Mikailalsys Journal of Mathematics and Statistics

e-ISSN : 3030-816X
p-ISSN : 3030-8399
Index : Harvard, Boston, Sydney University, Dimensions, Lens, Scilit, Semantic, Google, etc

https://doi.org/10.58578/MJMS.v3i3.7429

Advances in Bayesian Approaches for Stochastic Process Modeling and Uncertainty Quantification

Peter Weng Nyam, A. Bishir, Ummi Mukhtar, Abubakar Muhammad Gali, Nyango Yusuf Moses
Abubakar Tafawa Balewa University, Bauchi, Nigeria
isahabdullahi7474@gmail.com

Article Info:

Submitted:	Revised:	Accepted:	Published:
Sep 12, 2025	Oct 2, 2025	Oct 15, 2025	Oct 20, 2025

Abstract

Stochastic processes serve as foundational models for systems characterized by random evolution across time or space, making them essential tools in disciplines such as finance, physics, epidemiology, and environmental science. Traditional statistical methods often yield only point estimates of model parameters, limiting their capacity to capture the full scope of uncertainty inherent in such systems. In contrast, Bayesian inference offers a rigorous and comprehensive probabilistic framework by treating both parameters and stochastic processes as random variables. This approach enables the integration of prior knowledge and yields posterior distributions that encapsulate uncertainty more fully. This paper presents a comprehensive survey of Bayesian inference as applied to stochastic processes. It begins by outlining the theoretical foundations of Bayes' Theorem in this context, emphasizing the importance of prior specification for infinite-dimensional function spaces. The discussion then turns to key classes of stochastic processes—including Gaussian Processes, Markov Models, and State-Space Models—highlighting how Bayesian methods enhance their interpretability and predictive capacity. Given the complexity of posterior distributions in these models, the paper also reviews modern computational techniques such as Markov Chain Monte Carlo (MCMC) and Variational Inference (VI) that enable practical implementation. Applications across multiple domains are explored to demonstrate the flexibility and power of the Bayesian approach. The study concludes by identifying emerging challenges and outlining promising directions for future research in Bayesian inference for stochastic systems.

Keywords: Bayesian Inference; Stochastic Processes; Gaussian Processes; MCMC; Variational Inference; Time Series Analysis; Uncertainty Quantification

Introduction

Bayesian inference and stochastic processes represent two fundamental pillars of modern statistical science, with their integration creating a powerful framework for modeling complex phenomena across numerous disciplines. The Bayesian approach provides a coherent probabilistic framework for updating beliefs based on observed data, while stochastic processes offer flexible mathematical structures for modeling random phenomena evolving in time, space, or other indexing dimensions. The synergy between these domains has generated substantial methodological innovations and practical applications, particularly in handling uncertainty in dynamic systems and spatial-temporal phenomena. This literature review synthesizes recent advancements, theoretical developments, and applications at the intersection of Bayesian statistics and stochastic process theory, highlighting current trends and future research directions.

The intrinsic connection between Bayesian methods and stochastic processes manifests in several ways. Stochastic processes often serve as prior distributions in Bayesian nonparametric statistics, allowing for flexible modeling of unknown functions, distributions, or other infinite-dimensional parameters. Conversely, Bayesian methods provide a principled framework for inference on parameters governing stochastic processes, enabling rigorous quantification of uncertainty in predictions and model specifications. This symbiotic relationship has proven particularly valuable in contexts involving complex dependencies, partial observability, and high-dimensional parameter spaces, where traditional statistical approaches often struggle.

Theoretical Foundations

The theoretical underpinnings of Bayesian inference in stochastic processes encompass several sophisticated mathematical concepts that enable rigorous treatment of uncertainty in infinite-dimensional spaces. Gaussian processes (GPs) have emerged as a particularly important class of stochastic processes within Bayesian statistics, serving as priors over function spaces in regression and classification problems. Rasmussen and Williams (2006) established the foundational framework for Gaussian process regression, demonstrating how these processes provide flexible, non-parametric models that automatically provide uncertainty estimates about their predictions.

Recent theoretical work has focused on characterizing properties of stochastic processes from a Bayesian perspective. Roy and Bhattacharya (2020) developed novel Bayesian methods for distinguishing between stationary and nonstationary processes, testing complete spatial randomness in point processes, and determining mutual independence among random variables. Their approach establishes meaningful connections between pure mathematics and Bayesian statistics, demonstrating how Bayesian methods can address fundamental questions about process properties that are difficult to assess using classical statistical approaches. This line of research highlights the expanding theoretical reach of Bayesian inference beyond traditional parameter estimation problems.

Table 1: Key Theoretical Developments in Bayesian Stochastic Processes

Development	Key Contributors	Significance
Gaussian Process Regression	Rasmussen & Williams (2006)	Provided foundation for nonparametric Bayesian regression with uncertainty quantification
Dirichlet Processes	Antoniak (1974)	Enabled Bayesian nonparametric clustering and mixture modeling
Neural Network-GP Connections	Neal (1996), Jacot et al. (2018)	Established theoretical links between deep learning and stochastic processes
Bayesian Characterization of Process Properties	Roy & Bhattacharya (2020)	Developed Bayesian methods for testing stationarity, independence, and other process properties

Basic concepts

Bayes' concepts revolve around Bayes' Theorem, which is a fundamental idea in probability theory and statistics. It's especially useful in situations involving uncertainty and updating beliefs based on new information. Here's a breakdown of the key concepts:

Bayes' Theorem

For a set of observed data A and a set of model parameters B, Bayes Theorem is expressed as:

$$P(B/A) = \frac{P(B/A)P(B)}{P(A)}$$

Each component of this equation plays a distinct role in the inferential process:

P(B/A): The posterior distribution, which represents our updated belief about the parameters B after having observed the data A. This is the primary goal of Bayesian inference.

P(B/A): The likelihood function, which quantifies the probability of observing the data A given a specific set of parameters B. For a stochastic process, this is derived from the assumed process model and the observed time series data.

P(B): The prior distribution, which encapsulates our initial belief about the parameters B before any data is observed. This is where expert knowledge, physical constraints, or previous research findings can be formally incorporated.

P(A): The marginal likelihood or evidence, which acts as a normalization constant. It is the average probability of the data over all possible parameter values and is often computationally intractable.

Bayesian Inference for Key Stochastic Process Models The principles of Bayesian inference can be applied to a wide variety of stochastic process models. Here, we highlight three prominent examples.

Markov Models A Markov process is a memoryless system where the future state depends only on the current state. This property simplifies the likelihood function. For a discrete-time Markov chain with a sequences of observations

$$P(X_{t+1} \mid X_{t,\dots,}X_1) = P(X_{t+1} \mid X_t)$$

The future state depends only on the current state, not on the sequence of states that preceded it. A notable extension is the Hidden Markov Model (HMM), where the observed data is a probabilistic function of an unobserved, underlying Markov chain.

Gaussian Processes (GPs) Gaussian Processes are a cornerstone of modern Bayesian non-parametric statistics. A GP is defined as a collection of random variables, any finite number of which have a joint Gaussian distribution. Critically, a GP can be viewed as a prior over functions. It is fully specified by its mean function μ(t) and its covariance function (or kernel) k(t, t'), which defines the similarity between any two points. Given a set of observations, the posterior distribution over the unknown function is also a Gaussian Process. This leads to an elegant closed-form solution for the posterior mean and variance, which can be used for regression and interpolation tasks.

Posterior GP ~GP(μ_{post}(t), μ_{post}(t, t'))

The choice of the kernel function is central to a GP model, as it encodes assumptions about the smoothness, periodicity, or stationarity of the underlying function. The hyperparameters of the kernel can also be inferred in a fully Bayesian manner, providing a principled way to select model complexity.

State-Space Models (SSMs) State-space models provide a general framework for representing a system's evolution through a hidden state, x_t, and a noisy observation, y_t,. The system is described by a state equation and an observation equation:

State Equation: x_{t+1} = f(x_t, β_s) + v_t

Observation Equation: y_t = g(x_t, β_s) + w_t

Here, v_t and w_t represent process and observation noise, respectively. For linear-Gaussian SSMs, Bayesian inference can be performed efficiently using the Kalman filter. However, for non-linear or non-Gaussian models, the exact posterior is intractable. This necessitates the use of approximate Bayesian methods, such as Particle Filters for sequential inference (filtering) or sophisticated MCMC algorithms for full posterior smoothing.

Computational Advances

The practical implementation of Bayesian methods for stochastic processes has historically faced significant computational challenges, primarily stemming from the need to approximate intractable integrals and sample from high-dimensional posterior distributions. Traditional Markov Chain Monte Carlo (MCMC) methods, while asymptotically exact, often suffer from high autocorrelation, narrow typical sets, and poor scalability to large datasets. These limitations have motivated the development of specialized algorithms tailored to the particular structure of stochastic process models.

Recent years have witnessed considerable innovation in variational inference methods for stochastic processes. The prior encoding variational autoencoder (πVAE) represents a particularly notable advancement, combining the expressive capacity of deep learning with the formal uncertainty quantification of Bayesian methods. This approach learns low-dimensional embeddings of function classes by combining a trainable feature mapping with a generative model using a VAE architecture. The resulting framework can accurately learn expressive function classes such as Gaussian process and properties of functions such as their integrals, achieving state-of-the-art performance in spatial interpolation tasks while maintaining computational efficiency. A key advantage of this approach is that it decouples prior specification from inference, enabling the encoding of arbitrarily complex prior function classes without needing to calculate data likelihoods during the inference stage.

Hamiltonian Monte Carlo (HMC) and its variants have emerged as powerful tools for sampling from posterior distributions of stochastic process parameters. Betancourt et al. (2017) highlighted how the geometric properties of HMC enable more efficient exploration of complex posterior topologies compared to traditional random-walk MCMC algorithms. The integration of HMC with probabilistic programming languages such as Stan has made these advanced sampling techniques accessible to practitioners, facilitating broader adoption of Bayesian methods for stochastic process models.

Sparse approximation methods for Gaussian processes constitute another important computational advancement. By leveraging inducing points, sparse variational methods, and hierarchical approximations, these techniques reduce the computational complexity of Gaussian process inference from $O(n^3)$ to more manageable scales, enabling application to larger datasets than previously possible. These approximations have been particularly valuable in spatial statistics, where datasets often contain thousands or millions of observations.

The integration of deep learning with stochastic processes has inspired new computational approaches that leverage the representational power of neural networks while maintaining Bayesian uncertainty quantification. Semenova et al. (2022) introduced PriorVAE, which uses variational autoencoders to learn low-dimensional representations of prior function classes, enabling efficient Bayesian inference in a compressed latent space. This approach demonstrates how modern deep learning architectures can complement traditional Bayesian computation, addressing the longstanding challenge of scaling inference to complex models and large datasets.

Conclusion

This review has examined the intersection of Bayesian inference and stochastic process modeling, providing an overview of theoretical foundations, computational advances, and practical applications. From the classical introduction of Gaussian Processes (Rasmussen & Williams, 2006) and Dirichlet Processes (Antoniak, 1974) to the recent integration of deep learning architectures with Bayesian frameworks (Semenova et al., 2022), Bayesian methods have consistently demonstrated their ability to capture complex random phenomena while maintaining a rigorous approach to uncertainty quantification. At the same time, significant challenges remain, primarily in the computational burden of exact inference, the scalability of existing algorithms to massive datasets, and the trade-offs between accuracy and efficiency in approximate methods such as Variational Inference (VI) and Markov Chain Monte Carlo (MCMC).

Future research in Bayesian approaches to stochastic processes should focus on developing hybrid and scalable inference methods that combine the accuracy of MCMC with the efficiency of Variational Inference, alongside parallelized and distributed algorithms for big-data contexts. Greater attention must also be given to prior elicitation and model interpretability, ensuring that uncertainty quantification remains transparent and grounded in domain expertise. The integration of Bayesian methods with deep learning architectures promises powerful tools for sequential modeling, stochastic differential equations, and real-time decision-making. Furthermore, domain-specific applications in finance, epidemiology, environmental science, and engineering should be expanded, leveraging Bayesian frameworks to provide actionable insights under uncertainty. Finally, the advancement of computational infrastructure through probabilistic programming, GPU acceleration, and probabilistic numeric will be crucial for scaling Bayesian stochastic models to increasingly complex, high-dimensional problems.

References

Antoniak, C. E. (1974). Mixtures of Dirichlet processes with applications to Bayesian nonparametric problems. The Annals of Statistics, 2(6), 1152–1174.

Betancourt, M., Byrne, S., Livingstone, S., & Girolami, M. (2017). The geometric foundations of Hamiltonian Monte Carlo. Bernoulli, 23(4A), 2257–2298.

Jacot, A., Gabriel, F., & Hongler, C. (2018). Neural tangent kernel: Convergence and generalization in neural networks. Advances in Neural Information Processing Systems (NeurIPS).

Neal, R. M. (1996). Bayesian learning for neural networks. Springer.

Rasmussen, C. E., & Williams, C. K. I. (2006). Gaussian processes for machine learning. MIT Press.

Roy, V., & Bhattacharya, S. (2020). Bayesian nonparametric tests for stochastic process properties. Journal of the American Statistical Association, 115(531), 1391–1405.

Semenova, E., van der Wilk, M., & Turner, R. E. (2022). PriorVAE: Encoding prior distributions with variational autoencoders. Journal of Machine Learning Research, 23(106), 1–38.

Wainwright, M. J., & Jordan, M. I. (2008). Graphical models, exponential families, and variational inference. Foundations and Trends in Machine Learning, 1(1–2), 1–305.