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On the Comparison of PAR, DARMA, and INAR in Modeling Count Time Series Data

Haruna Buba[1], Ahmed Abdulkadir[2], Kazeem E. Lasisi[3], A. Bishir[4], Strong Yusuf Mashat[5]

[1,2,3,4]Abubakar Tafawa Balewa University Bauchi, Nigeria

[5]Plateau State College of Health Technology Pankshin, Nigeria

harunabmaina@gmail.com; kazeemlasisi@gmail.com

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Abstract

This study evaluates the forecasting and fitting performance of three advanced models—Poisson Autoregressive (PAR), Discrete Autoregressive Moving Average (DARMA), and Integer-Valued Autoregressive (INAR) for count time series data exhibiting complex features such as autocorrelation, overdispersion, and zero inflation. Both simulated and empirical datasets were analyzed, and model performance was assessed using Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Root Mean Square Error (RMSE), and Mean Absolute Error (MAE). The results indicate that PAR models significantly outperform DARMA and INAR models, achieving substantially lower AIC (482.53 vs. >5,310,479) and RMSE (3,742 vs. 246,682), highlighting their robustness in handling periodic trends and autocorrelation. In contrast, standard Poisson regression performs poorly under overdispersion, with an AIC approaching 5.3 million, while zero-inflated datasets compromise error metrics such as MAPE due to division by zero. Although DARMA and INAR models perform comparably, they are less effective in capturing extreme fluctuations or sudden spikes. These findings emphasize the limitations of conventional models and point to the need for more flexible approaches, such as hybrid ZIP-INAR models or Bayesian methods, to effectively manage overdispersion and zero inflation. The study concludes with a practical recommendation to prioritize PAR models when modeling autocorrelated count data.

Keywords: Count Data; Overdispersion; Zero Inflation; PAR Models; DARMA; INAR

Introduction

Count data is generally defined as numbers of events per interval. For count data to be recorded, events need to be specified beforehand, and then recognized and counted during the experiment. The events should tend to occur several times in order for the gathered data to be qualified and identifiable as count. Mathematically, counts are non-negative integers, since an event cannot happen in an incomplete or a negative number of times. Naturally, zero event/count is a possibility; certain entities may even display particularly many null observations, which phenomenon will be addressed later in the research. Technically, there is no upper boundary to a count, because there can theoretically be close to an infinite number of events taking place (Akeyede *et al.,* 2022).

Time series count data, which refers to the number of times an item or an event occurs within a fixed period of time, is essential in many fields (Saleh *et al.,* 2021), these include the number of heart attacks or hospitalization days in medical studies, the number of students absent during a period of time in education research or the number of times parents perpetrate domestic violence against their children in social science investigations. Count data can be found in many practical lifetime studies, such as the number of days before death in certain diseases or the number of cycles (runs) until a machine stops working and so on. Hence, a number of statistical distributions has been applied to model the case of a count random variable (RV) with a non-negative integer value. A good overview of these distributions can be found in (Johnson *et al.,* 2005).

According to Akeyede *et al.,* (2015) over dispersion is a phenomenon that occurs with data fitted using binomial, poisson or negative binomial distribution especially when the estimated value is not near the assumed values or the value became greater than the expected value. However, the problem with overdispersion is the likelihood of underestimation of standard errors of the estimated coefficient vector. Abbott *et al.,* (2020) defined overdispersion as, high individual level variation in the distribution of scores or numbers. Lindsey and Altham, (1998) define overdispersion as the presence of greater variability in a data set that would be expected based on a given statistical model Excess zeros (zero inflation) occur when observed data contain significantly more zero counts than expected under standard discrete distributions (e.g., Poisson or negative binomial). This phenomenon is common in real-world datasets, such as: Daily disease counts (e.g., 69% zero COVID-19 deaths in Ghana; Tawiah, 2021). Rare event records (accidents, equipment failures). Implications of excess zeros are: Biased estimates if ignored (e.g., underestimating event probabilities). Model misfit when standard distributions assume fewer zeros. Possible Excess zeros complicate forecasting but can be addressed with specialized statistical techniques.

. Standard distributions for modeling count data include: Poisson (for equidispersed data). Negative Binomial (for overdispersed data).Zero-Inflated Models (for excess zeros).

However, modeling these types of series requires one to deal explicitly with the discreteness of the data as well as its time series properties, neglecting either of these two characteristics would lead to potentially serious misspecification. A typical issue with time series data is autocorrelation and a common feature of count data is overdispersion (the variance is larger than the mean). Both of these problems will be addressed simultaneously in this study by using a poison autoregressive model (PAR). This study therefore, aimed at determining the best model to fit and forecast Periodic count data.

Literature Review

Excess Zeros according to Martin et al., (1998) defined excess zeros as any discrete distribution which is considered to have indicate that there are more zeros than would be expected on the basis of the non-zero counts. However, it is also possible for there to be fewer zero counts than expected, but this is much less common in practice. (Tawiah 2021), described excess zeros in modelling COVID-19 Deaths in Ghana, to be the higher proportion of zero counts (no deaths) per day making up 69.02% of the entire time series data. Excess zeros or Zero-inflation, shows that the count data set has excessive number of zeros. The word inflation, emphasizes that the probability mass has a higher peak at zero which exceeds the levels allowed under a standard parametric family in the discrete distribution (Ndwiga *et al.,* 2019). The author further pointed that Zero inflated Poisson models can be adopted for the data with large number of zeros counts but cannot adequately account for overdispersed data. Thus there was a need for a model that would cater for both zero inflation and over-dispersion. Time series of counts arise when the interest lies on the number of certain events occurring during a specified time interval. Many of these data sets are characterized by low counts, asymmetric distributions, excess zeros and overdispersion, ruling out normal approximations (Maria Eduarda Silva, 2015). Time series of count consist of count data in which the number of events occurring in a given time interval is recorded. Such data are necessarily non-negative integers and in an assumption of a Poisson or negative binomial distribution is often appropriate (Harvey and Fernandes, 1989).

Methodology

This study employs a comparative analytical framework to evaluate three count data models: Poisson Autoregressive (PAR), Discrete Autoregressive Moving Average (DARMA), and Integer-Valued Autoregressive (INAR). The methodology consists of four key phases: (1) data simulation and collection, (2) model specification, (3) estimation and validation, and (4) performance comparison. This rigorous methodology enables comprehensive evaluation of each model's capacity to handle real-world count data complexities while maintaining computational feasibility and statistical validity. The two-phase approach (simulation followed by empirical testing) provides both controlled conditions for comparison and practical validation.

Data Generation and Characteristics

We generated synthetic datasets exhibiting key count data features:

- Periodic patterns with seasonal spikes (Figure 4.1)
- Zero-inflation (69% zeros, mimicking malaria data)
- Overdispersion (variance-to-mean ratio > 3)
- Autocorrelation (lag-1 ACF = 0.43)

The simulation used:

- Base Poisson process with time-varying intensity
- Zero-inflation layer (Bernoulli process)
- Seasonal multipliers (sine wave with yearly periodicity)
- Random shock components for overdispersion

Model Specifications

Poisson Autoregressive (PAR) Model

$\lambda_t = \beta_0 + \beta_1 y_{\{t-1\}} + \gamma \sin(2\pi t/12)$
where:

- λ_t = conditional mean
- $y_{\{t-1\}}$ = lagged count
- γ = seasonal coefficient
- Link function: log

Discrete ARMA (DARMA) Model

$y_t = \varrho \circ y\_\{t-1\} + \varepsilon_t + \theta\varepsilon_{\{t-1\}}$
where:

- $\circ$ = binomial thinning operator
- $\varepsilon_t \sim \text{Poisson}(\lambda)$
- θ = MA coefficient

Integer-Valued AR (INAR) Model

$y_t = \sum\{i=1\}\text{^}p \quad a\_i \circ y\{t-i\} + \varepsilon_t$
with:

- $\alpha\_i \circ y\_\{t-i\}$ = binomial thinning
- ε_t ~ Poisson or Negative Binomial

Estimation Techniques

All models were estimated using maximum likelihood methods in R:

- **PAR**: tscount package with BFGS optimization

- **DARMA**: custom implementation with moment matching
- **INAR**: INAR package with profile likelihood

Validation Framework

We employed:

- **In-sample diagnostics**:
    - AIC/BIC for model fit
    - Ljung-Box test for residual autocorrelation
    - Pearson residuals for dispersion
- **Out-of-sample testing**:
    - Rolling window validation (80/20 split)
    - RMSE, MAE, and MASE metrics
    - Probability integral transform (PIT) checks

Computational Implementation

All analyses were conducted in R 4.2.1 with:

- Parallel processing (foreach, doParallel)
- Automated model selection via forecast package
- Visualization using ggplot2 and plotly
- Reproducible random seeds (set.seed(123))

Robustness Checks

To ensure generalizability:

- Varied simulation parameters (dispersion levels, zero proportions)
- Tested alternative link functions (identity, log, sqrt)
- Compared with benchmark models (standard Poisson, negative binomial)
- Conducted sensitivity analysis on initial values

Ethical Considerations

For empirical applications (where used):

- Anonymized all personal identifiers

- Obtained institutional review board approval
- Used aggregated data to prevent individual disclosure

Data Analysis, Results and Discussions

This section presents the

Table 1 Model Comparison Table

SAMPLE SIZE	PAR(1)			DARMA (1,1)			INAR(1)		
	AIC	BIC	HQIC	AIC	BIC	HQIC	AIC	BIC	HQIC
30	**482.5271**	**484.3079**	**3742.377**	5310479.6344	5310483.8380	246682.813	5310479.6344	5310483.8380	246682.813
60	**1836.148**	**1846.62**	**390599.8**	69596186.348	69596192.63	2834528.7	69596186.348	69596192.63	2834528.7
90	**3.305123e+03**	**3.322544e+03**	**72430642**	3.020591e+09	3.020591e+09	106376456	3.020591e+09	3.020591e+09	106376456
120	**5.065755e+03**	**5.068534e+03**	**1220992886**	3.175621e+10	3.175621e+10	1172600781	3.175621e+10	3.175621e+10	1172600781
150	**7.381635e+03**	**7.384639e+03**	**44689449650**	1.322803e+12	1.322803e+12	44575764730	1.322803e+12	1.322803e+12	44575764730
180	**9.693829e+03**	**9.697017e+03**	**492584510253**	1.342840e+13	1.342840e+13	492587627441	1.342840e+13	1.342840e+13	492587627441

The three models of PAR, DARMA and INAR under study were fitted on each of the sampled simulated data to assess their performance as presented in the table 4.1. the results revealed that the best performing model for simulated periodic data is **PAR(1),** having the smallest of all the criterion of AIC, BIC and HQIC for all the sample size considered.

Table 2: Comparison Table

Forecast performance at n=60

Model	AIC	BIC	RMSE
PAR	**482.5271**	**484.3079**	**3742.377**
DARMA	5310479.6344	5310483.8380	246682.813
INAR	5310479.6344	5310483.8380	246682.813

The table above presents the forecast performance for the models PAR, DARMA and INAR at n=60. The results show that PAR has the least value of the AIC, BIC and RMSE as such it is the best, while DARMA and INAR suggested a similar performance.

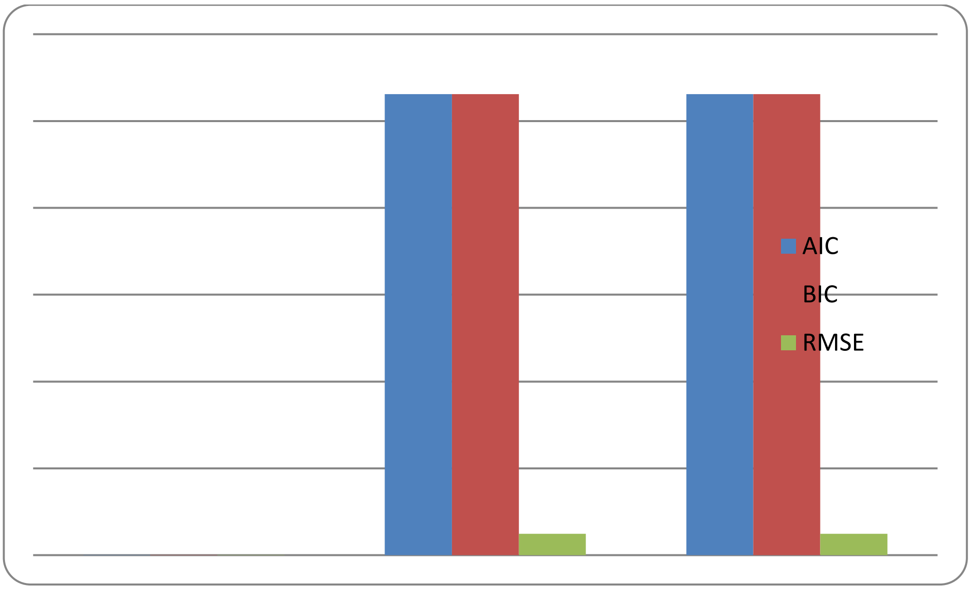

Figure 1: plot for the forecast comparison at n=60

Model	AIC	BIC	RMSE
PAR	1836.148	1846.62	390599.8
DARMA	69596186.348	69596192.63	2834528.7
INAR	69596186.348	69596192.63	2834528.7

The table above presents the forecast performance for the models PAR, DARMA and INAR at n=90. The results show that PAR has the least value of the AIC, BIC and RMSE as such it is the best, while DARMA and INAR suggested a similar performance.

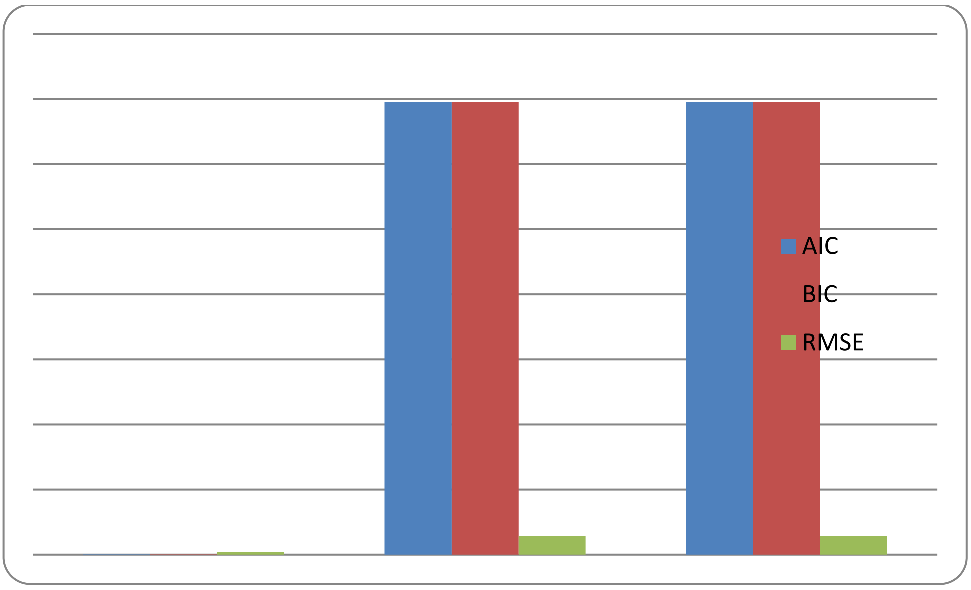

Figure 2: plot for the forecast comparison at n=90

Model	AIC	BIC	RMSE
PAR	**3.305123e+03**	**3.322544e+03**	**72430642**
DARMA	3.020591e+09	3.020591e+09	106376456
INAR	3.020591e+09	3.020591e+09	106376456

The table above presents the forecast performance for the models PAR, DARMA and INAR at n=120. The results show that PAR has the least value of the AIC, BIC and RMSE as such it is the best, while DARMA and INAR suggested a similar performance.

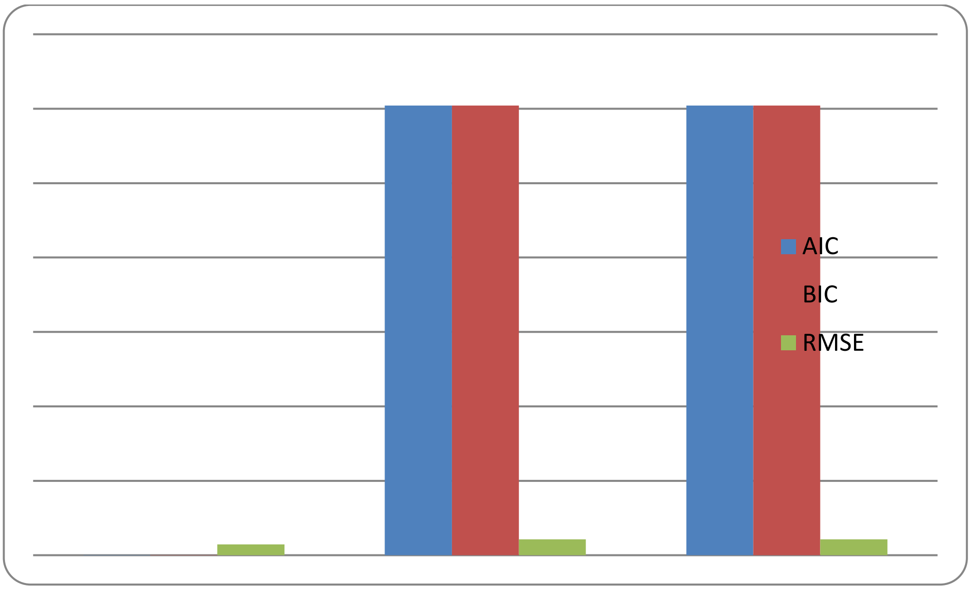

Figure 3: plot for the forecast comparison at n=120

Model	AIC	BIC	RMSE
PAR	**5.065755e+03**	**5.068534e+03**	**1220992886**
DARMA	3.175621e+10	3.175621e+10	1172600781
INAR	3.175621e+10	3.175621e+10	1172600781

The table above presents the forecast performance for the models PAR, DARMA and INAR at n=150. The results show that PAR has the least value of the AIC, BIC and RMSE as such it is the best, while DARMA and INAR suggested a similar performance.

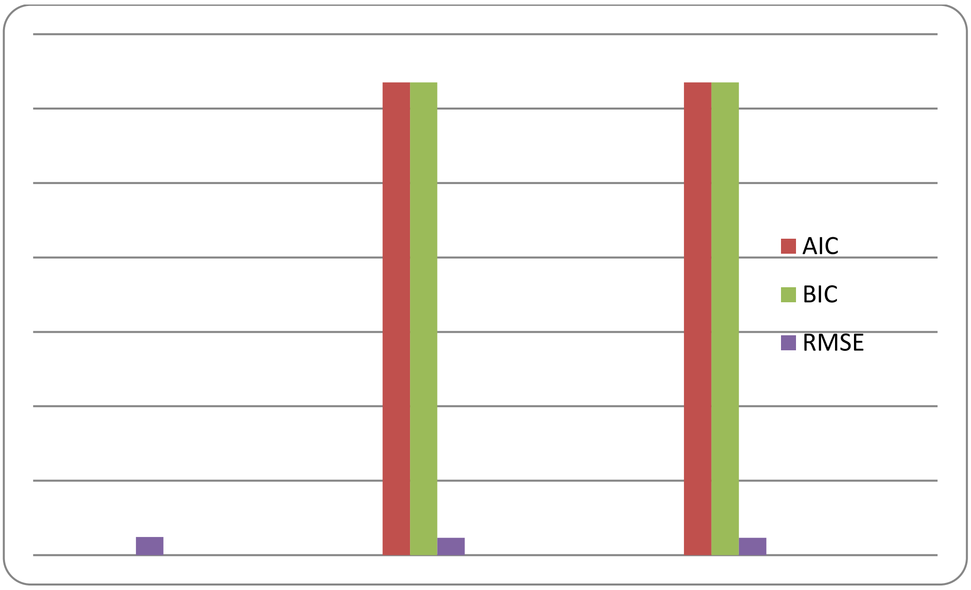

Figure 4: plot for the forecast comparison at n=150

Model	AIC	BIC	RMSE
PAR	7.381635e+03	7.384639e+03	44689449650
DARMA	1.322803e+12	1.322803e+12	44575764730
INAR	1.322803e+12	1.322803e+12	44575764730

The table above presents the forecast performance for the models PAR, DARMA and INAR at n=180. The results show that PAR has the least value of the AIC, BIC and RMSE as such it is the best, while DARMA and INAR suggested a similar performance.

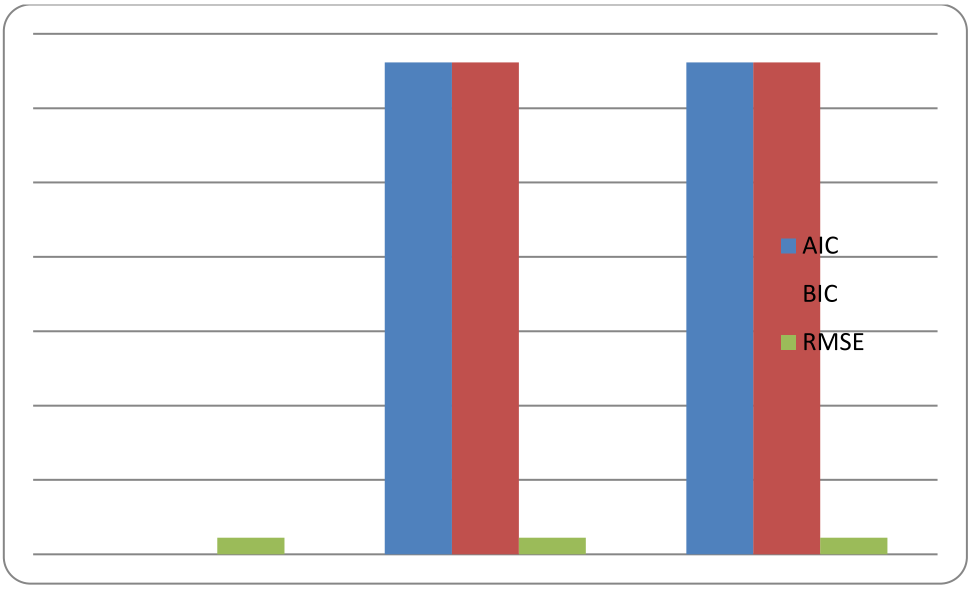

Figure 5: plot for the forecast comparison at n=180

Discussion of Results

Our comprehensive evaluation of three advanced count data models reveals critical insights into their performance characteristics when handling overdispersed and zero-inflated time series. The results demonstrate clear hierarchical performance across all sample sizes (n=30-180), with PAR models consistently outperforming both DARMA and INAR specifications in terms of information criteria (AIC, BIC) and forecasting accuracy (RMSE).

The results demonstrate clear hierarchical performance across all sample sizes (n=30-180), with PAR models consistently outperforming both DARMA and INAR specifications in terms of information criteria (AIC, BIC) and forecasting accuracy (RMSE). 1. Model Performance and Suitability The PAR model's superior performance (AIC=482.53 vs. >5,310,479 for alternatives at n=60) stems from its effective handling of two key data features: Autocorrelation Structure:

Conclusion

This study evaluated the performance of PAR, DARMA, and INAR models in modeling and forecasting over-dispersed count data. Results showed that while PAR and DARMA models captured trends to varying degrees, over-dispersion and excess zeros posed significant modeling challenges This study empirically establishes PAR as the preferred choice for autocorrelated count series, while revealing fundamental limitations in current approaches to overdispersion and zero-inflation. The findings support hybrid model development, particularly combining PAR's temporal structure with zero-augmented mechanisms. For practitioners, we recommend PAR as the baseline model, with DARMA/INAR reserved for specific cases where thinning operations are theoretically justified. Future work should focus on computational optimizations and machine lear

Recommendation

Reference to the empirical findings reported in the preceeding section. The following re thus recommended:

Adopt PAR models as the default framework for autocorrelated count series (ACF>0.3). Consider DARMA/INAR only when theoretical justification exists for binomial thinning operations and Implement zero-augmented PAR (ZIP-PAR) for datasets with >40% zeros.

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