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Mathematical Modeling to Reduce Disordered Cell Division

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Abstract

The Fibonacci sequence is a sequence of numbers that gets closer and closer to the golden ratio when divided by the number before it. The golden ratio has been recognized since antiquity as the order relation that gives the best harmony and proportions in many formations in art and nature. Fibonacci polynomials, which are obtained with the help of these number sequences, are a mathematical modelling developed to be used in many branches of science. The correlation data of the 3-step and 4-step Fibonacci polynomials obtained from the division accelerations of cells, which are standardly indexed to irregular division, and the development of Fibonacci polynomials were obtained. In this correlation, it is seen that the aggregation in the interval [0,1] is closer to 0 in the 3-step Fibonacci polynomial, while it moves away from 0 in the 4-step Fibonacci polynomial. The 4-step Fibonacci polynomial obtained here represents the division modelling of any cell indexed to irregular division. In order to ensure the digitizability of the obtained 3-step and 4-step Fibonacci polynomials, the coefficients of these polynomials are converted into a

BINARY code system, then ENTROPI values are calculated by taking polynomials that can take values less than 1 in the interval [0,1] according to the definition of probability density functions and irregular division comparisons are made by obtaining scatter plots.

Keywords: Fibonacci polynomial; Entropy; Probability density function; Binary code

INTRODUCTION

Importance of Fibonacci polynomials having applications in many different branches of nature, science and art (Hoggatt & Bicknell, 1973; Koshy, 2001), a modeling was obtained by comparing the correlational data of entropy (Anderson, 2008; Bulut, 2017) values over the probability density function for the irregular division of a cell from Fibonacci polynomials.

As a result of the detailed literature review, logical design, modeling and scientific methods used in mathematical operations were used in order to approach the studies on the irregular division of a cell from a different perspective. In our literature reviews, it is seen that Fibonacci and Lucas number sequences are popular and there are many encryption and decryption algorithms based on these sequences. Moreover, in the following years, different encryption algorithms have been created with more general versions of these number sequences (Basu & Prasad, 2009; Dişkaya et al., 2022; Kuloğlu & Özkan, 2023a; Kuloğlu & Özkan, 2023b; Uçar et al., 2019).

In this sense, in addition to adding a mathematical dimension to the study, it is aimed to associate Fibonacci polynomials with the aforementioned importance of Fibonacci polynomials, which will be useful to study on this subject.

First studied by Catalan in 1883

$$F_n(x) = xF_{n-1}(x) + F_{n-2}(x), n \geq 2 \quad (1)$$

and Fibonacci polynomials with initial values $F_1(x) = 1, F_2(x) = x$ have been considered in more generalized forms (Kuloğlu et al., 2022; Kuloğlu et al., 2023) in the following years.

With the help of Fibonacci polynomials, multinomial Fibonacci polynomials were created, and 3-step and 4-step Fibonacci polynomials were obtained, which have higher validity and reliability of modeling with increasing number of terms.

The probability density functions of the terms obtained over Fibonacci polynomials are obtained with the formula given below ("Probability Density Function," 2024):

$$P(a \leq y \leq b) = \int_a^b f(y)dy \tag{2}$$

Here, due to the property of the probability density function, the function obtained in the defined region the property $f(y) \geq 0$ must hold.

Entropy, a statistical measure of disorder and clutter in a given data set, was introduced by Claude E. Shannon (Shannon, 1948) and is widely used in many fields such as data analysis, data mining (ICT), machine learning and classification, artificial prediction mechanisms, and thermodynamic applications.

In addition, the concept of entropy can be used to examine the complexity in the concept of "ICT", which is used as a measure of the average amount of information in data as an integral part of information technology.

The entropy values used in the calculation of all these data can be calculated from the values of the probability density functions in the range $0 \leq y \leq 1$ with the formula given below:

$$E = -\sum_{i=1}^{N} P_i \log_2 P_i \tag{3}$$

Here, P_i denotes the probability of finding the $i.$ data class in the whole class and N denotes the number of elements or data in the cluster (Gray, 1990). Other methods for calculating entropy values can be found with the following formulas.

$$E = -\left(\log_2 \prod_{i=1}^{N} P_i^{P_i}\right) \tag{4}$$

$$E = -\left(\log_2 \left(\frac{x_1}{N}\right)^{\frac{x_1}{N}} . \left(\frac{x_2}{N}\right)^{\frac{x_2}{N}} \dots \left(\frac{x_n}{N}\right)^{\frac{x_n}{N}}\right) \tag{5}$$

As can be seen from these equations, there are many methods to calculate entropy values. The entropy value increases in direct proportion to the fact that this value has a positive real number value greater than 0 and the irregularity in the data set increases. In short, this means that the further the data moves away from 0, the more irregularity increases. If the dataset has a perfectly regular distribution, the entropy value can be 0.

In line with all the explanations mentioned above, when the entropy values of 3-step and 4-step Fibonacci polynomials are compared, it is seen that 4-step polynomials tend to move away from the value 0, while 3-step polynomials tend to approach the value 0. By switching from 4-step Fibonacci polynomials to 3-step Fibonacci polynomials, a downward acceleration in the speed of cells indexed to irregular division was obtained with the modeling.

In this study, a study has been carried out for the division modeling of any cell indexed to irregular division through a modeling consisting of Fibonacci polynomials. Since Fibonacci numbers are used to determine the possible SUPPORT and RESISTANCE levels in any numerical data of an asset, it is aimed to create Fibonacci polynomials where each of these numbers can be used.

MAIN RESULTS

The main objective is to create a model representing an irregular or partially regular cell with the help of Fibonacci numbers and Fibonacci polynomials, which symbolize the closest approach to perfection in many fields. In order to digitize the coefficients of the polynomials of these models, these numbers were converted into BINARY code system.

In this study, 3-step and 4-step Fibonacci polynomials were created by increasing the number of steps in order to increase the validity and reliability of Fibonacci polynomials, which were first defined by Catalan as an improvement technique with the mathematical modeling to be created on cells indexed to irregular division. Then, the values of the probability density functions of each of these polynomials in the range [0,1] were examined and only the polynomials with values less than 1 and 1 were considered.

The entropy (uncertainty), which is defined as the unit of measure used in scientific studies and which expresses the disorder of the elements in any data group with real number values, and all polynomials that fall within the range that can be defined as the probability density functions mentioned above, first calculated their own and then total entropy values, and graphs showing their value ranges were created.

Finally, by creating scatter plots with the values of 3-step and 4-step polynomials according to Fibonacci numbers, it is aimed to see the partial regular distributions of these polynomials in a concrete way. Based on all these explanations, let us construct the 3-step and 4-step Fibonacci polynomials we have mentioned.

3-step Fibonacci polynomials

$$f_0^{(3)}(x) = 0, f_1^{(3)}(x) = 1, f_2^{(3)}(x) = x$$

with initial values

$$f_{n+3}^{(3)}(x) = x^2 f_{n+2}^{(3)}(x) + x f_{n+1}^{(3)}(x) + f_n^{(3)}(x), n \geq 0$$

is an equation defined by the recurrence relation given as. The first few 3-step Fibonacci polynomials are

$$f_3^{(3)}(x) = x^2(x) + x(1) + 0$$

$$= x^3 + x$$

$$f_4^{(3)}(x) = x^2(x^3 + x) + x.x + 1$$

$$= x^5 + x^3 + x^2 + 1$$

$$f_5^{(3)}(x) = x^2(x^5 + x^3 + x^2 + 1) + x(x^3 + x) + x$$

$$= x^7 + x^5 + 2x^4 + 2x^2 + x.$$

If we arrange the coefficients according to mod2

$$f_5^{(3)}(x) = x^7 + x^5 + x$$

obtained. Similarly, if we arrange and write the other polynomials according to mod2, the following polynomials are obtained:

$$f_6^{(3)}(x) = x^9 + x^7 + x^6 + x^4 + x^3,$$

$$f_7^{(3)}(x) = x^{11} + x^9 + x^3 + 1,$$

$$f_8^{(3)}(x) = x^{13} + x^{11} + x^{10} + x^8 + x^5 + x^4 + x^2 + x,$$

$$f_9^{(3)}(x) = x^{15} + x^{13} + x^9 + x^4 + x,$$

$$f_{10}^{(3)}(x) = x^{17} + x^{15} + x^{14} + x^{12} + x^{11} + x^5 + x^3 + x^2 + 1,$$

$$f_{11}^{(3)}(x) = x^{19} + x^{17} + x^{11} + x^8 + x^7 + x^5 + x^2 + x,$$

$$f_{12}^{(3)}(x) = x^{21} + x^{19} + x^{18} + x^{16} + x^{13} + x^{12} + x^{10} + x^7 + x^6 + x^4,$$

$$f_{13}^{(3)}(x) = x^{23} + x^{21} + x^{17} + x^{12} + x^{11} + x^5 + 1,$$

$$f_{14}^{(3)}(x) = x^{25} + x^{23} + x^{22} + x^{20} + x^{19} + x^7 + x.$$

4-step Fibonacci polinomials

$$f_0^{(4)}(x) = 0, f_1^{(4)}(x) = 1, f_2^{(4)}(x) = x, f_3^{(4)}(x) = 1 + x$$

with initial values

$$f_{n+4}^{(4)}(x) = x^3 f_{n+3}^{(4)}(x) + x^2 f_{n+2}^{(4)}(x) + x f_{n+1}^{(4)}(x) + f_n^{(4)}(x), n \geq 0$$

is an equation defined by the recurrence relation given as. Similarly, the first few 4-step Fibonacci polynomials are

$$f_4^{(4)}(x) = x^4 + x,$$

$$f_5^{(4)}(x) = x^7 + x^4 + x^3 + 1,$$

$$f_6^{(4)}(x) = x^{10} + x^7 + x^2,$$

$$f_7^{(4)}(x) = x^{13} + x^{10} + x^9 + x^6 + x^5 + x + 1,$$

$$f_8^{(4)}(x) = x^{16} + x^{13} + x^5 + x^3,$$

$$f_9^{(4)}(x) = x^{19} + x^{16} + x^{15} + x^{12} + x^8 + x^6 + x^4 + x^3 + x^2 + 1,$$

$$f_{10}^{(4)}(x) = x^{22} + x^{19} + x^{14} + x^9 + x^3 + x,$$

$$f_{11}^{(4)}(x) = x^{25} + x^{22} + x^{21} + x^{18} + x^{17} + x^{13} + x^{12} + x^9 + x^8 + x^4 + x^2 + x + 1,$$

$$f_{12}^{(4)}(x) = x^{28} + x^{25} + x^{17} + x^{15} + x^{12} + x^9 + x,$$

$$f_{13}^{(4)}(x) = x^{31} + x^{28} + x^{27} + x^{24} + x^{20} + x^{18} + x^{16} + x^{14} + x^{11} + x^8 + x^2 + 1,$$

$$f_{14}^{(4)}(x) = x^{34} + x^{31} + x^{26} + x^{21} + x^{18} + x^{13} + x^{10} + x^2.$$

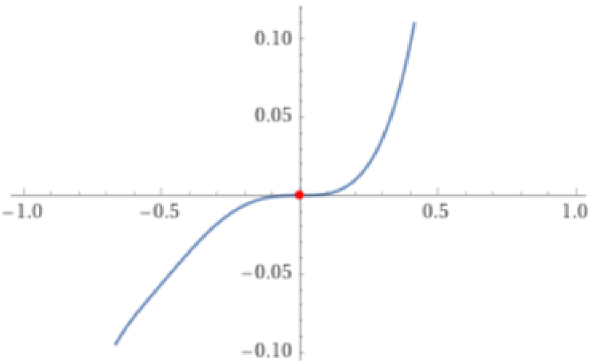
In these formulas, the 3-step formula shows more regular and stable divisive modeling than the 4-step formula. Moreover, based on the definition of probability density function, only

$f_6^{(3)}(x), f_{12}^{(3)}(x), f_{14}^{(3)}(x)$ and $f_6^{(4)}(x), f_{12}^{(4)}(x), f_{14}^{(4)}(x)$ polynomials are considered in this study.

Now let us calculate the probability density functions of the 3-step and 4-step Fibonacci polynomials:

$$f_6^{(3)}(x) = x^9 + x^7 + x^6 + x^4 + x^3$$

In order to talk about the probability density function of this polynomial, it must take a value greater than 0 in the 1st region of the coordinate system where x is positive. The graph below shows that the polynomial is defined in the mentioned region and is greater than 0:



$$P(0 \le x \le 1) = \int_0^1 (x^9 + x^7 + x^6 + x^4 + x^3)dx$$

$$= \frac{x^{10}}{10} + \frac{x^8}{8} + \frac{x^7}{7} + \frac{x^5}{5} + \frac{x^4}{4} = \frac{1}{10} + \frac{1}{8} + \frac{1}{7} + \frac{1}{5} + \frac{1}{4} = \frac{229}{280}.$$

Here we denote the probability density function of the polynomial $f_6^{(3)}(x)$ by $P_6\left[f_6^{(3)}(x)\right]$;

$$P_6\left[f_6^{(3)}(x)\right] = \frac{229}{280}$$

is obtained.

Now let us calculate the entropy value so that the disorder corresponding to this polynomial can be expressed in real numbers:

$$E_6^{(3)} = -P_6\left[f_6^{(3)}(x)\right] \log_2 P_6\left[f_6^{(3)}(x)\right]$$

Here $E_6^{(3)}$ represents the entropy value corresponding to the polynomial $f_6^{(3)}(x)$.

$$E_6^{(3)} = -P_6\left[f_6^{(3)}(x)\right] \log_2 P_6\left[f_6^{(3)}(x)\right]$$

$$= -\frac{229}{280}\log_2\left(\frac{229}{280}\right)$$

This value is $0.2 < E_6^{(3)} < 0.3$. The graph below shows the entropy range of $E_6^{(3)}$, thus concretising the range of values mentioned.

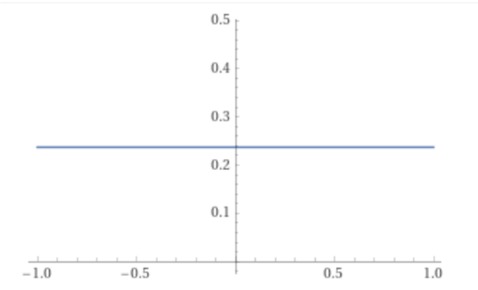


Figure 2. Entropy value range of the polynomial $f_6^{(3)}(x)$

Similarly,

$f_6^{(4)}(x) = x^{10} + x^7 + x^2$.

The existence of the probability density function of this polynomial is similar to the previous 3-step Fibonacci polynomials and the graph below shows that this polynomial is defined in region 1.

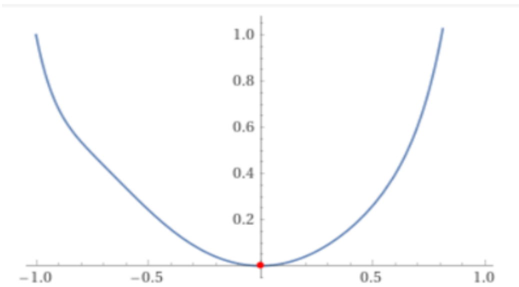


Figure 3. Graph of the polynomial $f_6^{(4)}(x)$.

$$P(0 \leq x \leq 1) = \int_0^1 (x^{10} + x^7 + x^2)dx$$

$$= \frac{x^{11}}{11} + \frac{x^8}{8} + \frac{x^3}{3} = \frac{1}{11} + \frac{1}{8} + \frac{1}{3} = \frac{145}{264}.$$

If we denote the probability density function of the polynomial $f_6^{(4)}(x)$ by $P_6\left[f_6^{(4)}(x)\right]$, the following value is obtained.

$$P_6\left[f_6^{(4)}(x)\right] = \frac{145}{264}$$

Let us calculate the entropy value so that the disorder corresponding to this polynomial can be expressed in real numbers:

$$E_6^{(4)} = -P_6\left[f_6^{(4)}(x)\right] \log_2 P_6\left[f_6^{(4)}(x)\right]$$

Here $E_6^{(4)}$ represents the entropy value corresponding to the polynomial $f_6^{(4)}(x)$.

$$E_6^{(4)} = -P_6\left[f_6^{(4)}(x)\right] \log_2 P_6\left[f_6^{(4)}(x)\right]$$

$$= -\frac{145}{264} \log_2 \left(\frac{145}{264}\right)$$

This value is $0.4 < E_6^{(4)} < 0.6$. The graph below shows the entropy range of $E_6^{(4)}$, thus concretising the range of values mentioned.

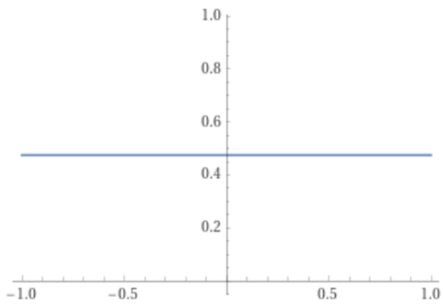


Figure 4. Entropy value range of the polynomial $f_6^{(4)}(x)$

Based on the data in Figure 2 and Figure 4, it is seen that the disorder values in the system are closer to 0 for the $f_6^{(3)}(x)$ polynomial, while they are far from 0 for the $f_6^{(4)}(x)$ polynomial. This means that the entropy distribution in $f_6^{(3)}(x)$ polynomial is more regular and stable, while the entropy distribution in $f_6^{(4)}(x)$ polynomial is more irregular.

4-step Fibonacci polynomial: Another reason for taking the mild regeneration cycle of the 3-step Fibonacci polynomial as a reference is shown by the more regular entropy range of the 4-step Fibonacci polynomial, which is in the positive direction of the tamed cellular modelling of its aggressive cycle.

Similarly, let us calculate and graphically represent the probability density functions of the polynomials $f_{12}^{(3)}(x)$ and $f_{12}^{(4)}(x)$ and the corresponding entropy values.

$$f_{12}^{(3)}(x) = x^{21} + x^{19} + x^{18} + x^{16} + x^{13} + x^{12} + x^{10} + x^7 + x^6 + x^4$$

The graph below shows that the polynomial is defined in the mentioned region:

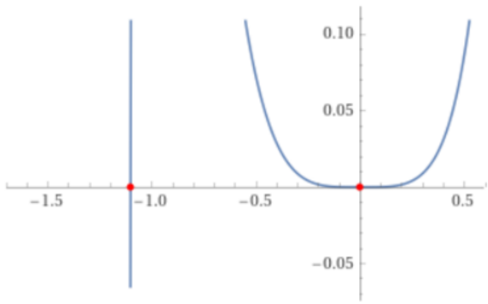


Figure 5. Graph of the polynomial $f_{12}^{(3)}(x)$.

$$P(0 \le x \le 1) = \int_0^1 (x^{21} + x^{19} + x^{18} + x^{16} + x^{13} + x^{12} + x^{10} + x^7 + x^6 + x^4)dx$$

$$= \frac{x^{22}}{22} + \frac{x^{20}}{20} + \frac{x^{19}}{19} + \frac{x^{17}}{17} + \frac{x^{14}}{14} + \frac{x^{13}}{13} + \frac{x^{11}}{11} + \frac{x^8}{8} + \frac{x^7}{7} + \frac{x^5}{5}$$

$$= \frac{1}{22} + \frac{1}{20} + \frac{1}{19} + \frac{1}{17} + \frac{1}{14} + \frac{1}{13} + \frac{1}{11} + \frac{1}{8} + \frac{1}{7} + \frac{1}{5} = \frac{2364209}{2586584}.$$

$$P_{12}\left[f_{12}^{(3)}(x)\right] = \frac{2364209}{2586584}$$

$$E_{12}^{(3)} = -P_{12}\left[f_{12}^{(3)}(x)\right] \log_2 P_{12}\left[f_{12}^{(3)}(x)\right]$$

Here $E_{12}^{(3)}$ represents the entropy value corresponding to the polynomial $f_{12}^{(3)}(x)$.

$$E_{12}^{(3)} = -P_{12}\left[f_{12}^{(3)}(x)\right] \log_2 P_{12}\left[f_{12}^{(3)}(x)\right]$$

$$= -\frac{2364209}{2586584} \log_2 \left(\frac{2364209}{2586584}\right)$$

This value is $0.10 < E_{12}^{(3)} < 0.15$. The graph below shows the entropy range of $E_{12}^{(3)}$, thus concretising the range of values mentioned.

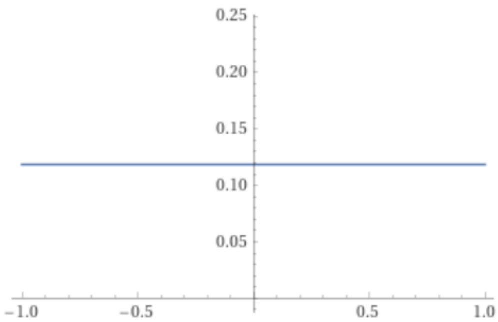


Figure 6. Entropy value range of the polynomial$f_{12}^{(3)}(x)$.

$$f_{12}^{(4)}(x) = x^{28} + x^{25} + x^{17} + x^{15} + x^{12} + x^{9} + x$$

The graph below shows that the polynomial $f_{12}^{(4)}(x)$ is defined in the mentioned region:

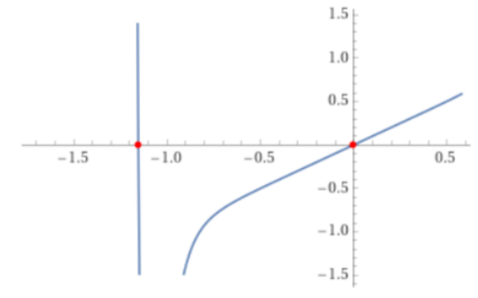


Figure 7. Graph of the polynomial $f_{12}^{(4)}(x)$.

$$P(0 \leq x \leq 1) = \int_0^1 (x^{28} + x^{25} + x^{17} + x^{15} + x^{12} + x^{9} + x)dx$$

$$= \frac{x^{29}}{29} + \frac{x^{26}}{26} + \frac{x^{18}}{18} + \frac{x^{16}}{16} + \frac{x^{13}}{13} + \frac{x^{10}}{10} + \frac{x^{2}}{2} = \frac{1}{29} + \frac{1}{26} + \frac{1}{18} + \frac{1}{16} + \frac{1}{13} + \frac{1}{10} + \frac{1}{2}$$
$$= \frac{235589}{271440}.$$

$$P_{12}\left[f_{12}^{(4)}(x)\right] = \frac{235589}{271440}$$

$$E_{12}^{(4)} = -P_{12}\left[f_{12}^{(4)}(x)\right] \log_2 P_{12}\left[f_{12}^{(4)}(x)\right]$$

$E_{12}^{(4)}$ represents the entropy value corresponding to the polynomial $f_{12}^{(4)}(x)$.

$$E_{12}^{(4)} = -P_{12}\left[f_{12}^{(4)}(x)\right] \log_2 P_{12}\left[f_{12}^{(4)}(x)\right]$$

$$= -\frac{235589}{271440}\log_2\left(\frac{235589}{271440}\right)$$

This value is $0.15 < E_{12}^{(4)} < 0.20$.

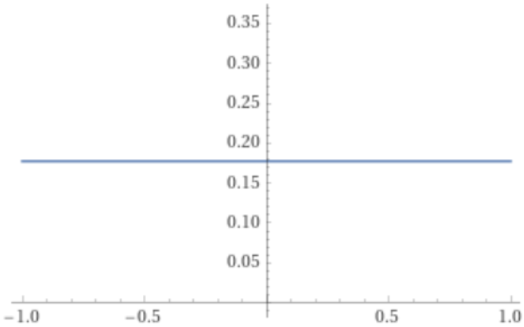


Figure 8. Entropy value range of the polynomial $f_{12}^{(4)}(x)$.

Based on the data in Figure 6 and Figure 8, it is seen that the disorder values in the system are closer to 0 for the $f_{12}^{(3)}(x)$polynomial, while they are far from 0 for the $f_{12}^{(4)}(x)$ polynomial. This means that the entropy distribution for the polynomial $f_{12}^{(3)}(x)$ is more regular and stable, while the entropy distribution for the polynomial $f_{12}^{(4)}(x)$is more irregular.

Similarly, let us calculate the probability density functions of the polynomials $f_{14}^{(3)}(x)$ and $f_{14}^{(4)}(x)$ and the corresponding entropy values and show them graphically.

$$f_{14}^{(3)}(x) = x^{25} + x^{23} + x^{22} + x^{20} + x^{19} + x^7 + x.$$

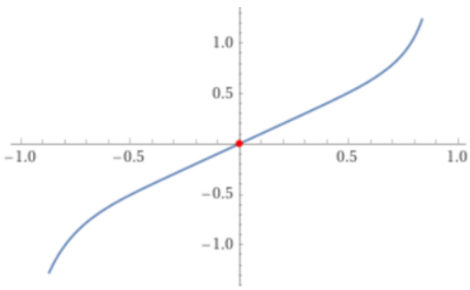


Figure 9. Graph of the polynomial $f_{14}^{(3)}(x)$

$$P(0 \le x \le 1) = \int_0^1 (x^{25} + x^{23} + x^{22} + x^{20} + x^{19} + x^7 + x)dx$$

$$= \frac{x^{26}}{26} + \frac{x^{24}}{24} + \frac{x^{23}}{23} + \frac{x^{21}}{21} + \frac{x^{20}}{20} + \frac{x^{8}}{8} + \frac{x^{2}}{2} = \frac{1}{16} + \frac{1}{24} + \frac{1}{23} + \frac{1}{21} + \frac{1}{20} + \frac{1}{8} + \frac{1}{2}$$
$$= \frac{35423}{41860}.$$

$$P_{14}\left[f_{14}^{(3)}(x)\right] = \frac{35423}{41860}$$

Let us calculate the entropy value so that the disorder corresponding to this polynomial can be expressed in real numbers:

$$E_{14}^{(3)} = -P_{14}\left[f_{14}^{(3)}(x)\right] \log_2 P_{14}\left[f_{14}^{(3)}(x)\right]$$
$$= -\frac{35423}{41860} \log_2 \left(\frac{35423}{41860}\right)$$

This value is $0.2 < E_{14}^{(3)} < 0.3$.

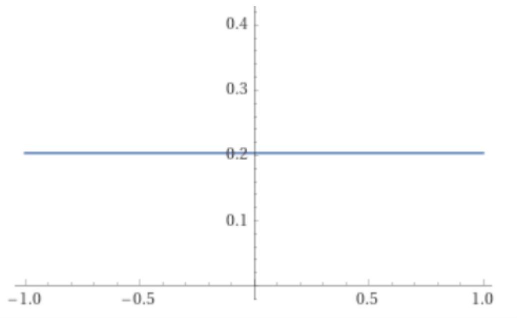


Figure 10. Entropy value range of the polynomial $f_{14}^{(3)}(x)$.

$$f_{14}^{(4)}(x) = x^{34} + x^{31} + x^{26} + x^{21} + x^{18} + x^{13} + x^{10} + x^{2}.$$

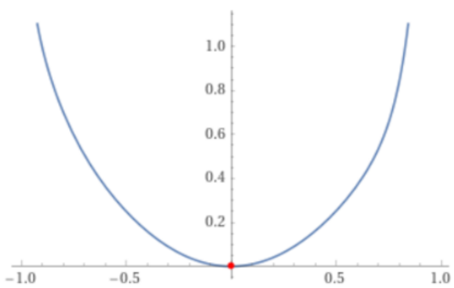


Figure 11. Graph of the polynomial $f_{14}^{(4)}(x)$.

$$P(0 \leq x \leq 1) = \int_0^1 (x^{34} + x^{31} + x^{26} + x^{21} + x^{18} + x^{13} + x^{10} + x^2)dx$$

$$= \frac{x^{35}}{35} + \frac{x^{32}}{32} + \frac{x^{27}}{27} + \frac{x^{22}}{22} + \frac{x^{19}}{19} + \frac{x^{14}}{14} + \frac{x^{11}}{11} + \frac{x^3}{3}$$

$$= \frac{1}{35} + \frac{1}{32} + \frac{1}{27} + \frac{1}{22} + \frac{1}{19} + \frac{1}{14} + \frac{1}{11} + \frac{1}{3} = \frac{623543}{902880}.$$

$$P_{14}\left[f_{14}^{(4)}(x)\right] = \frac{623543}{902880}$$

Let us calculate the entropy value so that the disorder corresponding to this polynomial can be expressed in real numbers:

$$E_{14}^{(4)} = -P_{14}\left[f_{14}^{(4)}(x)\right] \log_2 P_{14}\left[f_{14}^{(4)}(x)\right]$$

$$= -\frac{623543}{902880} \log_2 \left(\frac{623543}{902880}\right)$$

This value is $0.2 < E_{14}^{(4)} < 0.4$.

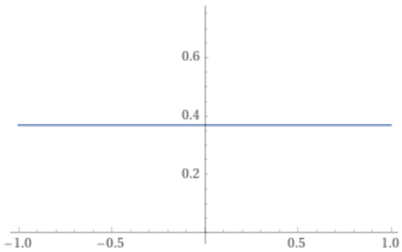


Figure 12. Entropy value range of the polynomial $f_{14}^{(4)}(x)$.

Based on the data in Figure 10 and Figure 12, it is seen that the disorder values in the system are closer to 0 for the $f_{14}^{(3)}(x)$ polynomial, while they are far from 0 for the $f_{14}^{(4)}(x)$ polynomial. This means that the entropy distribution for the polynomial $f_{14}^{(3)}(x)$ is more regular and stable, while the entropy distribution for the polynomial $f_{14}^{(4)}(x)$is more irregular.

In order to convert the modelling representing 4-step Fibonacci polynomials with irregular division into 3-step Fibonacci polynomials, the following equations are obtained if the characteristic equations of these polynomials are formed:

$$a^4 - x^3a^3 - x^2a^2 - xa - 1 = 0$$

and

$$b^3 - x^2 b^2 - xb - 1 = 0$$

Here, for all values to be given to x, the product of the roots obtained in both 3 and 4-step Fibonacci polynomials does not change, and while the product of the roots is 1 in the 3-step Fibonacci polynomial, it is -1 in the 4-step Fibonacci polynomial. This situation shows us the existence of the transition between roots and indirectly the transition between polynomials.

Finally, when the total entropy value ranges of the 3-step and 4-step polynomials are compared, it is clearly seen that the 3-step polynomials are more regularly divided, that is, closer to 0, while the divisions in the 4-step polynomials are more unstable and irregular.

The equations given below show the total entropy values of the 3-step and 4-step polynomials:

$$E_T^{(3)} = \sum_{i=6,12,14} -P_i\left[f_i^{(3)}(x)\right] \log_2 P_i\left[f_i^{(3)}(x)\right]$$

$$= \left(-P_6\left[f_6^{(3)}(x)\right] \log_2 P_6\left[f_6^{(3)}(x)\right] - P_{12}\left[f_{12}^{(3)}(x)\right] \log_2 P_{12}\left[f_{12}^{(3)}(x)\right] - P_{14}\left[f_{14}^{(3)}(x)\right] \log_2 P_{14}\left[f_{14}^{(3)}(x)\right]\right)$$

$$0.4 < E_T^{(3)} < 0.6$$

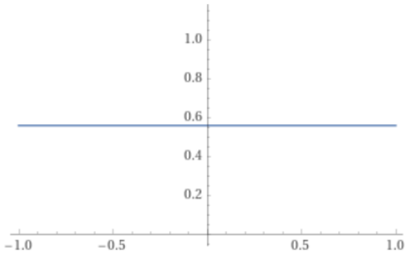
and

$$E_T^{(4)} = \sum_{i=6,12,14} -P_i\left[f_i^{(3)}(x)\right] \log_2 P_i\left[f_i^{(3)}(x)\right]$$

$$= \left(-P_6\left[f_6^{(4)}(x)\right] \log_2 P_6\left[f_6^{(4)}(x)\right] - P_{12}\left[f_{12}^{(4)}(x)\right] \log_2 P_{12}\left[f_{12}^{(4)}(x)\right] - P_{14}\left[f_{14}^{(4)}(x)\right] \log_2 P_{14}\left[f_{14}^{(4)}(x)\right]\right)$$

$$1 < E_T^{(4)} < 1.5$$

Here $E_T^{(3)}$ and $E_T^{(4)}$ represents the total Entropy value of the 3-step and 4-step polynomials. The graphs given below show the proximity-distance of the scatter plots of the total entropy values comparatively to the value 0:



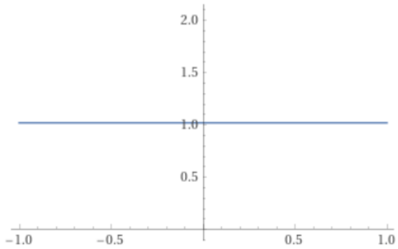


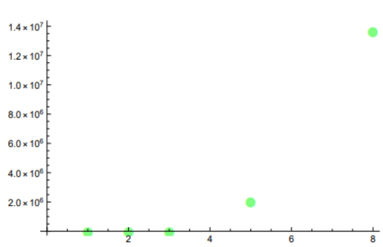
Figure 13. Entropy value range of $E_T^{(3)}$ **Figure 14.** Entropy value range of $E_T^{(4)}$

In line with the above-mentioned information, scatter plots are used in the Cartesian coordinate system to display the values of two variables, and by displaying a variable on each axis, the presence or absence of a relationship or correlation between two variables is more clearly addressed by scatter plots. Based on this information, comparative interpretations of 3-step polynomials, 4-step polynomials and both 3-step and 4-step polynomials with scatter plots used to functionalise the existing meaning are shown in the graphs below, and it can be easily said that the presence of a transition from 4-step polynomials to 3-step polynomials can represent more regular and stable cell division in real terms.

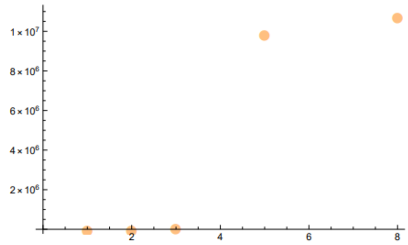
In order to make the correlational relationships more reliable in showing these graphs

Fibonacci numbers were utilised. In addition, the values corresponding to each Fibonacci number were reduced at an appropriate rate in order to see the stability of the distribution more clearly and understandably and to ensure that the data are in the graphic area. For the same purpose in the graphs, the first five sequences of Fibonacci numbers, 1,2,3,5,8, were used.

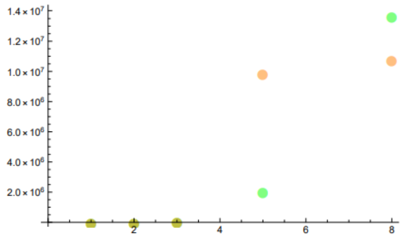
Let us now show the scatter plots corresponding to the Fibonacci polynomials $f_6^{(3)}(x)$ and $f_6^{(4)}(x)$:



(a) 3-step



(b) 4-step

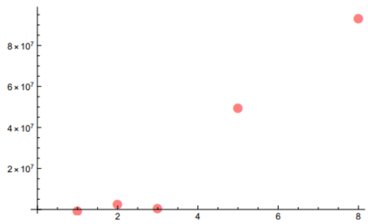


(c) 3-step and 4-step

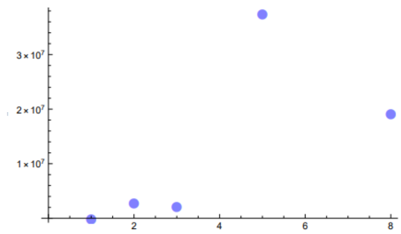
Figure 15. Scatter plots of $f_6^{(3)}$ and $f_6^{(4)}$ polynomials.

As can be seen from the graphs, the distribution represented in Figure 15 (a) is more regular and stable.

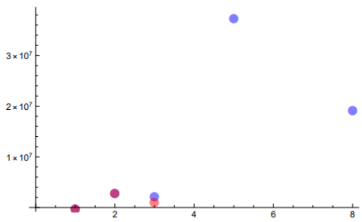
Similarly, let's show it which the scatter plots corresponding to the Fibonacci polynomials $f_{12}^{(3)}$ and $f_{12}^{(4)}$:



(a) 3-step



(b) 4-step

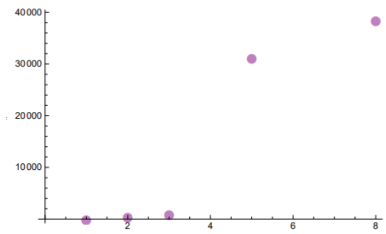


(c) 3-step and 4-step

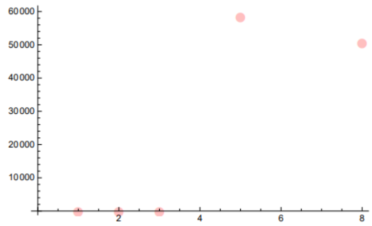
Figure 16. Scatter plots of $f_{12}^{(3)}$ and $f_{12}^{(4)}$ polynomials.

As can be seen from the graphs, the distribution represented in Figure 16 (a) is more regular and stable.

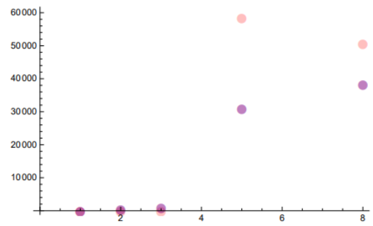
Finally, let us show the scatter plots corresponding to the Fibonacci polynomials $f_{14}^{(3)}$ and $f_{14}^{(4)}$:



(a) 3-step



(b) 4-step



(c) 3-step and 4-step

Figure 17. Scatter plots of $f_{14}^{(3)}$and $f_{14}^{(4)}$ polynomials.

As can be seen from the graphs, the distribution represented in Figure 17 (a) is more regular and stable.

In summary, the 4-step Fibonacci polynomial: As a further reason for taking the mild regeneration cycle of the 3-step Fibonacci polynomial as a reference, the tamed division modelling of the aggressive cycle of the 4-step Fibonacci polynomial results in positive improvements.

CONCLUSION

This study was carried out with the modeling of 3-step and 4-step Fibonacci polynomials on Entropy, which calculates the uncertainty, instability and the probability of occurrence of the unexpected situation with the help of the probability density function and shows the results accordingly.

In this study, based on the graphical data, we can see that the 3-step polynomial gives healthier results than the 4-step polynomial with the data obtained from the modeling based on the positive-negative correlational relationship between the polynomials.

In order to improve this study, the 3-step and 4-step Fibonacci polynomials can be applied to larger steps and comparisons can be made between the models by comparing different cell modeling.

For this purpose, more comprehensive models can be created by applying n-step Fibonacci polynomials to n-step Fibonacci polynomials in order to increase the reliability, to create many similar models and to ensure the transferability of the system to the digital environment.

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REFERENCES

Anderson, D. R. (2008). Information Theory and Entropy (pp. 51-82). Springer, ISBN-13: 978-1441979698, New York.

Basu, M., Prasad, B. (2009). The generalized relations among the code elements for Fibonacci coding theory. Chaos Solitons Fractals, 5 2517-2525.

Bulut, F. (2017). Bilgi Kuramındaki Entropi Kavramıyla İlgili Farklı Matematiksel Modeller. Bilge International Journal of Science and Technology Research, 1(2), 167-174.

Dişkaya, O., Avaroğlu, E., Menken, H., ve Emsal, A. (2022). A new Encryption Algorithm Based on Fibonacci Polynomials and Matrices. International Information and Engineering Technology Association, 39 (5), 1453-1462.

Gray R. M., (1990). Entropy and information. In Entropy and Information Theory, Publisher: Springer, (pp. 21-55) New York, ISBN-13: 978-1441979698

Hoggatt, J. V. E., Bicknell, M. (1973). Generalized Fibonacci polynomials, Fibonacci Quarterly, 11(5), 457-465.

Koshy, T. (2001). Fibonacci and Lucas Numbers with Applications, John, W., Sons, Inc., Canada.

Kuloğlu, B., Özkan, E., Kumarı, M., and Prsad, K. (2022). The k-Fibonacci group and periods of the k-step Fibonacci sequences: 10.37394/23206.21.95.

Kuloğlu, B., Özkan, E., and Marın, M. (2023). Fibonacci and Lucas Polynomials in n-gon, 31 (2), 127-140

Kuloğlu B., Özkan E. (2023). Creating Encryption and Decryption Algorithm Using Pell Numbers. EYFOR-14, Antalya, Türkiye, pp. 505.

Kuloglu, B., Özkan, E. (2023). Applications of Jacobsthal and Jacobsthal-Lucas Numbers in Coding Theory, ¨ Mathematica Montisnigri, Lvii, 54–64.

Probability density function. (2024, February 7). Wikipedia. Probability density function - Wikipedia

Shannon, CE (1948). Matematiksel bir iletişim teorisi. Bell Sistemi Teknik Dergisi. 27 (3): 379–423, 623–656. doi : 10.1002/j.1538-7305.1948.tb01338.

Uçar, S., Taş, N., and Özgür, N. H. (2019). A New Application to Coding Theory via Fibonacci and Lucas Numbers. Mathematical Sciences and Application E-Notes, 7 (1), 62-70.