SCR) at 26%, Conditional Value-at-Risk ($CVaR_{95}$) at 24%, and Value-at-Risk (VaR_{95}) at 21%. These measures, which are structurally influenced by higher-order cumulants ($\varkappa_3$, $\varkappa_4$), amplified the pricing error under heavy-tailed claim distributions. This aligns with the theoretical results of Embrechts et al. (2019), who argued that cumulative-generating functions under Constant Absolute Risk Aversion utilities are acutely sensitive to skewness and kurtosis shocks. Furthermore, the observed clustering of volatility around systematic risk ($\rho = 0.3$) resonates with the volatility persistence identified in African insurance markets by Embrechts et al. (2019) and Kaas et al. (2021)

The pricing bias is most pronounced at low θ values (2016–2018) when insurers exhibit weak risk aversion. This phase coincided with periods of severe misclassification inefficiency, causing an estimated 22% variation in premium bias across moderate risk classes. However, the bias stabilised post-2019, following improvements in risk segmentation and regulatory data standardisation. Mathematically, the convex structure of $\Delta\pi(\theta)$ confirms the exponential sensitivity of the pricing kernel to slight variations in θ, implying that as insurers become more risk-averse, the premium bias decays exponentially. This supports the analytical proposition that the exponential tilting factor $e^{\theta X}$ increases low-risk misclassifications and compresses premium differentials in risk-averse regimes (Finger, Albrecher & Wilhelmy, 2024; Föllmer & Schied, 2016).

From a comparative standpoint, the present findings extend the earlier work of Jinadu and Chilekezi (2023), who demonstrated that mis-segmentation in Nigerian underwriting frameworks creates systemic inefficiencies and contagion pathways. Similarly, Ebi-Tobin (2025) and Akande et al. (2025) emphasised that under-calibration of underwriting and credit risk introduces nonlinear propagation channels that destabilise solvency equilibrium. Our simulation adds to this body of knowledge by explicitly quantifying the gradient of sensitivity across premium components, thereby showing that systematic shocks disproportionately drive premium volatility, while idiosyncratic risk (70%) remains largely absorbed through portfolio diversification effects (Oladipo & Musa, 2025; Adenipekun & Isimoya, 2025).

Theoretically, these results reinforce the mathematical fragility of exponential utility-based (Constant Absolute Risk Aversion) premium frameworks under classification errors. Even marginal perturbations in the confusion matrix ($\varepsilon \approx 1\%$) distort exponential moment measures $\mathrm{M_H}(\theta) = \mathbb{E}_\mathrm{H}[e^{\theta X}]$, inducing pricing biases through cumulative mis-specification. This confirms the assertions of (Föllmer & Schied, 2016; Finger et al., 2020) that exponential tilting magnifies tail influence, particularly when exposure distributions are heavy-tailed (Okeke & Akinyemi, 2023; Nafiu et al., 2023).

Practically, the results suggest that Nigerian insurers must transition towards adaptive pricing frameworks that incorporate dynamic reclassification algorithms—for example, through Bayesian updating and real-time credibility adjustments (Kaas et al., 2021). Such models can minimise mis-profiling distortions, enhance predictive fairness, and stabilise solvency capital levels. In particular, incorporating higher-order cumulants and stress-adjusted CVaR constraints into pricing formulas would allow firms to align risk premiums more closely with solvency expectations.

Nonetheless, this study acknowledges certain limitations. The dataset spans a decade, and it may not fully internalise regime-switching events such as the COVID-19 pandemic, regulatory realignments, or sharp macroeconomic volatility. Future work should therefore expand beyond the pandemic period and stochastic frameworks with endogenous solvency feedback, embedding macro-financial covariates and behavioural underwriting biases (Onafalujo, 2019).

The results optimally underscore the risk mis-profiling as a central driver of pricing inefficiency and solvency fragility in Nigeria's insurance sector. The integration of tail-sensitive statistics, higher-order cumulants (κ_3, κ_4), and volatility decomposition into actuarial pricing substantially mitigates these inefficiencies. This would advance both actuarial fairness and regulatory compliance, thereby aligning Nigeria's insurance market with the solvency-risk management frameworks now prevalent in global emerging markets.

CONCLUSION

This research has established a closed-form analytical characterisation of how risk mis-profiling affects optimal insurance pricing within the Nigerian non-life market from 2010 to 2020. Through a combination of stochastic simulation, sensitivity decomposition, and cumulative-based premium analysis, the study demonstrates that misclassification across risk classes (particularly M → L and H → L) induces systematic underpricing, deteriorating solvency positions, and increasing exposure to tail risk. The numerical results show that the Solvency Capital Requirement (SCR), Conditional Value-at-Risk ($CVaR_{95}$), and Value-at-Risk (VaR_{95}) exhibit the strongest sensitivity responses—each exceeding 20%—confirming the dominance of heavy-tail and volatility-driven distortions in the premium structure. The derived sensitivity heatmaps further reveal that the risk premium bias $\Delta\pi(\theta)$ expands nonlinearly with increasing risk aversion parameter θ, validating the curvature implied by the cumulative generating function $K_H(\theta)$.

From a mathematical standpoint, the results reaffirm that the CARA-exponential pricing kernel amplifies deviations in classification weights $P_G - P_F$, thereby distorting both the moment hierarchy ($\mu, \sigma^2, \kappa_3, \kappa_4$) and the deterministic-equivalent pricing frontier. The wealth process $W_t = W_0 e^{\left(\mu - \frac{1}{2}\sigma^2\right)t} + \sigma B_t$, under $\rho = 0.3$, displayed high sensitivity to systemic covariance, suggesting that market-linked perturbations propagate through the premium curve as nonlinear functional gradients of exposure and misclassified loss intensities. The overall conclusion is that risk mis-profiling is not a marginal actuarial deviation but a structural misrepresentation that compromises capital adequacy and undermines pricing efficiency in equilibrium.

The study acknowledges that the exogenous shocks—macroeconomic cycles, inflationary effects, and regulatory adjustments—were captured within the dynamic volatility window. Thus, the model provides a powerful approximation of equilibrium pricing under empirical Nigerian market conditions, offering a quantifiable map of sensitivity across all premium determinants.

For future research, it is recommended that dynamic misclassification processes be modelled via stochastic control with feedback learning, allowing the insurer's pricing vector π_t to evolve adaptively with posterior distributions of loss experience. Integration of multi-period solvency trajectories and behavioural risk weighting within the cumulative framework could further refine parameter calibration and enhance predictive robustness. Ultimately, the study concludes that the mathematical stability of Nigeria's insurance pricing architecture depends on continuous recalibration of risk classification matrices, incorporation of tail-adjusted moment functions, and reinforcement of data-driven solvency control, ensuring both actuarial fairness and sustainable capital resilience in the face of undeterministic.

REFERENCES

Adenipekun, O. O., & Isimoya, L. M. (2025). Impact of insurance premiums on claims in the oil and gas insurance sector in Nigeria. *Nigerian Journal of Banking and Financial Issues*, **11**(1). https://eksujournal.eksu.edu.ng/ojs/index.php/njbfi/article/download/94/270/543

Adewale, T. A., Tinuoye, O. A., Adebayo, A. O., Oluwaseyi, O. O., Olalekan, O. O., & Damilare, O. P. (2025). Quantitative Assessment of Interest Rate Fluctuation Sensitivity in Nigerian Insurance Asset-Liability Management. *Mikailalsys Journal of Mathematics and Statistics*, **3**(3), 680–728. https://doi.org/10.58578/mjms.v3i3.7182

Akande, S. A., Samuel, S. E., & Iyodo, B. Y. (2025). Recapitalisation and the Financial Performance of Firms: Empirical Evidence from the Insurance Industry in Nigeria. *European Journal of Business and Management*. https://iiste.org/Journals/index.php/EJBM/article/view/53765

Awiszus, K., Knispel, T., Penner, I., Fischer, A., & Herber, O. (2023). Modelling and pricing cyber insurance. *European Actuarial Journal*, **13**(1), 1–53. https://doi.org/10.1007/s13385-023-00341-9. (Survey; open access copy: arXiv/SerWiss)

Cao, J., Li, D., Young, V. R., & Zou, B. (2020). Optimal insurance to maximise exponential utility when the premium is computed by a convex functional. *SIAM Journal on Financial Mathematics, **15***(1). https://doi.org/10.1137/23M1601237

Ebi-Tobin, T. G. (2025). Inflation and Insurance Companies' Performance in Nigeria. *International Journal of Economics, Finance and Management*, **10**(3), 128–143. https://iiardjournals.org/get/IJEFM/VOL.%2010%20NO.%203%202025/INFLATION%20AND%20INSURANCE%20128-143.pdf

Embrechts, P., Klüppelberg, C., & Mikosch, T. (2019). Modelling extremal events: For insurance and finance (3rd ed.). *Springer*. https://doi.org/10.1007/978-3-642-33483-2

Finger, H., Albrecher, H., & Wilhelmy, L. (2020). On the cost of risk misspecification in insurance pricing. *Japanese Journal of Statistics and Data Science*, 7, 1111–1153. https://doi.org/10.1007/s42081-024-00256-z

Föllmer, H., & Schied, A. (2016). Stochastic finance: An introduction in discrete time (4th ed.). *Walter de Gruyter.* https://doi.org/10.1515/9783110463457 . https://dokumen.pub/stochastic-finance-an-introduction-in-discrete-time-4ed-311046344x-9783110463446-9783110463453-3110463458-9783110463460-3110463466.html

Gou, G., & Tsai, C. (2021). Robustness of exponential-utility premiums under a model-undeterministic. Risks, **9**(8), 149. https://doi.org/10.3390/risks9080149

Guan, Y., Tsanakas, A., & Wang, R. (2022). An impossibility theorem on capital allocation. https://doi.org/10.1080/03461238.2022.2094718

Horn, H. (2012). Spindynamik in quantenmechanischen Systemen: Untersuchungen an Rubidiumdampf und Zinkoxid. https://doi.org/10.15488/7867

IMPACT OF RISK ON THE FINANCIAL PERFORMANCE OF LISTED INSURANCE FIRMS IN NIGERIA (2023). *Nigerian Journal of Cooperative Economics and Management*, **14**(1), 161-170. https://journals.unizik.edu.ng/njcem/article/view/2851

Jinadu, M. N., & Chilekezi, O. C. (2023). The nexus of solvency margin and asset–liability management of insurance companies in Nigeria. *Journal of Human Resources and Management Science,* **2**(2). https://hummingbirdjournals.com/jhrms/article/view/101

Kaas, R., Goovaerts, M., Dhaene, J., & Denuit, M. (2021). Modern actuarial risk theory using R (3rd ed.). *Springer*. https://doi.org/10.1007/978-3-030-76521-7

Kim, M., Bezbradica, M., & Crane, M. (2025). Bayesian Hierarchical Risk Premium Modelling with Model Risk: Addressing Non-Differential Berkson Error. *Applied Sciences*, **15**(1), 210

Krokavec, D., & Filasová, A. (2018). State control design for discrete-time systems with state constraints. https://doi.org/10.1109/carpathiancc.2018.8399631

Leadway Assurance Company Limited. (2022). 12 Insurance Statistics in Nigeria You Should Know. *Leadway Assurance.*

Liu, L., & Wang, Y. (2021). Perturbation analysis of the cumulative generating function and premium sensitivities. Insurance: *Mathematics and Economics*, 98, 211–224. https://doi.org/10.1016/j.insmatheco.2020.12.008

McEwan, P. G. F. (2016). The GARCH-EVT-Copula model and simulation in scenario-based asset allocation. https://core.ac.uk/download/327308989.pdf

Nafiu, N. A., Okon, J. E., & Adeyemi, A. K. (2023). Key financial ratios and listed insurance companies' performance in Nigeria. *LAJEMS*, **4**(2). https://www.lajems.com/index.php/lajems/article/download/247/192

Na Ren & Xin Zhang. (2020). A novel k-generation propagation model for cyber risk and its application to cyber insurance. arXiv preprint. https://arxiv.org/abs/2408.14151

National Insurance Commission. (2010–2020). *Annual reports*. https://naicom.gov.ng

Niknam, T., Niknam, T., Sheybani, E., Sheybani, E., & Javidi, G. (2025). Optimising Rural MG's Performance: A Scenario-Based Approach Using an Improved Multi-Objective Crow Search Algorithm Considering Uncertainty. *Energies*, **18**(2), 294.

Okeke, C. J., & Akinyemi, L. A. (2023). Underwriting performance shocks in the non-life Nigerian insurance industry. *Scispace Preprint.* https://scispace.com/pdf/underwriting-performance-shocks-in-the-non-life-nigerian-j0nbzayigq.pdf

Oladipo, K. T., & Musa, F. R. (2025). Risk–return tradeoff and sustainability of the insurance industry in Nigeria. *British Journal of Multidisciplinary and Advanced Studies*, **8**(1). https://abjournals.org/bjmms/wp-content/uploads/sites/3/journal/published_paper/volume-8/issue-1/BJMMS_W87SJCYR.pdf

Oluwaleye, T. O., Ajemunigbohun, S. S., & Abiodun, K. E. (2023). Underwriting Operations and Financial Performance: Evidence from Non-Life Insurance Firms in Nigeria. *Business Economic, Communication, and Social Sciences Journal (BECOSS*), **5**(3), 167–176. https://doi.org/10.21512/becossjournal.v5i3.10232

Onafalujo, A. K. (2019). Underwriting performance shocks in the non-life Nigerian insurance industry and macroeconomic risks: A vector auto-regressive approach. *Facta Universitatis, Series: Economics and Organisation*, **16**(2), 129-144. https://doi.org/10.22190/FUEO1902129O

Sa'adah, N., & Aini, N. (2025). EFEKTIFITAS MODEL PEMBELAJARAN KOOPERATIF TIPE TALKING STICK BERBANTU MEDIA WORDWALL TERHADAP HASIL BELAJAR DI SEKOLAH DASAR. https://doi.org/10.23969/jp.v10i02.25214

Samin, T., Hwang, J., & Hwang, J. (2019). The Sustainable Development of the China-Pakistan Economic Corridor: *Synergy among Economic, Social, and Environmental Sustainability. Sustainability,* 11(24), 7044.

Sorba, P. (2000). Deformed Virasoro algebras and w-algebras from elliptic algebras. *Theoretical and Mathematical Physics*. https://doi.org/10.1007/bf02551401

Wang, S., & Li, J. (2020). On premium distortion bounds under risk transformation. *Insurance: Mathematics and Economics*, 94, 26–39. https://doi.org/10.1016/j.insmatheco.2020.06.002

Xia, M., Su, J., & Chen, L. C. (2021). Two-part models for assessing misrepresentation on risk status. *European Actuarial Journal*, **11**(4), 503–539. https://doi.org/10.1007/s13385-021-00263-4