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MATHEMATICAL MODEL OF COUNTERACT BANDITS TERRORISM IN NIGERIA

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Abstract

This paper explores a mathematical model for analyzing the dynamics of banditry and terrorism, focusing on equilibrium states and stability conditions. Key concepts include the reproduction number R_b, which serves as a threshold parameter indicating the potential for spread or decline of these activities. The study finds that a banditry-free equilibrium is locally asymptotically stable when $R_b < 1$, suggesting that the activities will decline over time. In contrast, $R_b \geq 0$ indicates the possibility of globally stable coexistence equilibrium, where banditry and terrorism persist at endemic levels. The model also identifies invariant regions, ensuring that the system's trajectories remain within certain bounds, and guarantees unique and positive solutions, essential for real-world applicability. These findings provide critical insights for developing comprehensive strategies to mitigate banditry and terrorism, highlighting the importance of addressing both root causes and immediate symptoms

Keywords: Lyapunov function, Bandit, Terrorism, Lipschitz condition, Stability analysis

Introduction

Banditry and terrorism have become significant threats to the security and stability of Nigeria, particularly in the northern regions. These criminal activities not only result in loss of lives and property but also disrupt economic activities and displace communities. To address these challenges, it is essential to understand the dynamics of these issues through comprehensive and systematic analysis. Mathematical modeling offers a powerful tool to study the spread and impact of banditry and terrorism, providing insights into their underlying mechanisms and informing effective policy responses. Over the years, several researchers have applied mathematical models to analyze various aspects of criminal activities and their societal impacts. For instance, the works of Adamu and Umar (2019) focused on the dynamics of insurgency in Northern Nigeria, highlighting the factors contributing to its persistence. Owolabi *et al.* (2020) investigated the economic implications of terrorism, using models to predict future trends. Similarly, Oladipo and Alabi (2021) explored the role of socioeconomic factors in the spread of banditry, offering a framework for understanding its drivers.

Moreover, studies by Ibrahim and Musa (2018) and Suleiman *et al.* (2017) have emphasized the importance of integrating local community dynamics into models to capture the complexity of banditry and terrorism in Nigeria. Onwubiko and Okeke (2020) provided a comparative analysis of different intervention strategies, using mathematical models to assess their potential effectiveness. Ekundayo and Afolabi (2022) examined the role of governmental policies in curbing terrorism, utilizing stochastic models to account for uncertainties in policy implementation. The use of epidemiological models, as seen in the work of Bakare *et al.* (2023), has also been instrumental in understanding the transmission dynamics of violent extremism. Akanbi and Bello (2019) applied compartmental models to study the recruitment processes of terrorist groups, while Adeola *et al.* (2021) explored the impact of social media on radicalization using network theory. Fagbohun and Adefila (2020) highlighted the influence of migration patterns on the spread of banditry, employing spatial models to illustrate the geographical spread.

Other notable contributions include the work of Ojo and Olatunde (2018) on the role of poverty in sustaining criminal networks, and Nwankwo *et al.* (2019) on the demographic analysis of terrorism-prone regions. Tijani and Lawal (2022) investigated the psychological factors influencing participation in terrorist activities, using agent-based models to simulate individual decision-making processes. Obi and Nwachukwu (2020) focused on the impact of international cooperation on counter-terrorism efforts, while Abdulkarim *et al.* (2021) studied the effectiveness of military interventions. Additionally, Kehinde and Salami (2019) explored the economic costs of terrorism, incorporating macroeconomic models to analyze the broader economic implications. Bassey and Udoh (2022) used game theory to study the strategic interactions between terrorist groups and government forces. Afolayan and Akinola (2018) provided a comprehensive review of policy interventions, utilizing scenario analysis to predict outcomes under different policy scenarios.

Model Description

These paper population interests are Susceptible (S), Moderate Bandit (M), Terrorist Bandit (T) and Deserted (D). The susceptible population is enhance per capital recruitment rate Ψ of birth or immigration and shrink due to contact rate at which susceptible are recruited to adopt the extremist ideology at $\frac{\beta_1 T}{N}S$ rate, natural death rate at μ and public enlightenment at η rate. Moderate Bandit population step up as a result of contact of extremist bandit $\frac{\beta_1 T}{N}S$ and reduced due to natural death rate at μ and $\beta_2 M$ conversion rate from moderate Bandit to Terrorist bandit. while, $\beta_2 M$ the rate at which extremist bandit becomes terrorist bandit, and reduced due to natural death, military intelligence, terrorist natural risk and of their activities induced death or suicide death rate of counter terror activities and fears induces terrorist bandit move to deserted population at rate μ, e, δ and γ respectively. And due to the risk can desert terrorist population at rate ϵ, fear or conscience move to deserted population at rate γ and natural death rate μ. The model assumptions are as follows: We assume susceptible individuals can be reduced through public enlightenment, Terrorist bandit can as while reduced using military intelligence approach, Moderate and terrorist bandits are assume to have frequent contacts with susceptible individuals, We have the susceptible population has same average natural death rate and all the ordinary differential equation are positive.

Model equation

The counter terrorism model equation of banditry terrorist of Nigeria is mathematically represented as

$$\frac{dS}{dt} = \psi - \frac{\vartheta_1 T}{N} S - (\mu + \eta) S$$

$$\frac{dM}{dt} = \frac{\vartheta_1 T}{N} S - \vartheta_2 M - \mu M \qquad (1.0)$$

$$\frac{dT}{dt} = \vartheta_2 M - (\gamma + \mu + e + \theta) T$$

$$\frac{dD}{dt} = (\gamma + e) T - \mu D$$

This normalized the total model population

$$N = S + M + T + D$$

And initial conditions $S(0) > 0, M(0) > 0, T(0) > 0, D(0) > 0$.

Invariant of Banditry Terrorism model in Nigeria

Considering Banditry model equation (1) above,

Gives the rate of change of total model population as $N = S + M + T + D,$

Where $\frac{dN}{dt} = \psi - \mu N - \theta T \le \psi - \mu N.$

The state variables and their parameters all have usual scourge style and implications. Thus, the feasible region of banditry terrorism systems in Nigeria is $\phi = \left\{ (S, M, T, D) \varepsilon R_+^4 \ : \ N \le \frac{\psi}{\mu} \right\}$ is positively invariant and attracts all positive solutions of the system R_+^4 therefore the model equation (1) is mathematical well posed in the domain. Thus, in this domain, it is sufficient to consider the dynamics of the flow generated by the system

Existence of banditry terrorist free equilibrium

The model disease free equilibrium point of the bandit terrorism can be obtain when setting the right hand side of the system (1) to zero ; then solving the model resultants equations with ideal of Akpienbi *et al.* (2021) gives $E_0 = \left(D = 0, M = 0, S = \frac{\psi}{\mu + \eta}, T = 0 \right)$

Feasibility region and positivity solution of the banditry model

Theorem1. The feasibility region and positivity of solution of banditry in Nigeria, if $\mathrm{S}(\mathrm{t}),\ \mathrm{M}(\mathrm{t}),\ \mathrm{T}(\mathrm{t}),\ \mathrm{D}(\mathrm{t})$ have positive initial condition, were there exists a region

$$\Omega = \left\{ \mathrm{S}(\mathrm{t}),\ \mathrm{M}(\mathrm{t}),\ \mathrm{T}(\mathrm{t}),\ \mathrm{D}(\mathrm{t}) \in R_{+}^{4} : 0 < \mathrm{S}(\mathrm{t}) + \mathrm{M}(\mathrm{t}) + \mathrm{T}(\mathrm{t}) + \mathrm{D}(\mathrm{t}) \le \frac{\psi}{\mu} \right\}$$

for all $t, \mathrm{S}(\mathrm{t}) \ge 0,\ \mathrm{M}(\mathrm{t}) \ge 0,\ \mathrm{T}(\mathrm{t}) \ge 0,\ \mathrm{D}(\mathrm{t}) \ge 0.$

Proof. The banditry human population model parameters are assumed to be positive for all time t. We consider $N(t) = \mathrm{S}(\mathrm{t}) + \mathrm{M}(\mathrm{t}) + \mathrm{T}(\mathrm{t}) + \mathrm{D}(\mathrm{t})$

$$\frac{\mathrm{N(t)}}{dt} = \frac{\mathrm{S}(\mathrm{t})}{dt} + \frac{\mathrm{M}(\mathrm{t})}{dt} + \frac{\mathrm{T}(\mathrm{t})}{dt} + \frac{\mathrm{D}(\mathrm{t})}{dt}$$

$= \psi - \mu N(t) - \lim\limits_{t \to \infty} \frac{dN(t)}{dt} = \psi - \mu N(t)$, and it implies that $N(t) \le \frac{\psi}{\mu}$, and model dynamics are $\mathrm{N}(\mathrm{t}) = \frac{\psi}{\mu}(1 - \mathrm{e}^{-\mu t}) + \mathrm{N}_0 \mathrm{e}^{-\mu t}.$ (1.1)

The region of feasibility of the model is mathematically posed and extensive that the region Ω is positive invariant with respect to system $\mathbb{R}_{+}^{4}$. Meaning that, feasible solutions of banditry model equation of human population enter and remain in the region.

Positivity of solutions

Theorem 2. The banditry terrorist model governed in equation (1) is positive given the initial condition then, the solutions $\mathrm{S}(\mathrm{t}),\ \mathrm{M}(\mathrm{t}),\ \mathrm{T}(\mathrm{t}),\ \mathrm{D}(\mathrm{t})$ are positive for $t > 0$

Proof. . The first equation of the banditry model (1.0) gives;

$\frac{dS}{dt}$ [illegible] $\left([illegible] \frac{[illegible]_1 T}{N} \right) S$ [illegible] $\left([illegible] S\ [illegible] S\ [illegible] \frac{[illegible]_1 T}{N} \right) S$

Thus,

$\frac{dS}{dt}$ [illegible] $\left([illegible] S\ [illegible] S\ [illegible] \frac{[illegible]_1 T}{N} \right) S$

Using separation of variable approach and Let $\frac{\beta_1 T}{N} = p$

$\frac{dS}{S} = \geq -\left(\mu+\eta+p\right)dt$,

Integrate both sides of the inequality yields

$\int \frac{dS}{S} \geq -\int\left(\mu+\eta+p\right)dt$

$S(t) \geq -\left(\mu+\eta+p\right)t + C$

$$S(t) \geq C\exp\left(-((\mu+\eta+p)dt)t\right) \tag{1.2}$$

Then, at $t=0$ applying the initial condition which gives; $S(t) \geq 0.$ similarly, it can be shown that $S(t), T(t), M(t)$ and $D(t) > 0$ for all $t \geq 0$.

Banditry terrorism existence and uniqueness solution

The model problem is mathematically well posed and the unique solution is essential to eradication bandit menace. Lipschitz condition is considered to test the model existence and uniqueness.

Theorem 3. Suppose the banditry model (1.0) is written in compact form $f_i\left(t,x\right), x\left(t_0\right) = x_0, i = 1,2,3,4$ and the unique solution exists; then, (1.0) satisfies the Lipschitz condition

Proof. Rewrite model equation (1.0) as

$f_1\left(t,S\right) = \psi - \frac{\beta_1 T}{N}S - \left(\mu+\eta\right)S$

$f_2\left(t,M\right) = \frac{\beta_1 T}{N}S - \beta_2 M - \mu M$

$f_3\left(t,T\right) = \beta_2 M - \left(\omega+\mu+e+\theta\right)T$

$f_4\left(t,D\right) = \left(\omega+e\right)T - \mu D$,

The model equation (1.0) can be presented in the form

$\left|F_i\left(t,x_2\right) - F_i\left(t,x_1\right)\right| \leq \left|x_2 - x_1\right|,\ X_i^{'} = F_i\left(t,x\right), x\left(t_0\right) = x_0, i = 1,2,3,4$

And by illustration

$\left|F_1\left(t,S\right) - F_1\left(t,S\right)\right| = \left|-\left(\frac{\beta_1 T}{N}+\mu+\eta\right)S_2 - \left(-\left(\frac{\beta_1 T}{N}+\mu+\eta\right)\right)S_1\right|$

$= \left|-\left(\frac{\beta_1 T}{N}+\mu+\eta\right)\left(S_2 - S_1\right)\right| \leq \left|-\left(\frac{\beta_1 T}{N}+\mu+\eta\right)\right|\left|S_2 - S_1\right|,$

$$\left|F_2(t,M)-F_2(t,M)\right|=\left|-(\beta_2+\mu)M_2-\left(-(\beta_2+\mu)\right)M_1\right|$$

$$\leq\left|-\left(-(\beta_2+\mu)\right)M_2\right|\left|M_2-M_1\right|,$$

$$\left|F_3(t,T)-F_3(t,T)\right|=\left|-(\omega+\mu+e+\theta)T_2-\left(-\left((\omega+\mu+e+\theta)\right)T_1\right)\right|$$

$$\leq\left|-\left(-(\omega+\mu+e+\theta)\right)T_2\right|\left|T_2-T_1\right|$$

$$\left|F_4(t,D)-F_4(t,D)\right|=\left|-(\mu)D_2-(-\mu)D_1\right|$$

$$\leq\left|-\left(-(\mu)\right)D_2\right|\left|D_2-D_1\right|$$

Ordinary deferential equation system is said to exist and has unique solution when it satisfies the Lipschitz condition provided is continuous and bounded (Henry, 2021). Therefore, the proposed banditry terrorism model is said to be mathematically well-posed.

Theorem 4. Let Ω be denoted the region stated in theorem 1 and bounded in $0 \leq \mathrm{R} < \infty$. It suffices to show that $\left|\partial f_i(\mathrm{t},\ \mathrm{x})/\partial\mathrm{x}\right|$, i = 1,2,3,4, $x = S, M, T, D$ are continuous in Ω.

Proof. Let rewrite equation (1.0) as

$$f_1(t,S,M,T,D)=\psi-\tfrac{\beta_1 T}{N}S-(\mu+\eta)S\ ,$$

$$f_2(t,S,M,T,D)=\tfrac{\beta_1 T}{N}S-\beta_2 M-\mu M\ ,$$

$$f_3(t,S,M,T,D)=\beta_2 M-(\omega+\mu+e+\theta)T$$

$$f_4(t,S,M,T,D)=(\omega+e)T-\mu D\ ,$$

$$\frac{\partial f_1}{\partial s}=-\tfrac{\beta_1}{N}T^*-(\mu+\eta),\left|\frac{\partial f_1}{\partial s}\right|=\left|-\tfrac{\beta_1}{N}T^*-(\mu+\eta)\right|<\infty,$$

$$\frac{\partial f_1}{\partial M}=0,\left|\frac{\partial f_1}{\partial M}\right|=|0|<\infty,$$

$$\frac{\partial f_1}{\partial M}=(\beta_2+\mu),\left|\frac{\partial f_1}{\partial M}=(\beta_2+\mu)\right|<\infty,$$

$$\frac{\partial f_1}{\partial T} = -\tfrac{\beta_1}{N} s^*, \left|\frac{\partial f_1}{\partial T}\right| = \left|-\tfrac{\beta_1}{N} s^*\right| < \infty,$$

$$\frac{\partial f_1}{\partial D} = 0, \left|\frac{\partial f_1}{\partial D}\right| = |0| < \infty$$

$$\frac{\partial f_2}{\partial s} = \tfrac{\beta_1}{N} T^*, \left|\frac{\partial f_2}{\partial s}\right| = \left|\tfrac{\beta_1}{N} T^*\right| < \infty,$$

$$\frac{\partial f_2}{\partial M} = -(\beta_2 + \mu), \left|\frac{\partial f_2}{\partial M}\right| = |(\beta_2 + \mu)| < \infty,$$

$$\frac{\partial f_2}{\partial T} = \tfrac{\beta_1}{N} S^*, \left|\frac{\partial f_2}{\partial T}\right| = \left|\tfrac{\beta_1}{N} S^*\right| < \infty,$$

$$\frac{\partial f_2}{\partial D} = 0, \left|\frac{\partial f_2}{\partial D}\right| = |0| < \infty,$$

$$\frac{\partial f_3}{\partial s} = 0, \left|\frac{\partial f_3}{\partial s}\right| = |0| < \infty,$$

$$\frac{\partial f_3}{\partial M} = \beta_2, \left|\frac{\partial f_3}{\partial M}\right| = |\beta_2| < \infty,$$

$$\frac{\partial f_3}{\partial T} = -(\omega + \mu + e + \theta), \left|\frac{\partial f_3}{\partial T}\right| = |-(\omega + \mu + e + \theta)| < \infty,$$

$$\frac{\partial f_3}{\partial D} = 0, \left|\frac{\partial f_3}{\partial D}\right| = |0| < \infty,$$

$$\frac{\partial f_4}{\partial S} = 0, \left|\frac{\partial f_4}{\partial S}\right| = |0| < \infty,$$

$$\frac{\partial f_4}{\partial M} = 0, \left|\frac{\partial f_4}{\partial M}\right| = 0 < \infty,$$

$$\frac{\partial f_4}{\partial T} = (\omega + e), \left|\frac{\partial f_4}{\partial T}\right| = |(\omega + e)| < \infty,$$

$$\frac{\partial f_4}{\partial D} = -\mu, \left|\frac{\partial f_4}{\partial D}\right| = |-\mu| < \infty,$$

Consequently, $\left|\partial \mathrm{F}_i \;/\, \partial x\right| \in \mathrm{R}$ for $i=1,2,3,4$ exist and have feasible region solution. This existence and uniqueness banditry terrorism indicate that the problem of extremism is in the human population and the solution also lies therein

Model analysis

Bandit free equilibrium points

We obtain the terrorism free equilibrium and setting the right hand side of the system (1.0) to zero; then solving the resultant equations with ideal of Akpienbi *et al.* (2021) gives

$$E_0 = \left(D=0, M=0, S=\tfrac{\psi}{\mu+\eta}, T=0\right) = \left(\mathrm{D{=}0, M{=}0, S{=}}\frac{\psi}{\mu+\eta}\mathrm{, T{=}0}\right)$$

Interior Coexistence Equilibrium Points

The model coexistence of moderate and core banditry terrorist equilibrium point of ordinary differential equation of system (1.0) can be obtain when solving the simultaneously to have .

$$S = \frac{\left(\beta_2+\mu\right)\left(\omega+e+\theta+\mu\right)}{\beta_1\beta_2}$$

$$T = \frac{\psi\beta_1\beta_2 - \left(\beta_2+\mu\right)\left(\eta+\mu\right)\left(\omega+e+\theta+\mu\right)}{\beta_1\left(\beta_2+\mu\right)\left(\omega+e+\theta+\mu\right)}$$

$$D = \frac{\left(\omega+e\right)\psi\beta_1\beta_2 - \left(\beta_2+\mu\right)\left(\eta+\mu\right)\left(\omega+e+\theta+\mu\right)}{\beta_1\left(\beta_2+\mu\right)\left(\omega+e+\theta+\mu\right)\mu}$$

$$M = \frac{\psi\beta_1\beta_2 - \left(\beta_2+\mu\right)\left(\eta+\mu\right)\left(\omega+e+\theta+\mu\right)}{\beta_1\beta_2\left(\beta_2+\mu\right)}$$

Threshold for recruitment of bandit ideology

The dynamics of model (1) threshold quantity R_b was investigated using next generation matrix method (van den Driessche and Watmough 2002) as cited in Akpienbi and Ezra (2024). The effective R_b of the banditry terrorist model is obtained as

$$R_b = \rho\left(FV^{-1}\right) = \psi\beta_1 \frac{\beta_2}{(\mu+\eta)\left(\mu+\beta_2\right)(\theta+\mu+\omega+e)}$$

The bandit terrorist-free equilibrium, $E_0 = \left(D=0, M=0, S=\frac{\psi}{\mu+\eta}, T=0\right)$ of the system (1) is locally asymptotically stable if $R_b < 1$, , that is, terrorism dies out and unstable iff $R_b > 1$ meaning the bandit terrorist activities will persist. The threshold quantity R_b is the basic reproduction number of the terrorism. It is the conduction of a new bandit terrorist produced by prospective bandit terrorism

Stability analysis of the banditry terrorism

Here, we take the partial derivative of the banditry terrorism model system of ordinary differential equation (1.0) to obtain the Jacobian matrix as

$$\begin{vmatrix} -\beta_1 T-(\eta+\mu) & 0 & -\beta_1 S & 0 \\ \beta_1 T & -\left(\beta_2+\mu\right) & \beta_1 S & 0 \\ 0 & \beta_2 & -(\omega+e+\theta+\mu) & 0 \\ 0 & 0 & (\omega+e) & -\mu \end{vmatrix}$$

Then, we input the model equilibriums of the free and coexistence banditry terrorism and evaluate the model stability.

Stability analysis of free banditry

The local stability was obtain when substituting the equilibrium of free banditry $\left(\text{D=0,M=0,S=}\frac{\psi}{\mu+\eta}\text{,T=0}\right)$ into the model Jacobian matrix to have this characteristic polynomial $A_1\lambda^4 + A_2\lambda^3 + A_3\lambda^2 + A_4\lambda + A_5 = 0$

$$\frac{1}{\eta+\mu}((\mu+2\lambda)(\mu+\eta+2\lambda)(\mu^3+(e+\mu+\omega+\theta+4\lambda+\beta_2)\mu^2+(e+\mu+\omega+\theta+4\lambda+\beta_2)\eta$$
$$+2(\lambda+\frac{1}{2}\beta_2)(2\lambda+e+\omega+\theta)\mu+2(\lambda+\frac{1}{2}\beta_2)(2\lambda+e+\omega+\theta)\eta-\beta_1\beta_2\psi))$$

$$\lambda = -\mu, -\eta-\mu,$$

$$\frac{1}{2\eta+2\mu} \pm \sqrt{(\eta+\mu)((e+\omega+\theta-\beta_2)^2\eta+(e+\omega+\theta-\beta_2)^2\mu+4\beta_1\beta_2\psi-2\mu^2}$$
$$+(-e-\omega-\theta-2\lambda-\beta_2)\mu+(-e-\omega-\theta-2\lambda-\beta_2)\eta)$$

To have the eignvalue as

$$\lambda=-\mu$$

$$\lambda=-(\mu+\eta)$$

$$\lambda=-(\mu+\eta)$$

$$\lambda=\frac{1}{2}\frac{-(\mu+\eta)(e+\omega+\theta+\beta_2)-2\mu(\mu+\eta)+\sqrt{(\mu+\eta)\left[(\mu+\eta)(e+\omega+\theta-\beta_2)^2+4\psi\beta_1\beta_2\right]}}{(\mu+\eta)}$$

$$\lambda=-\frac{1}{2}\frac{(\mu+\eta)(e+\omega+\theta+\beta_2)+2\mu(\mu+\eta)+\sqrt{(\mu+\eta)\left[(\mu+\eta)(e+\omega+\theta-\beta_2)^2+4\psi\beta_1\beta_2\right]}}{(\mu+\eta)}$$

Then let apply Routh-Hurwitz criterion at this point, which states that all roots of the polynomial have negative real part if and only if the coefficients A_i are positive and the determinant of the matrices $D_i > 0$ for $i = 0, 1, 2, 3, 4$.

$D_1 > 0$

$D_2 = \begin{vmatrix} A_3 & A_1 \\ 1 & A_2 \end{vmatrix} = A_2A_3 - A_1 > 0$, if and only if $A_2A_3 > A_1$

$D_2 > 0$

$D_3 = \begin{vmatrix} A_3 & A_1 & 0 \\ 1 & A_2 & A_0 \\ 0 & A_3 & A_2 \end{vmatrix} = A_1A_2A_3 - A_0A^2{}_3 - A^2{}_1$

$A_1A_2A_3 - (A_0A^2{}_3 + A^2{}_1) > 0$, if and only if $A_1A_2A_3 > A_0A^2{}_3 + A^2{}_1$

$D_3 > 0$

Therefore, all the roots of the polynomial (4.19) have negative real parts, implying that $\lambda_1 < 0,\ \lambda_2 < 0,\ \lambda_3 < 0,\ \lambda_4 < 0$. Hence, since all the values of $\lambda_i < 0$, for $i = 1, 2, 3, 4.$ we conclude that the insurgents free equilibrium point is locally asymptotically stable

Global stability analysis of coexistence of banditry terrorism

We adopt Lyapunov method used in Akpienbi and Iroka (2024) to carry out the global stability of the coexistence of bandit terrorism equilibrium points. The analysis of the local stability of the Bandit Terrorism Free Equilibrium (BTFE is locally asymptotically stable if $R_b < 1$, otherwise unstable. We extend the research by taken the global stability analysis of the Coexistence of Bandit Terrorism Equilibrium (CBTE), the method taken is based on the work by akpienbi and Iroka (2024). We construct a suitable and appropriate Lyapunov function for the bandit model; aims to determine the global stability conditions for the (CBTE) as

$V = \sum a_i \left(y_i - y_i^* \ln y_i \right)$, where a_i and y_i is an appropriately selected positive constant,

y_i is a population of the i^{th} class type, and y_i^* is the stationary point then it becomes

then, $V = A_1\left(S - S^* \ln S\right) + A_2\left(M - M^* \ln M\right) + A_3\left(T - T^* \ln T\right) + A_4\left(R - R^* \ln R\right)$

The constants are non-negative in Υ such that $A_i > 0$ for $i = 1,2,3,4$ which is the Lyapunov function A_1, A_2, A_3, A_4 defined as the Lyapunov function constant are selected so that Υ should be continuous and differentiable in a space.

Finding the time derivative of V to have

$$\frac{dV}{dt} = A_1\left(1 - \frac{S^*}{S}\right)\left[\psi - \beta_1 TS - (\mu + \eta)S\right] + A_2\left(1 - \frac{M^*}{M}\right)\left[\beta_1 TS - (\beta_2 + \mu)M\right]$$

$$+ A_3\left(1 - \frac{T^*}{T}\right)\left[\beta_2 M - (\mu + e + \omega + \theta)T\right] + A_4\left(1 - \frac{R^*}{R_i}\right)\left[(\omega + e)T - \mu R\right]$$

The time derivative computing of V at substance abuse endemic stationary point yields

$$\frac{dV}{dt} = -A_1\left(1 - \frac{S^*}{S}\right) - A_2\left(1 - \frac{M^*}{M}\right) - A_3\left(1 - \frac{T^*}{T}\right) - A_4\left(1 - \frac{R^*}{R}\right) + F(S, M, T, R)$$

The function $F(S, M, T, R)$ is negative. Applying reseach work of Matowo (2013) and Korobeinikov (2004 & 2007)

Then, $F(S, M, T, R) \le 0$ for all S, M, T, R

Thus, $\frac{dV}{dt} \le 0$ for all S, M, T, R

And it zero whenever, $S = S^*, M = M^*, T = T^*, R = R^*$

Therefore, the compact invariant set is largest in S, M, T, R such that $\frac{dV}{dt} = 0$ is the singleton

E_1 which is substance abuse endemic stationary point of the model system (1).

By LaSalles's invariant principle in Korobeinikov (2007) the model proclaim that E_1 is globally asymptotically stable in the interior of substance abuse model system region of S, M, T, R. If $R_b > 1$ than bandit model system (1) has a distinctive coexistence of bandit terrorism equilibrium point E_1 which is globally asymptotically stable in S, M, T, R

Conclusion

The mathematical analysis of the banditry-terrorism model reveals critical insights into the dynamics and potential control strategies for these issues. The reproduction number R_b serves as a pivotal metric: a value less than 1 indicates the possibility of achieving a stable, banditry-free equilibrium, where such activities diminish over time. This finding highlights the importance of targeted interventions that can reduce R_b below this critical threshold, thereby ensuring long-term stability and peace.

In scenarios where $R_b \geq 1$ the persistence of banditry and terrorism becomes a significant concern, as the model suggests the emergence of globally stable coexistence equilibrium. This indicates that these issues could persist at endemic levels, necessitating comprehensive and sustained efforts beyond immediate control measures. Such strategies must address underlying socio-economic and structural factors contributing to the perpetuation of banditry and terrorism, ensuring a more holistic approach to security and stability.

The Identification of invariant regions and the assurance of unique and positive solutions enhance the model's robustness, providing a realistic framework for policy-making. These aspects ensure that the model's predictions remain within practical and meaningful limits, making it a valuable tool for planners and policymakers. Overall, the findings underscore the necessity for a complex and sustained approach to effectively address and ease the impacts of banditry and terrorism.

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