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DERIVATION OF TWO PARAMETERS POISSON RANI DISTRIBUTION AND ITS PROPERTIES

Bamigbala Olateju Alao[1], Pantuvo Tsoke Peter[2], Ikrimat Aliyu Babando[3], Abdulganiy Abdullahi Gatta[4]

[1,2]Federal University, Wukari Taraba State, Nigeria

[3]College of Education, Zing, Taraba State, Nigeria

[4]Modibbo Adama University, Yola, Adamawa State, Nigeria

bamigbala@fuwukari.edu.ng

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Abstract

This study introduces the Two Parameters Poisson Rani Distribution (TPPRD). The probability distribution of TPPRD is derived by assuming that the parameters of the Poisson distribution follow the Two Parameters Rani Distribution, resulting in the formation of the TPPRD. The study derives some of its fundamental properties and demonstrates that TPPRD is a special-case distribution capable of handling overdispersed count data. Additionally, the maximum likelihood estimators are used to derive equations for estimating the parameters of the Two Parameters Poisson Rani Distribution.

Keywords: Overdispersed, Count, Parameters, Properties, Estimators

Introduction

Count data distributions refer to statistical models that describe data consisting of non-negative integer values, typically representing the frequency of events or occurrences within a specific time period or space. These distributions are especially useful for modeling scenarios where the outcome is a count of discrete events. One of the most widely used models for count data is the Poisson distribution, which predicts the number of events in a fixed interval, assuming that events occur independently at a constant average rate.

Although the Poisson distribution has been instrumental in modeling count data, its assumption that the mean and variance are equal limits its practical application. This limitation becomes problematic when dealing with overdispersion—a situation where the variance exceeds the mean. Overdispersion can lead to the underestimation of uncertainty, inaccurate parameter estimates, and biased standard errors. Such issues can negatively impact model selection, reduce interpretability, and compromise the reliability of predictions (Lawal, 2022; Noemi & Cinzia, 2022).

Recent research has shown significant progress in the development and adaptation of discrete probability distributions, with a particular emphasis on managing over-dispersed count data. Scientists have systematically explored the fusion of different distributions to develop more robust models, frequently using the Poisson distribution as their starting point. Significant contributions to the field include innovations such as the Weighted Negative Binomial-Poisson Lindley (Zamani, *et al.,* 2018), the Poisson Weighted Pranav Distribution (Showkat, *et al.,* 2022), and the Poisson Generalized Lindley distributions (Yupapin, 2023), all demonstrating enhanced capabilities in handling over-dispersed data compared to conventional approaches. The field has also seen the introduction of specialized models such as the Zero-inflated Poisson-Lindley regression model (Emrah, 2018) and the Bivariate Poisson-Lindley distribution (Zamani, *et al.,* 2015) to tackle specific analytical challenges. Additionally, the Discrete Poisson Bilal (Mohamed, *et al.,* 2023), Poisson Extended Exponential (Maya, *et al.,* 2022), and Poisson-Gold distributions (Ahmad & Amjad, 2021) have emerged as viable new options for count data modeling.

In this study, we will introduce a novel discrete probability distribution by combining the Poisson distribution with a two-parameter Rani distribution through a mixture approach. The proposed distribution, termed the two parameter Poisson-Rani Distribution (TPPRD), will be systematically analyzed to understand its mathematical properties and potential applications in modeling count data.

Methods

The Proposed Probability Mass Function of Two Parameters Poisson Rani Distribution

If $y \sim P(\lambda)$, then the probability mass function (pmf) of Poisson distribution is given by

$$f(y/\lambda) = \frac{\lambda^y e^{-\lambda}}{y!} \qquad y = 1,2,3,\ldots \qquad \lambda > 0 \tag{1}$$

The Two Parameters Rani Distribution (TPRD) by Al-omari, Adi and Ameur (2021) is given by

$$f(y;\theta,\alpha) = \frac{\theta^5}{\alpha\theta^5 + 24}\left(\alpha\theta^5 + y^4\right)e^{-\theta y} \qquad y > 0, \quad \theta > 0, \quad \alpha > 0 \tag{2}$$

Therefore, parameter λ follows TPRD given by,

$$f_{TPRD}(\lambda;\theta,\alpha) = \frac{\theta^5}{\alpha\theta^5 + 24}\left(\alpha\theta + \lambda^4\right)e^{-\theta\lambda} \qquad \lambda > 0, \quad \theta > 0 \tag{3}$$

Therefore, Two Parameters Poisson Rani Distribution (TPPRD), which is unconditional distribution is as follows

$$P_Y(y) = \int_0^\infty f(y/\lambda) f_{TPRD}(\lambda;\theta,\alpha)\, d\lambda = \int_0^\infty \frac{\lambda^y e^{-\lambda}}{y!} \cdot \frac{\theta^5}{\alpha\theta^5 + 24}\left(\alpha\theta + \lambda^4\right)e^{-\theta\lambda} d\lambda \tag{4}$$

$$P_Y(y) = \frac{\theta^5}{y!\left(\alpha\theta^5 + 24\right)}\left[\int_0^\infty \left(\alpha\theta\lambda^y + \lambda^{4+y}\right)e^{-(1+\theta)\lambda} d\lambda\right]$$

$$P_Y(y) = \frac{\theta^5}{y!\left(\alpha\theta^5 + 24\right)}\left[\frac{\alpha\theta\Gamma(y+1)}{(\theta+1)^{y+1}} + \frac{\Gamma(y+5)}{(\theta+1)^{y+5}}\right]$$

$$P_Y(y) = \frac{\theta^5}{\left(\alpha\theta^5 + 24\right)}\left[\frac{y^4 + 10y^3 + 35y^2 + 50y + 24 + \alpha\theta(1+\theta)^4}{(1+\theta)^{y+5}}\right] \qquad y = 0,1,2,\ldots; \ \theta > 0$$

(5)

Which is the pmf of $(TPPRD(y;\theta,\alpha))$

The corresponding cumulative distribution frequency (CDF) of ($TPPRD\left(y;\theta,\alpha\right)$) is obtained as:

$$F\left(y\right)=\sum_{t=0}^{y}P_{Y}\left(y\right)=\sum_{t=0}^{y}\frac{\theta^{5}}{\left(\alpha\theta^{5}+24\right)}\left[\frac{\alpha\theta\left(1+\theta\right)^{4}+t^{4}+10t^{3}+35t^{2}+50t+24}{\left(1+\theta\right)^{t+5}}\right]$$

$$F\left(y\right)=1-\frac{\left(\begin{array}{l}\alpha\theta^{9}+4\alpha\theta^{8}+6\alpha\theta^{7}+4\alpha\theta^{6}+\alpha\theta^{5}+y^{4}\theta^{4}+14y^{3}\theta^{3}+71y^{2}\theta^{4}+154y\theta^{4}+120\theta^{4}+\\4y^{3}\theta^{3}+48y^{2}\theta^{3}+188y\theta^{3}+240\theta^{3}+12y^{2}\theta^{2}+108y\theta^{2}+240\theta^{2}+24y\theta+120\theta+24\end{array}\right)}{\left(\alpha\theta^{5}+24\right)\left(\theta+1\right)^{y+5}} \quad (6)$$

Fig.1: pmf plot of Two Parameters Poisson Rama Distribution

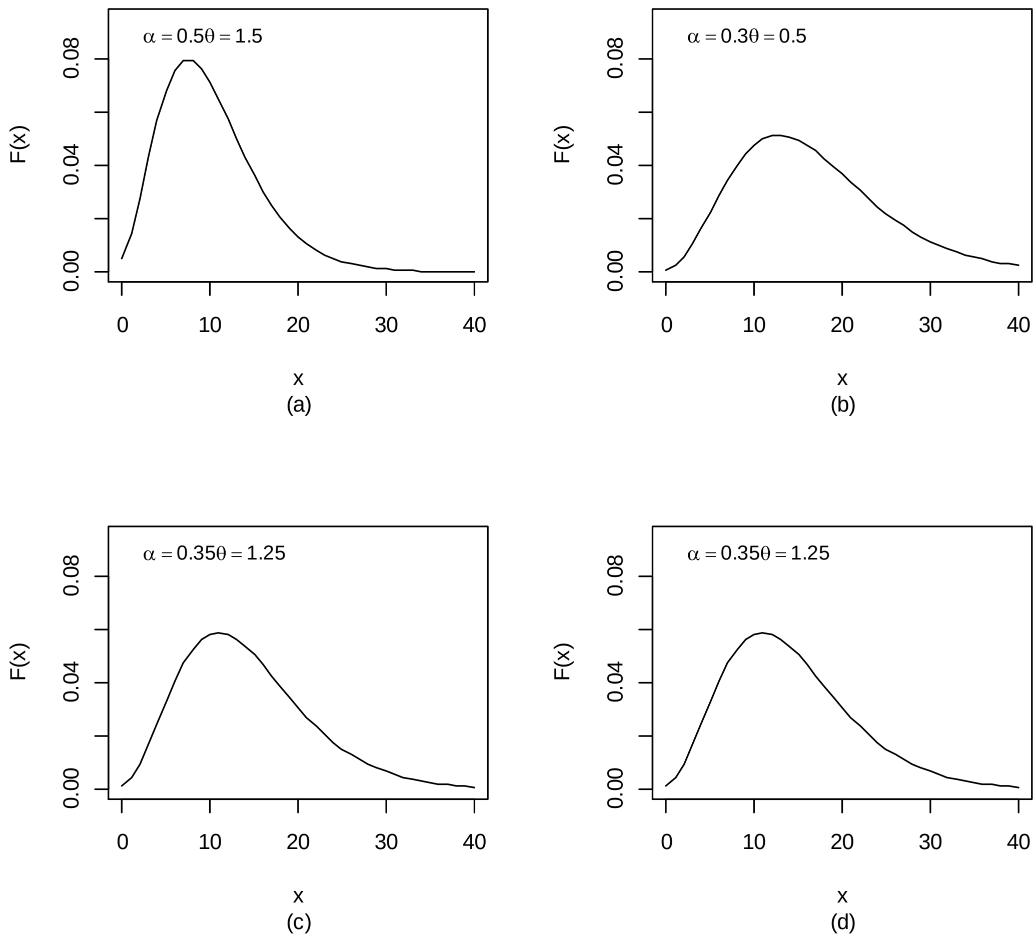
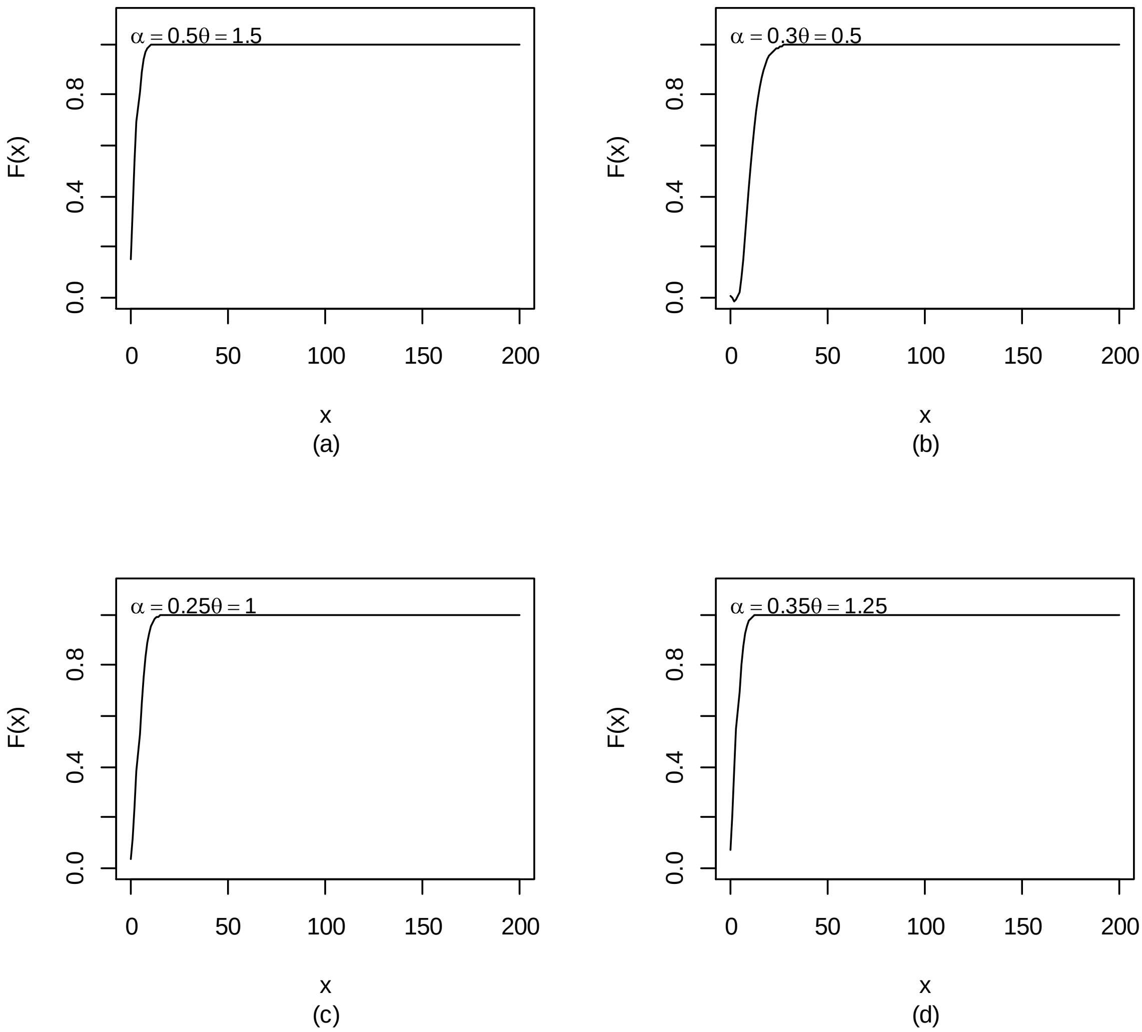


Fig.2: cdf plot of Two Parameters Poisson Rani Distribution



Statistical Properties of $TPPRD(y;\theta,\alpha)$

Moments of $TPPRD(y;\theta,\alpha)$

The r^{th} moment of the $TPPRD(y;\theta,\alpha)$ is denoted by $\mu_{(r)}'$ and can obtained by:

$$\mu_{(r)}' = E\left(y^r\right) = \sum_{y=0}^{\infty} y^r P_Y\left(y\right) \tag{7}$$

$$\mu_{(r)}' = \frac{\theta^5}{\alpha\theta^5 + 24}\left[\int_0^{\infty} y^r \sum_{y=0}^{\infty} \frac{\lambda^y e^{-\lambda}}{y!}.\left(\alpha\theta+\lambda^4\right)e^{-\theta\lambda} d\lambda\right]$$

$$\mu_{(r)}' = \frac{\theta^5}{\alpha\theta^5 + 24}\left[\int_0^{\infty} \lambda^r \left(\sum_{x=r}^{\infty} \frac{\lambda^{y-r} e^{-\lambda}}{(y-r)!}\right)\left(\alpha\theta + \lambda^4\right) e^{-\theta\lambda} d\lambda\right]$$

Taking $u = (y-r)$, we get

$$\mu_{(r)}' = \frac{\theta^5}{\alpha\theta^5 + 24}\left[\int_0^{\infty} \lambda^r \left(\sum_{u=0}^{\infty} \frac{\lambda^{u} e^{-\lambda}}{(u)!}\right)\left(\alpha\theta + \lambda^4\right) e^{-\theta\lambda} d\lambda\right]$$

Where $\sum_{u=0}^{\infty} \frac{\lambda^u e^{-\lambda}}{u!} = 1$

$$\mu_{(r)}' = \frac{\theta^5}{\alpha\theta^5 + 24}\left[\int_0^{\infty} \lambda^r \left(\alpha\theta + \lambda^4\right) e^{-\theta\lambda} d\lambda\right]$$

$$\mu_{(r)}' = \frac{\theta^5}{\alpha\theta^5 + 24}\left[\int_0^{\infty} \left(\alpha\theta\lambda^r + \lambda^{r+4}\right) e^{-\theta\lambda} d\lambda\right]$$

$$\mu_{(r)}' = \frac{\theta^5}{\alpha\theta^5 + 24}\left[\int_0^{\infty} \alpha\theta\lambda^r e^{-\theta\lambda} d\lambda + \int_0^{\infty} \lambda^{r+4} e^{-\theta\lambda} d\lambda\right]$$

$$\mu_{(r)}' = \frac{\theta^5}{\alpha\theta^5 + 24}\left[\frac{\alpha\theta\Gamma(r+1)}{\theta^{r+1}} + \frac{\Gamma(r+5)}{\theta^{r+5}}\right]$$

$$\mu_{(r)}' = \frac{\theta^5 r!}{\alpha\theta^5 + 24}\left[\frac{\alpha\theta^5 + (r+4)(r+3)(r+2)(r+1)}{\theta^{r+5}}\right]$$

$$\mu_{(r)}' = \frac{r!}{\alpha\theta^5 + 24}\left[\frac{\alpha\theta^5 + (r+4)(r+3)(r+2)(r+1)}{\theta^{r}}\right] \quad (8)$$

Taking $r = 1,2,3,4$ in equation (8), the first 4 factorial moments about origin of $TPPRD(y;\theta,\alpha)$ is given below

$$\mu_{(1)}' = \frac{\alpha\theta^5 + 120}{\theta\left(\alpha\theta^5 + 24\right)} \quad (9)$$

$$\mu_{(2)}' = \frac{2(\alpha\theta^5 + 360)}{\theta^2\left(\alpha\theta^5 + 24\right)} \quad (10)$$

$$\mu_{(3)}' = \frac{6(\alpha\theta^5 + 840)}{\theta^3\left(\alpha\theta^5 + 24\right)} \tag{11}$$

$$\mu_{(4)}' = \frac{24(\alpha\theta^5 + 1680)}{\theta^4\left(\alpha\theta^5 + 24\right)} \tag{12}$$

The first four moment about origin of the $TPPRD\left(y;\theta,\alpha\right)$ are obtained with relationship with equation (9) to (12) as shown below

$$M_1 = \mu_1^{'} = \mu^{'}_{(1)} = \frac{\alpha\theta^5 + 120}{\theta\left(\alpha\theta^5 + 24\right)} \tag{13}$$

$$M_2 = \mu_2^{'} = \mu^{'}_{(2)} + \mu^{'}_{(1)} = \frac{\alpha\theta^6 + 2\alpha\theta^5 + 120\theta + 720}{\theta^2\left(\alpha\theta^5 + 24\right)} \tag{14}$$

$$M_3 = \mu_3^{'} = \mu^{'}_{(3)} + 3\mu^{'}_{(2)} + \mu^{'}_{(1)} = \frac{\alpha\theta^7 + 6\alpha\theta^6 + 6\alpha\theta^5 + 120\theta^2 + 2160\theta + 5040}{\theta^3\left(\alpha\theta^5 + 24\right)} \tag{15}$$

$$M_4 = \mu_4^{'} = \mu^{'}_{(4)} + 6\mu^{'}_{(3)} + 7\mu^{'}_{(2)} + \mu^{'}_{(1)}$$

$$\mu_4^{'} = \frac{\alpha\theta^8 + 14\alpha\theta^7 + 36\alpha\theta^6 + 24\alpha\theta^5 + 120\theta^3 + 5040\theta^2 + 30240\theta + 40320}{\theta^4\left(\alpha\theta^5 + 24\right)} \tag{16}$$

Moment about mean of the $TPPRD\left(y;\theta,\alpha\right)$ are obtained with relationship with equation (13) to (16) as shown below:

$$\mu_1 = \frac{\alpha\theta^5 + 120}{\theta\left(\alpha\theta^5 + 24\right)} \tag{17}$$

$$\mu_2 = \mu_2^{'} - \mu_1^{'2} = \frac{\alpha\theta^{11} + 2\alpha^2\theta^{10} + 144\alpha\theta^6 + 528\alpha\theta^5 + 2880\theta + 2880}{\theta^2\left(\alpha\theta^5 + 24\right)^2} \tag{18}$$

$$\mu_3 = \mu_3^{'} - 3\mu_1^{'}\mu_2^{'} + 2\left(\mu_1^{'}\right)^3 = \frac{2\alpha^3\theta^{15} + 7344\alpha^2\theta^{10} + 4150638\alpha\theta^5 + 114719040}{\theta^3\left(\alpha\theta^5 + 24\right)^3} \tag{19}$$

$$\mu_4 = \mu_4^{'} - 4\mu_3^{'}\mu_1^{'} + 6\mu_2^{'}\left(\mu_1^{'}\right)^2 - 3\left(\mu_1^{'}\right)^4$$

$$\mu_4 = \frac{9\alpha^4\theta^{20} + 23904\alpha^3\theta^{15} + 528768\alpha^2\theta^{10} + 11114496\alpha\theta^5 + 34836480}{\theta^4\left(\alpha\theta^5 + 24\right)^4} \tag{20}$$

Variance of $TPPRD(y;\theta,\alpha)$

$$Var(y) = \sigma^2 = \frac{\alpha^2\theta^{11} + 2\alpha^2\theta^{10} + 144\alpha\theta^6 + 528\alpha\theta^5 + 2880\theta + 2880}{\theta^2\left(\alpha\theta^5 + 24\right)^2} \tag{21}$$

Standard Deviation of $TPPRD(y;\theta,\alpha)$

$$\sigma = \frac{\sqrt{\alpha^2\theta^{11} + 2\alpha^2\theta^{10} + 144\alpha\theta^6 + 528\alpha\theta^5 + 2880\theta + 2880}}{\theta\left(\alpha\theta^5 + 24\right)}$$

Coefficient of Variation (CV) of $TPPRD(y;\theta,\alpha)$

$$CV = \frac{\sigma}{\mu_1} = \frac{\sqrt{\alpha^2\theta^{11} + 2\alpha^2\theta^{10} + 144\alpha\theta^6 + 528\alpha\theta^5 + 2880\theta + 2880}}{\alpha\theta^5 + 120}$$

Index of Dispersion (ID) of $TPPRD(y;\theta,\alpha)$

$$ID = \frac{\sigma^2}{\mu_1} = \frac{\alpha^2\theta^{11} + 2\alpha^2\theta^{10} + 144\alpha\theta^6 + 528\alpha\theta^5 + 2880\theta + 2880}{\theta\left(\alpha\theta^5 + 24\right)\left(\alpha\theta^5 + 120\right)}$$

Coefficient of Kurtosis (K_s) of $TPPRD(y;\theta,\alpha)$

$$K_s = \frac{\mu_4}{\mu_2^{2}} = \frac{9\alpha^4\theta^{20} + 23904\alpha^3\theta^{15} + 528768\alpha^2\theta^{10} + 11114496\alpha\theta^5 + 34836480}{\alpha^2\theta^{11} + 2\alpha^2\theta^{10} + 144\alpha\theta^6 + 528\alpha\theta^5 + 2880\theta + 2880}$$

Coefficient of Skewness (S_k) of $TPPRD(y;\theta,\alpha)$

$$S_k = \frac{\mu_3}{\mu_2^{3/2}} = \frac{2\alpha^3\theta^{15} + 7344\alpha^2\theta^{10} + 4150638\alpha\theta^5 + 114719040}{\alpha^2\theta^{11} + 2\alpha^2\theta^{10} + 144\alpha\theta^6 + 528\alpha\theta^5 + 2880\theta + 2880}$$

Probability generating function (Pgf) of $TPPRD(y;\theta,\alpha)$

If $Y \sim TPPRD(y,\theta,\alpha)$ then the probability generating function $P_Y(t)$ is defined as:

$$P_Y(t) = E(t) = \sum_{y=0}^{\infty} t^y P_Y(y)$$

$$P_Y(t)=\frac{\theta^5}{(\alpha\theta^5+24)}\sum_{y=0}^{\infty}t^y\left(\frac{\alpha\theta(1+\theta)^4+y^4+10y^3+35y^2+50y+24}{(1+\theta)^{y+5}}\right)$$

$$P_Y(t)=\frac{\theta^5}{(\alpha\theta^5+24)}\left[\begin{array}{l}\alpha\theta(1+\theta)^4\sum_{y=0}^{\infty}\left(\frac{t}{1+\theta}\right)^y+\sum_{y=0}^{\infty}y^4\left(\frac{t}{1+\theta}\right)^y+10\sum_{y=0}^{\infty}y^3\left(\frac{t}{1+\theta}\right)^y\\+35\sum_{y=0}^{\infty}y^2\left(\frac{t}{1+\theta}\right)^y+50\sum_{y=0}^{\infty}y\left(\frac{t}{1+\theta}\right)^y+24\sum_{y=0}^{\infty}\left(\frac{t}{1+\theta}\right)^y\end{array}\right]$$

$$P_Y(t)=\frac{\theta^5}{(\alpha\theta^5+24)(\theta+1)^4}\left[\begin{array}{l}\frac{\left(\begin{array}{l}11\theta^3t+41\theta^2t^2+33\theta^2t-19\theta t^3+82\theta t^2+\\33\theta t-9t^4-19t^3+41t^2+11t\end{array}\right)}{(\theta-t+1)^5}\\+\frac{(85\theta t-15t^2+85t)}{(\theta-t+1)^3}+\frac{(1+\theta)^4(\alpha\theta+24)}{(\theta-t+1)}\end{array}\right]$$

Moment generating function (Mgf) of $TPPRD(y;\theta,\alpha)$ denoted by $M_Y(t)$ is given by

$$M_Y(t)=\sum_{y=0}^{\infty}e^{ty}P_Y(y;\theta,\alpha)$$

$$M_Y(t)=\frac{\theta^5}{(\alpha\theta^5+24)(\theta+1)^4}\left[\begin{array}{l}\frac{\left(\begin{array}{l}11\theta^3e^t+41\theta^2e^{2t}+33\theta^2e^t-19\theta e^{3t}+82\theta e^{2t}+\\33\theta e^t-9e^{4t}-19e^{3t}+41e^{2t}+11e^t\end{array}\right)}{(\theta-e^t+1)^5}\\+\frac{(85\theta e^t-15e^{2t}+85e^t)}{(\theta-e^t+1)^3}+\frac{(1+\theta)^4(\alpha\theta+24)}{(\theta-e^t+1)}\end{array}\right]$$

Characteristics generating function (Cgf) of $TPPRD(y;\theta,\alpha)$ denoted by $C_Y(t)$ is given by

$$C_Y(t)=\sum_{y=0}^{\infty}e^{ity}P_Y(y;\theta,\alpha)$$

$$M_Y(t)=\frac{\theta^5}{(\alpha\theta^5+24)(\theta+1)^4}\left[\frac{\begin{pmatrix}11\theta^3e^{it}+41\theta^2e^{2it}+33\theta^2e^{it}-19\theta e^{3it}+82\theta e^{2it}+\\33\theta e^{it}-9e^{4it}-19e^{3it}+41e^{2it}+11e^{it}\end{pmatrix}}{(\theta-e^{it}+1)^5}+\frac{(85\theta e^{it}-15e^{2it}+85e^{it})}{(\theta-e^{it}+1)^3}+\frac{(1+\theta)^4(\alpha\theta+24)}{(\theta-e^{it}+1)}\right]$$

Maximum Likelihood Estimation of $TPPRD(y;\theta,\alpha)$

Let $y_1,y_2,\cdots,y_n$ be a random sample of size n from $TPPRD(y;\theta,\alpha)$ and its likelihood function (L) is given by:

$$L=\left(\frac{\theta^5}{\alpha\theta^5+24}\right)^n\prod_{i=1}^{n}\left(\frac{y_i^4+10y_i^3+35y_i^2+50y_i+24+\theta\alpha\ \theta+1^{\ 4}}{\theta+1^{\ y_i+5}}\right)$$

Taking In of both side

$$InL=5nIn\theta-nIn\ \alpha\theta^5+24\ +\sum_{i=1}^{n}In\begin{pmatrix}y_i^4+10y_i^3+35y_i^2+\\50y_i+24+\theta\alpha\ \theta+1^{\ 4}\end{pmatrix}-\sum_{i=1}^{n}In\ \theta+1^{\ y_i+5} \tag{22}$$

Differentiate equation (22) with respect to α

$$\frac{\partial InL}{\partial\alpha}=\frac{-n\theta^5}{\alpha\theta^5+24}+\sum_{i=1}^{n}\left(\frac{\theta\ 1+\theta^{\ 4}}{y_i^4+10y_i^3+35y_i^2+50y_i+24+\theta\alpha\ \theta+1^{\ 4}}\right)$$

Differentiate equation (22) with respect to θ

$$\frac{\partial InL}{\partial\theta}=\frac{5n}{\theta}-\frac{5n\alpha\theta}{\alpha\theta^5+24}+\sum_{i=1}^{n}\left(\frac{5\alpha\theta+\alpha\ \ 1+\theta^{\ 3}}{y_i^4+10y_i^3+35y_i^2+50y_i+24+\theta\alpha\ \theta+1^{\ 4}}\right)-\sum_{i=1}^{n}\left(\frac{y_i+5n}{\theta+1}\right)$$

Table 1: Dispersion Index of $TPPRD(y;\theta,\alpha)$ for varying $\alpha = 0.25, 0.5, 0.75\, and\ 1.0$ and $\theta = 0.25\, to\, 4.75$ with an interval of 0.5

$\alpha = 0.25$		$\alpha = 0.5$	
θ	**Dispersion Index**	θ	**Dispersion Index**
0.25	5.001723	0.25	5.005181
0.75	5.062725	0.75	5.125217
1.25	5.417467	1.25	5.765708
1.75	6.242442	1.75	6.581456
2.25	7.215174	2.25	6.06843
2.75	7.799372	2.75	4.809429
3.25	7.910856	3.25	3.906146
3.75	7.750183	3.75	3.401165
4.25	7.489504	4.25	3.114692
4.75	7.214193	4.75	2.937345
$\alpha = 0.75$		$\alpha = 1.0$	
θ	**Dispersion Index**	θ	**Dispersion Index**
0.25	5.010375	0.25	5.017305
0.75	5.187606	0.75	5.249891
1.25	6.065273	1.25	6.322622
1.75	6.616554	1.75	6.502512
2.25	5.136479	2.25	4.426312
2.75	3.476641	2.75	2.722253
3.25	2.57218	3.25	1.913156
3.75	2.140099	3.75	1.553754
4.25	1.922473	4.25	1.381988
4.75	1.801185	4.75	1.29079

Table 1 represents the dispersion index for two sets of values from a Poisson Rani distribution for varying $\alpha = 0.25, 0.5, 0.75\, and\ 1.0$ and $\theta = 0.25\, to\, 4.75$ with an interval of 0.5. The dispersion index reflects the degree of variability in the distribution, with values greater than 1 indicating overdispersion (variance > mean) and values less than 1 suggesting underdispersion (variance < mean).

For, $\alpha = 0.25$ the dispersion index starts high at 5.001723 for 0.25 and increases to a peak of 7.910856 at 3.25, showing overdispersion throughout the lower and middle ranges. After 3.25, the dispersion index begins to decrease, signaling a reduction in variability and indicating a trend towards a more stable distribution.

For $\alpha = 0.5$ starts with overdispersion (5.010375 to 6.065273) but shows a strong decrease in the dispersion index beyond 1.25, particularly after 2.75, the values drop to 1.29079 at 4.75.

For $\alpha = 0.75$, the dispersion index starts at 5.010375 at 0.25 and increases to 6.616554 at 1.75. This indicates overdispersion (variance greater than the mean), suggesting greater variability in the data than a typical Poisson distribution would predict.

Lastly, $\alpha = 1.0$, the dispersion index starts at 5.017305 at 0.25 and increases to 6.502512 at 1.75. This indicates overdispersion, where the variance is greater than the mean, suggesting greater variability than would be expected from a standard Poisson distribution. After peaking at 6.502512 at 1.75, the dispersion index begins to decrease significantly. At 2.25, it drops to 4.426312, and by 2.75, it falls further to 2.722253, indicating a reduction in the variability of the data.

Conclusion

The analysis of the Two Parameter Poisson-Rani Distribution (TPPRD) demonstrates its effectiveness in modeling count data, particularly in situations involving over-dispersion. Through the systematic examination of its statistical properties, including moments, coefficients of variation, skewness, kurtosis, and the index of dispersion, the study reveals the distribution's flexibility and robustness. The dispersion index analysis for varying parameters α and θ consistently shows values greater than 1, confirming the distribution's capability to handle over-dispersed data effectively. These findings suggest that the TPPRD can serve as a valuable alternative to traditional count data models, particularly when dealing with over-dispersed datasets.

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