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Fixed Point Theorems of Ćirić-Type Contraction

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Abstract

This paper introduces a novel class of Ćiríc-type contraction operators within the framework of complete metric-like spaces. The study establishes sufficient conditions under which fixed points exist for such mappings, expanding the theoretical foundation of fixed point theory. A carefully constructed, non-trivial comparative example is provided to illustrate the broader applicability and generality of the main result. Furthermore, several corollaries are derived, demonstrating that the proposed theorem not only encompasses but also unifies numerous existing fixed point theorems associated with Ćiríc-type contractions. The findings contribute to a deeper understanding of generalized contractive mappings and offer potential applications in related mathematical and applied contexts.

Keywords: Fixed point; Metric-like spaces; Quasi-contraction.

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Introduction

Fixed point theory is a branch of mathematics that deals with the study of functions that have points which remain invariant under those functions. In other words, a fixed point of a function is a point that is mapped to itself. It is expressed mathematically as: f(x) = x. One of the most famous results in fixed point theory is the Banach [4] fixed point theorem, which guarantees the existence and uniqueness of a fixed point for a contraction mapping on a complete metric space. This theorem has wide-ranging applications in differential equations, optimization problems, and dynamic systems. Since then, many other researchers have extended and generalized the Banach contraction principle in different ways. One is the formulation of new and more general contraction conditions (see e.g. [5, 9, 10]).

In another direction, the study of fixed point theorems has been extended to a variety of settings beyond the original context of metric and partial metric spaces leading to new and interesting results, by weakening the axioms of metric spaces. In 2012, Amini-Harandi [1] proposed the notion of metric-like space by relaxing axiom of non-negativity and small self-distances in partial metric space. Many other researchers have established fixed point results in this direction (see, e.g. [3, 7]). It is noted from the review of existing literature that little or no work has been done on C´iri´c-type contraction operator in the context of metric-like space. Hence, motivated by the idea in [6] and [8], we introduce in this manuscripts, a new concept of C´iri´c-type contraction operator in metric-like space and investigate the existence and uniqueness of fixed points of such operators. Substantial examples are presented to verify our proposed idea and compare them with other corresponding results. A few corollaries which collapse our new concepts to other famous idea in the literature have been presented and analyzed. Our proposed ideas herein extend the results of [6,8] and some references therein from complete metric space to σ-complete metric-like space. The paper is organized as follows: Section 1 presents the introduction and review of the related literature. In Section 2, the fundamental concepts needed in the sequel are collated. The main findings of the paper are discussed in Section 3 and Section 4 establishes some consequences of our acquired fixed point results.

Preliminaries

Throughout this work, we denote X as a non-empty set and $\mathbb{R}^+$, $\mathbb{N}$ respectively as the set of positive real numbers and set the of natural numbers.

Definition 2.1. [1] A mapping $\sigma : X \times X \to \mathbb{R}^+$ is referred to as an ML on X if for any $r, s, t \in X$, the following conditions hold:

$(\sigma 1)$ $\sigma(x, y) = 0 \Rightarrow x = y;$

$(\sigma 2)$ $\sigma(x, y) = \sigma(y, x);$

$(\sigma 3)$ $\sigma(x, z) \leq \sigma(x, y) + \sigma(y, z).$

The pair, (X, σ) is referred to as an MLS (metric-like space).

Definition 2.2. [1] In an MLS, a sequence $\{x_n\}_{n\in\mathbb{N}}$ is said to converge to an element $r \in X$ if $\sigma(x, x) = \lim_{n\to\infty} \sigma(x_n, x).$

Definition 2.3. [1] In an MLS, a sequence $\{x_n\}_{n\in\mathbb{N}}$ asserts to be a σ-Cauchy sequence if $\lim_{n,m\to\infty} \sigma(x_n, x_m)$ exists and non-infinite.

The MLS (X, σ) is complete if, for every σ- Cauchy sequence $\{x_n\}_{n\in\mathbb{N}}$, there is an element $x \in X$ so that $\lim_{n\to\infty} \sigma(x_n, x) = \lim_{n,m\to\infty} \sigma(x_n, x_m).$

Example 2.4 [1] Let $X = \{0,1\}$ and $\sigma(x, y) = \begin{cases} 2, & \text{if } x = y = 0; \\ 1, & \text{otherwise.} \end{cases}$

Then (X, σ) is a metric-like space, but since $\sigma(0,\ 0) \not\leq \sigma(0,\ 1)$, then (X, σ) is not a partial metric space.

Definition 2.5. [6] Let $F : X \to X$ be a self mapping on a metric space (X, d). Then, F is called a generalized Ćirić-type contraction mapping if it satisfies the following condition:

$$d(Fx, Fy) \leq \lambda(\mathrm{K}(x, y,))$$

for all $x,\ y \in X$, where $\lambda \in (0,\ 1)$ and

$$K(x,y) = \max\left\{\begin{matrix} d(x,y), d(x,Fx), d(y,Fy), \\ \frac{1}{2}[d(x,Fy)+d(y,Fx)] \end{matrix}\right\}.$$

Lemma 2.6. [2]. Let $(X,\ \sigma)$ be a metric-like space and let $\{x_n\}_{n\in N}$ be a sequence in X such that if $\{x_n\}_{n\in N}$ is not a σ-Cauchy sequence in $(X,\ \sigma)$. Then, there exists $\epsilon > 0$ and two sub-sequences $\{x_{m(k)}\}_{k\in N}$ and $\{x_{n(k)}\}_{k\in N}$ of $\{x_n\}_{n\in N}$, where n, m are positive integers with n(k) > m(k) > k such that

$$\sigma(x_{m(k)}, x_{n(k)}) \geq \epsilon \tag{2.1}$$

and

$$\sigma(x_{m(k)-1}, x_{n(k)}) < \epsilon. \tag{2.2}$$

Moreover, suppose that

$$\lim_{n\to\infty} \sigma(x_n, x_{n+1}) = 0.$$

Then, the following hold:

(1) $\lim_{k\to\infty} \sigma(x_{m(k)}, x_{n(k)}) = \epsilon;$

(2) $\lim_{k\to\infty} \sigma(x_{m(k)}, x_{n(k)+1}) = \epsilon;$

(3) $\lim_{k\to\infty} \sigma(x_{m(k-1)}, x_{n(k)}) = \epsilon;$

(4) $\lim_{k\to\infty} \sigma(x_{m(k)-1}, x_{n(k+1)}) = \epsilon.$

Ishak et al [8] obtained the following result in the context of partial metric space.

Definition 2.7. [8]. Let $F\ :\ X\ \to X$ be a self mapping on a partial metric space $(X,\ \rho)$. Then, F is called a generalized contraction mapping if it satisfies the following condition:

$$\rho(F\,x,\ F\,y)\ \leq\ \lambda(W(x,\ y))$$

for all $x,\ y \in X$, where $\lambda \in (0,\ 1)$ and

$$\mathrm{W}(x,y)=\max\left\{\begin{matrix}\rho(x,y),\rho(x,Fx),\rho(y,Fy),\\ \frac{1}{2}[\rho(x,Fy)+\rho(y,Fx)]\end{matrix}\right\}.$$

The main result of [8] is as follows.

Theorem 2.8. [8]. Let X be a complete metric space. If F is a generalized contraction,

then there exists a unique $z \in X$ such that $z = Tz$.

Main Results

In this section, we introduce the concept of C´iri´c-type contraction operator in the framework of metric-like space and examine the conditions for the existence of a fixed point of such operator.

Definition 3.1 Let (X, σ) be a metric-like space. A mapping $T : X \to X$ is called a Ćirić-type contraction operator, if it satisfies the following conditions:

$$\sigma(Tx,Tx) + \sigma(Tx,Ty) \leq \psi\left(M(x,y)\right) \qquad (3.1)$$

for all x, y ò X , where ψ ò Ψ and

$$M(x,y) = max\left\{\begin{matrix}\sigma(x,x)+\sigma(x,y),\ \sigma(y,y)+\sigma(x,y)+\sigma(y,Ty),\\ \frac{1}{2}[\sigma(x,x)+\sigma(x,Ty)+\sigma(y,Tx)]\end{matrix}\right\} \qquad (3.2)$$

The following is the main result of this paper.

Theorem 3.2 Let (X, σ) be a σ-complete metric-like space. If T is a Ćirić-type contraction operator, then there exists a unique u ò X u ∊ X such that $u = Tu$.

Proof. Starting from an arbitrary point $x = x_0$ò X , we will, construct a recursive sequence $\{x_n\}_{n\text{ò¥}}$ in the following manner:

$x_n = Tx_{n-1}$, for all n ò¥

It is observed that if $x_n = x_{n-1,} = Tx_{n-1}$, for some n ò¥ then x_n is a fixed point of T and the proof is finished. Hence, we assume that $x_n \neq x_{n-1}$ for all n ò¥ . By replacing $x = x_{n-1}$, and $y = x_n$ in equation (3.2) we obtain

$$M\left(x_{n-1},\ x_n\right) = \max\begin{Bmatrix}\sigma\left(x_{n-1,}x_{n-1}\right) + \sigma(x_{n-1},x_n),\ \ \sigma(x_n,x_n) + \sigma(x_n,Tx_n),\\ \frac{1}{2}\left[\sigma\left(x_{n-1,}\ x_{n-1}\right) + \sigma\left(x_{n-1,}\ Tx_n\right) + \sigma(x_n,Tx_{n-1})\right]\end{Bmatrix}$$

$$= max\begin{Bmatrix}\sigma\left(x_{n-1,}x_{n-1}\right) + \sigma(x_{n-1},x_n),\ \ \sigma(x_n,x_n) + \sigma(x_n,x_{n+1}),\\ \frac{1}{2}\left[\sigma\left(x_{n-1,}x_{n-1}\right) + \sigma\left(x_{n-1,}x_{n+1}\right) + \sigma(x_n,x_n)\right]\end{Bmatrix}. \qquad (3.3)$$

We observe that

$$\frac{1}{2}\left[\sigma\left(x_{n-1,}x_{n-1}\right) + \sigma\left(x_{n-1,}x_{n+1}\right) + \sigma(x_n,x_n)\text{-}\right.$$

$$\leq \frac{1}{2}\left[\sigma\left(x_{n-1,}x_{n-1}\right) + \sigma\left(x_{n-1,}x_n\right) + \sigma\left(x_{n,}x_{n+1}\right) + \sigma(x_n,x_n)\right)$$

$$\leq \max\left\{\sigma\left(x_{n-1,}x_{n-1}\right) + \sigma\left(x_{n-1,}x_n\right), \sigma(x_n,x_n) + \sigma\left(x_{n,}x_{n+1}\right)\right\}.$$

Hence (3.3) becomes

$$M\left(x_{n-1,}x_n\right) = \max\left\{\ \sigma\left(x_{n-1,}x_{n-1}\right) + \sigma\left(xx_{n-1,}x_n\right), \sigma(x_n,x_n) + \sigma\left(x_{n,}x_{n+1}\right)\right\}$$

Consequently (3.1) gives

$$\sigma(x_n,\ x_n) + \sigma(x_n,x_{n+1}) = \sigma\left(Tx_{n-1,}Tx_{n-1}\right) + \sigma\left(Tx_{n-1,}Tx_n\right)$$

$$\leq \psi(M(x_{n-1},x_n). \qquad (3.4)$$

If ,

$$\sigma(x_{n-1},\ x_{n-1}) + \sigma(x_{n-1},x_n) < \sigma(x_n,x_n) + (x_n,x_{n+1})$$

for some positive integer n, then it follows from (3.4) that

$$\sigma(x_n,\ x_n) + \sigma(x_n,x_{n+1}) \leq \psi\ (\sigma(x_n,x_n) + \sigma(x_n,x_{n+1}))$$

$$< \sigma(x_n,x_n) + \sigma(x_n,x_{n+1})$$

which is a contradiction.

Therefore,

$$\sigma(x_n,\ x_n) + \sigma(x_n,x_{n+1}) \leq \sigma\left(x_{n-1,}x_{n-1}\right) + \sigma\left(x_{n-1,}x_n\right) \qquad (3.5)$$

for n = 1, 2, 3, …

Hence,

$$M(x_{n-1},\, x_n) \;=\; \sigma(x_{n-1}, x_{n-1}) \;+\; \sigma(x_{n-1},\, x_n)\,.$$

From (3.4) we have

$$\sigma(x_n,\, x_n) \;+\; \sigma(x_n, x_{n+1}) \le \psi\; \left(\sigma\left(x_{n-1,} x_{n-1}\right) \;+\; \sigma\left(x_{n-1,} x_n\right)\right) \tag{3.6}$$

It follows from (3.5) that the sequence $\left\{\sigma\left(x_n,\, x_n\right) \;+\; \sigma\left(x_{n, n+1}\right)\right\}$ is bounded below and non-increasing. Therefore, $\sigma\left(x_n,\, x_n\right) \;+\; \sigma\left(x_{n,} x_{n+1}\right)) \;\to\; \eta$ as $n \;\to \infty$, for some $\eta \;\ge\; 0$.

If $\eta \;>\; 0$, then taking limit as $n \;\to\; \infty$ in (3.6) and using the continuity of ψ, we see that

$$\eta \;\le\; \psi\; (\eta) \;<\; \eta\,,$$

which is a contradiction. Thus $\lim\limits_{n\to\infty}(\sigma(x_n, x_n)) = \lim\limits_{n\to\infty}(\sigma(x_n, x_{n+1})) = 0$, from which we have

$$\lim_{n\to\infty}\left(\sigma(x_n, x_n)\right) = \lim_{n\to\infty}\left(\sigma(x_n, x_{n+n})\right) = 0 \tag{3.7}$$

Now, we prove that the sequence $\left\{x_n\right\}_{n\in ¥}$ is Cauchy. Assume that $\left\{x_n\right\}_{n\in ¥}$ is not Cauchy. Then by Lemma 2.6, there exists ò $>\; 0$ and subsequences $\left\{x_{n\;(k)}\right\}_{k\in ¥}$ and $\left\{x_{n\;(k)}\right\}_{k\in ¥}$ of $\left\{x_n\right\}_{n\in ¥}$ such that (2.1) and (2.2) hold.

From (3.2), we have

$$M(x_{n(k)},\, x_{m(k)}) \;=\; max \begin{Bmatrix} \sigma\left(x_{n(k),}\, x_{n(k)}\right) + \;\sigma\left(x_{n(k),}\;\; x_{m(k),}\right), \\ \sigma\left(x_{m(k),}\, x_{m(k)}\right) + \;\sigma\left(x_{m(k),}\, Tx_{m(k),}\right), \\ \frac{1}{2}\left[\sigma\left(x_{n(k),}\, x_{n(k)}\right) + \;\sigma\left(x_{n(k),}\, Tx_{m(k),}\right) + \sigma\left(x_{m(k),}\, Tx_{n(k),}\right)\right] \end{Bmatrix}$$

$$= \max \begin{Bmatrix} \sigma\left(x_{n(k),}\, x_{n(k)}\right) + \;\sigma\left(x_{n(k),}\;\; x_{m(k),}\right), \\ \sigma\left(x_{m(k),}\, x_{m(k)}\right) + \;\sigma\left(x_{m(k),}\, x_{m(k)+1,}\right), \\ \frac{1}{2}\left[\sigma\left(x_{n(k),}\, x_{n(k)}\right) + \;\sigma\left(x_{n(k),}\, x_{m(k)+1,}\right) + \sigma\left(x_{m(k),}\, x_{n(k)+1,}\right)\right] \end{Bmatrix} \tag{3.8}$$

As $k \to\; \infty$ in (3.8) applying Lemma 2.1 and using equation (3.7) yield

$$\lim_{n\to\infty}\; M\left(x_{n(k),}\, x_{m(k),}\right) = \in \tag{3.9}$$

From (3.1), we have

$$\sigma(x_{n(k)+1}, x_{n(k)+1}) + \sigma(x_{n(k)+1}, x_{m(k)+1}) \leq \psi\left(M\left(x_{n(k),} x_{m(k)}\right)\right) \tag{3.10}$$

Letting $k \to \infty$ in (3.10) and using Lemma 2.1, the continuity of ψ and by using equations (3.7) and (3.9), we obtain $ò \leq \psi\ (ò) < ò$, a contradiction. Therefore, $\{x_n\}_{n\in ¥}$ is a Cauchy sequence. The completeness of X implies that there exists $u \in X$.

Now, from (3.2), we obtain

$$M(x_n, u) = max\begin{Bmatrix} \sigma(x_{n,} x_n) + \sigma(x_{n,}\ u),\ \sigma(u,u) + \sigma(u,Tu) \\ \frac{1}{2}[\sigma(x_{n,} x_n) + \sigma(x_{n,}\ Tu), + \sigma(u, Tx_{n,})] \end{Bmatrix}$$

$$= max\begin{Bmatrix} \sigma(x_{n,} x_n) + \sigma(x_{n,}\ u),\ \sigma(u,u) + \sigma(u,Tu), \\ \frac{1}{2}[\sigma(x_{n,} x_n) + \sigma(x_{n,}\ Tu), + \sigma(u, x_{n+1})] \end{Bmatrix}.$$

From which we have

$$\lim_{n\to\infty} M(x_{n,} u) = max\begin{Bmatrix} \sigma(u,u), \sigma(u,u\ ) + \sigma(u,Tu), \\ \frac{1}{2}[\sigma(u,u\ ) + \sigma(u,Tu)] \end{Bmatrix}$$

$$= \sigma(u,\ u) + \sigma(u,\ Tu) \tag{3.11}$$

Therefore, from (3.1), we have

$$\sigma(x_{n+1}, x_{n+1}) + \sigma(x_{n+1}, Tu) = \sigma(Tx_n, Tx_n) + \sigma(Tx_n, Tu) \tag{3.12}$$

Letting $n \to \infty$ in (3.12) and employing the continuity of ψ and using equations (3.11) we have

$$\sigma(u,\ u) + \sigma(u,\ Tu) \leq \psi(\sigma(u,u) + \sigma(u,\ Tu))$$

$$< \sigma(u,u) + \sigma(u,\ Tu) \tag{3.13}$$

a contradiction and hence, $\sigma(u,u) + \sigma(u,Tu) = 0$. Therefore, $\sigma(u,u) = 0$ and $u = Tu$.

To show that the fixed point of T is unique, suppose that v is another fixed point of T with $x = u$ and $y = v$. Then, $v = Tu$. Now, using (3.1), we have

$$\sigma(u,\ u) + \sigma(v,u) = \sigma(Tu,Tu) + \sigma(Tu,Tv)$$

$$\leq \psi\left(M\left(u,v\right)\right)$$

$$= \psi\left(\sigma\left(u,u\right) + \sigma\left(v,u\right)\right) < \sigma\left(u,u\right) + \sigma\left(v,u\right),$$

a contradiction and hence, $\sigma\left(u,u\right) + \sigma\left(u,v\right) = 0$. Consequently

$u = v$.

Example 3.3. Let $X = R^{+}$ and $\sigma\left(x, y\right) = x + y$ for all $x, y \in X$. Then obviously, σ is a metric-like on X and $\left(X, \sigma\right)$ is complete. Also, σ is not a metric on X, since $\sigma\left(1, 1\right) = 2 \neq 0$. Define the self-map $T : X \to X$ by $Tx = x^{4}$, for all $x \in X$. In order to verify the property of T being a generalized C´iric´-type contraction operator; consider $\psi\left(t\right) = \frac{1}{2}$. If $x \neq y$, then

$$\sigma\left(Tx, Tx\right) + \sigma\left(Tx, Ty\right)$$

$$\begin{aligned}
&= \frac{x}{4} + \frac{x}{4} + \frac{x}{4} + \frac{y}{4} \\
&= \frac{3x}{4} + \frac{y}{4} \\
&< \frac{3x}{2} + \frac{y}{2} \\
&= \frac{1}{2}\left(3x + y\right) \\
&= \frac{1}{2}\left(x + x + x + y\right) \\
&= \psi(\sigma(x, x) + \sigma(x, y)) \\
&= \psi(\max\{\mathrm{M}\ (x, y)\}).
\end{aligned}$$

Consequently, all the hypotheses of Theorem 3.2 are satisfied. Thus, We conclude that the mapping T has a unique fixed point in X. However, since σ does not constitute a metric or partial metric on X, the corresponding results in [6,8] are inapplicable in this instance for determining a fixed point of T.

In what follows, we present some consequences of Theorem **3.2**

Corollary 3.4 Let $\left(X, \sigma\right)$ be a σ-complete metric-like space.

Suppose that the self-mapping T satisfies the following condition:

$$\sigma(Tx, Tx) + \sigma(Tx, Ty) \leq \psi\big(\sigma(x, x) + \sigma(x, y)\big),$$

for all x, y ò X, where ψ ò Ψ. Then, there exists a unique u ò X such that $u = Tu$.

Proof. By taking the $\max = \big(\sigma(x, x) + \sigma(x, y)\big)$, Theorem 3.2 can be applied to find a unique u ò X such that $u = Tu$.

Corollary 3.5 Let (X, σ) be a σ-complete metric-like space. Suppose that the self-mapping T satisfies the following conditions:

$$\sigma(Tx, Tx) + \sigma(Tx, Ty)$$

$$\leq \lambda\ \big(max\{\sigma(x, x) + \sigma(x, Tx),\ \sigma(y, y) + \sigma(y, Ty)\}\big),$$

for all x, y ò X, where $\lambda \in (0,1)$. Then, there exists a unique u ò X such that $u = Tu$.

Proof. By defining $\psi : [0, \infty) \to [0, \infty)$ as $\psi(t) < \lambda t$, for all $t > 0$ and $\lambda \in (0,1)$, Theorem 3.2 can be applied to find a unique u ò X such that $u = Tu$.

Corollary 3.5 Let (X, σ) be a σ-complete metric-like space. Suppose that the self-mapping T satisfies the following conditions:

$$\sigma(Tx, Tx) + \sigma(Tx, Ty) \leq \lambda\ \big(max\{M(x, y)\}\big)$$

for all x, y ò X, where $\lambda \in (0,1)$ and $M(x, y)$ is the same as in Definition 3.1. Then, there exists a unique u ò X such that $u = Tu$.

Proof. Put $\psi = \lambda$ in Theorem 3.2.

Conclusion

Recent years have been seen significant advancements in fixed point theory, particularly in the study of self mappings on metric spaces. Amini-Harandi generalized the concept of metric spaces by introducing metric-like spaces and establishing related fixed point results. This paper introduces C´iri´c-type contraction operators within the framework of metric-like spaces and investigates conditions for the existence of fixed points for such mappings. Non-trivial examples illustrate the generality of the proposed ideas. Moreover, several corollaries highlight the applicability of these results to various existing fixed point theorems.

Competing Interests

The authors declare that they have no competing interests.

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