Mikailalsys Journal of Mathematics and Statistics

e-ISSN : 3030-816X
p-ISSN : 3030-8399

Index : Harvard, Boston, Sydney University, Dimensions, Lens, Scilit, Semantic, Google, etc

https://doi.org/10.58578/MJMS.v2i3.3403

APPLICATION OF QUEUE THEORY IN CAFE SERVICES WITH ERLANG DISTRIBUTION

Bagus D. Ramadhani[1], Budi Cahyono[2], Joana K. Rahayu[3], Syifa M. Rahmah[4], Andrea Tri Rian Dani[5*]

Mulawarman University, Indonesia

bagusramadhani929@gmail.com; andreatririandani@fmipa.unmul.ac.id

Article Info:

Submitted:	Revised:	Accepted:	Published:
Jul 1, 2024	Jul 15, 2024	Jul 18, 2024	Jul 21, 2024

Abstract

As urban lifestyles evolve, culinary businesses, particularly cafes, have experienced rapid growth. This surge in popularity has led to an increase in customers and, consequently, longer queues. These extended wait times can frustrate customers and pose challenges to cafe management. To address this issue, we conducted a comprehensive evaluation and optimization of the service system at a Samarinda cafe using the Erlang distribution queuing system. Primary data was meticulously collected over six days, amounting to a total of 12 hours of observation. Kolmogorov-Smirnov distribution fitting tests were employed, revealing that customer service times adhered to an exponential distribution. The average customer arrival rate was determined to be 0.351 per minute, while the average service time was calculated at 5.546 minutes per customer. Our analysis confirmed that the system operates in a steady state with a utility value of 0.06, indicating sufficient service capacity to handle the current customer load. Therefore, the study concludes that the cafe's service system is currently optimal.

Keywords: Queuing; Erlang Distribution; Service; Steady State

INTRODUCTION

As time goes by and lifestyles change, especially in urban areas, the culinary business has experienced rapid growth. One of the most popular types of culinary businesses today is the cafe (Arif & Ekasari, 2020). This is evident from the many entrepreneurs opening cafes with various interesting concepts and ideas to attract customers from different segments. In fact, cafes are also starting to appear in smaller cities (Sihombing et al., 2021).

The rising popularity of cafes is inseparable from the culture of gathering or hanging out, which has become a habit for Indonesians. This habit stems from the desire for easy access to food and beverages. This trend encourages entrepreneurs to start culinary retail businesses in the form of cafes or restaurants. However, competition in the cafe industry is increasingly demanding owners to understand the specific desires of their target market and innovate to attract consumer interest. An attractive cafe ambiance and quality service can enhance customer satisfaction, potentially making them return and become loyal customers (Sholihah, 2020).

As the number of customers increases, a new challenge arises: queues. When a cafe opens, customers start arriving and queueing to place food or drink orders. Long queues can be detrimental to both customers and cafe managers. Therefore, optimizing the number of servers and improving the service system is expected to reduce queues and enhance service quality (Wahyudi et al., 2022).

Queues are situations where a group of people gathers to wait for their turn to receive a product or service (MZ et al., 2019). Queues occur when the customer arrival rate exceeds the service capacity. In a queuing system, customers arrive and wait to be served and leave the system once they are served (Wahyudi et al., 2022). Queues can indicate inadequate service as they cause consumers to wait (Purnomo et al., 2021). Queues can also occur due to insufficient service facility capacity compared to service demand (Melinda et al., 2018).

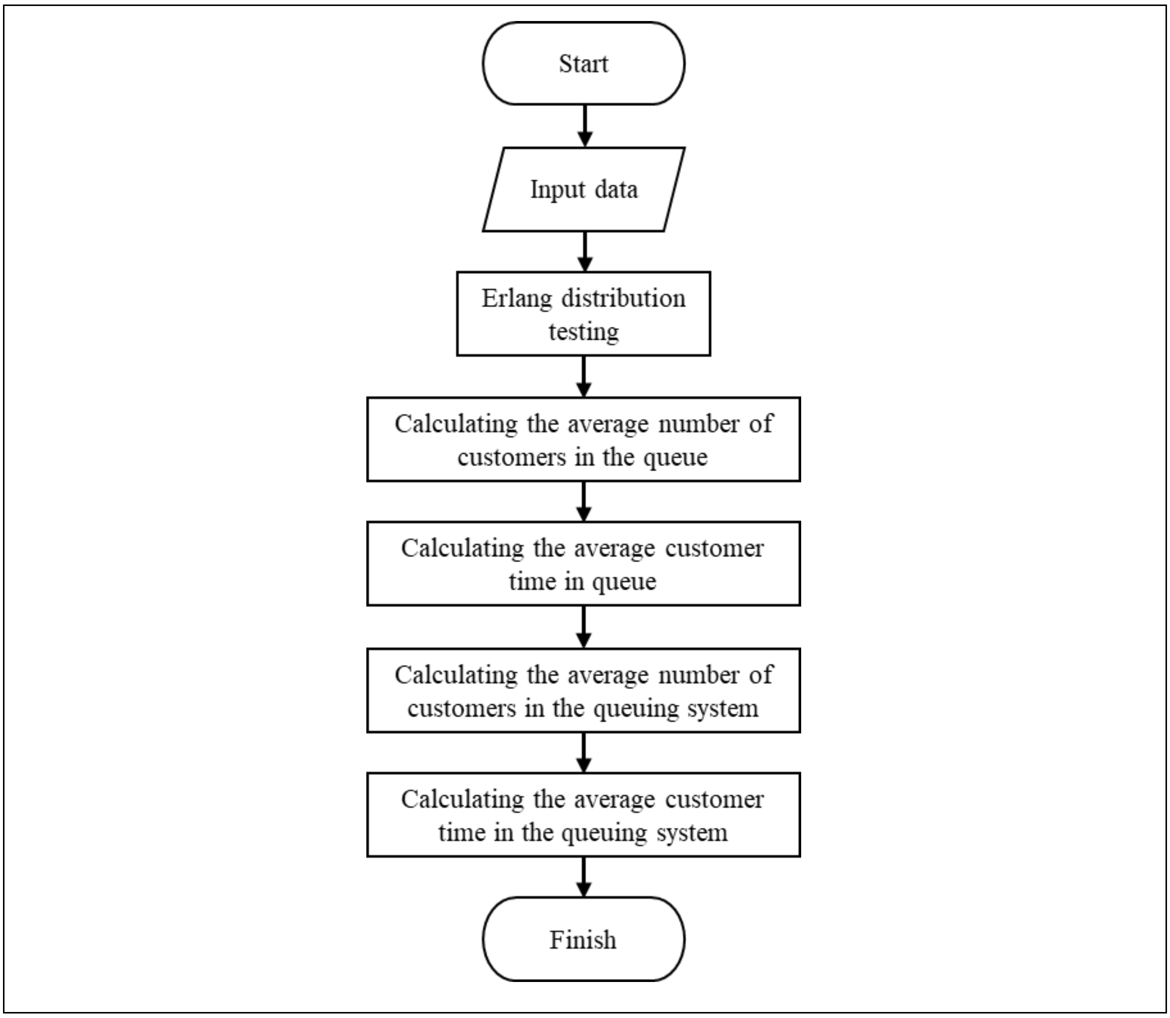
Queue problems are not only about long queues in service but also the absence of queues, which can cause losses. Queuing theory is the mathematical study of queues or situations where customers wait to receive service. The aim of this theory is to design and optimize service systems that meet customer needs and maintain cost balance. The basic components of the queuing process include customer arrivals, service, queue discipline, and queue structure. By understanding the basic components and their benefits, queuing theory can be applied in various sectors to improve service quality and operational efficiency (Bahar et al., 2018).

Considering this issue, research was conducted at a cafe in Samarinda to evaluate and optimize the service system used. This study uses the Erlang distribution queuing system as there are several stages in one order. The aim of this research is to evaluate and optimize the cafe's service system to enhance customer satisfaction while avoiding losses for the cafe owner. Thus, this research is expected to make a significant contribution to increasing the cafe's competitiveness.

METHODS

1. Data

The data used in this research is primary data. Primary data is data obtained from direct observation at Cafe X. Data collection was carried out for 6 days with a total observation time of 2 hours per day. The research began on May 27, 2024 to June 1, 2024 at 19.00 - 21.00 WITA.



2. Erlang Distribution Testing

The Erlang distribution is a particular form of the Gamma distribution where the parameters in the Gamma distribution must be positive integers. This causes the Erlang distribution to have the same characteristics as the Gamma distribution. Because of its similarity, the shape of the Erlang distribution curve follows the shape of the Gamma distribution curve (Warella et al., 2021).

According to Hillier and Liberman (1980), a random variable (T) will have an Erlang distribution if it has the following probability distribution function (PDF):

$$f(t)=\begin{cases}\dfrac{(\mu k)^k}{(k-1)!}t^{k-1}e^{-k\mu t} & ;\ for\ t\geq 0\\ 0 & ;others\end{cases} \tag{1}$$

with μ and k are parameters. Where the expected value and variance of the Erlang distribution are written as follows:

$$E(T) = \frac{1}{\mu} \quad (2)$$

$$Var(T) = \frac{1}{k\mu^2} \quad (3)$$

If $k = 1$, then the Erlang Distribution will be identical to the Exponential Distribution. Furthermore, if k approaches infinity (∞), the variance will approach zero, so that the service time which was originally a random variable will become constant (Kakiay, 2004).

In queuing theory, there are several things that need to be considered in managing queues to make them more effective and efficient. Assuming the queue model, and is the average or expectation of the arrival rate, symbolized for the service rate, and is the number of service stages, the implementation steps with the Erlang Distribution service time are as follows:

3. Average or Expected Arrival Rate

$$\lambda = \frac{\text{Total Customers}}{\text{Overall Time} \times \text{Number of Days}} \quad (4)$$

4. Service Rate

$$\mu = \frac{\text{Total Service Time}}{\text{Total Customers}} \quad (5)$$

5. Average or Expected Number of Customers in the Queuing System

$$L_s = \left(\frac{k+1}{2k}\right)\left(\frac{\lambda^2}{\mu(\mu-\lambda)}\right) \quad (6)$$

6. Average or Expected Time of Customers in Queue System

$$W_s = \frac{L_s}{\lambda} \quad (7)$$

7. Average or Expected Number of Customers in Queue

$$L_q = \left(\frac{k+1}{2k}\right)\left(\frac{\lambda^2}{\mu(\mu-\lambda)}\right) + \frac{\lambda}{\mu} \tag{8}$$

8. Average or Expected Time of Customers in Queue

$$W_q = \frac{L}{\lambda} \tag{9}$$

RESULTS

The data used in this study is primary data. Primary data is data obtained from direct observation at Cafe X. Data collection was carried out over 6 days with a total of hours for 2 hours of observation. The research will begin on 27 May 2024 until 1 June 2024 at 19.00 - 21.00 WITA. Data collection results are presented in Table 1.

Table 1. Observation data

Number	Day, Date	Working Hours		Customers
		19.00-20.00	20.00-21.00	
1.	Monday, 27 May 2024	11	12	23
2.	Tuesday, 28 May 2024	36	16	52
3.	Wednesday, 29 May 2024	11	13	24
4.	Thursday, 30 May 2024	13	21	34
5.	Friday, 31 May 2024	18	22	40
6.	Saturday, 1 June 2024	21	58	79
	Total Customers	110	142	252

Based on Table 1, the total number of customer arrivals was 252 during the six-day study. The study was based on the total observation time of 2 hours per day, which can be cumulated as 12 hours.

The distribution matching test carried out was a Kolmogorov-Smirnov test using R software. This test can determine whether the time between the arrival of customers distributed by Poisson and the time of customer service distributed Exponentially or not. Based on the output of the Kolmogorov-Smirnov test on the part time between customer arrivals using R software performed obtained p-value $=< 2,2 \times 10^{-16}$. Furthermore, since p-value $< \alpha$ is 0,05 then H_0 is rejected so that the time between customers arrival is not distributed by Poisson.

Based on the output of the Kolmogorov-Smirnov test on the part of customer service time using R software performed obtained p-value = 0,2025. Next, since p-valued $> \alpha$ is 0,05 then H_0 is received so that the time customer service data is distributed Exponential.

Based on the number of customer arrivals in Cafe X on the survey data can be calculated the average arrival of customers using the equation (4) that is:

$$\begin{aligned}\lambda &= \frac{\text{Total Customer}}{\text{Total Time} \times \text{Number of Day}} \\ &= \frac{252}{120 \times 6} \\ &= \frac{252}{720} \\ &= 0,35 \text{ customer/minute}\end{aligned}$$

So, the obtained average customer arrival is as much as 0,35 ≈ 1 customer per minute. Next, we can calculate the average time of customer service using the equation (5) with the result:

$$\begin{aligned}\mu &= \frac{\text{Total Service Time}}{\text{Total Customer}} \\ &= \frac{1396}{252} \\ &= 5,54 \text{ minutes/customer}\end{aligned}$$

So, the earned average customer service time is 5,54 ≈ 6 minutes per customer.

Steady-state is a condition where the properties of a system do not change over time or constant. The steady-state measurement of service system performance can be seen from utility values obtained from incoming and service speed data as follows:

$$\begin{aligned}\rho &= \frac{\lambda}{c \times \mu} \\ &= \frac{0,35}{1 \times 5,54} \\ &= 0,06\end{aligned}$$

Based on the utility value, it can be understood that the average arrival of customers does not exceed the service capacity. This indicates that the service system is balanced with the arrival rate so it does not require the addition of servers.

Next, we calculate the performance measurement of the system using the equations (6) to (9), using the help of the software R. Assuming the model $(M/E_k/1):(GD/\infty/\infty)$ and the number of phases (k) = 3 which means that there are three different stages that must be done all well in order for the order to be completed. The results of the calculation are presented in Table 2.

Table 2. Calculation of the loop analysis

Condition	Value
L_s	0,0660
W_s	0,0028
L_q	0,1886
W_q	0,0081

DISCUSSION

Arrests occur because the number of arrivals of customers exceeds the service facilities available. Long waiting time in the tray system can be detrimental to customers. If the number of customers that come in is only small in the pending system will cause unemployment time at the service facilities that are opened resulting in losses for the company. This problem also usually occurs in cafes. Therefore, the theory of the tray is used to solve the existing tray problem in order to optimize the service and number of servers to avoid unemployment time for the servers.

The time between arrivals at Café X is assumed to be distributed by Poisson with an arrival rate of 0,35 customers/minute, while the service time is distributed Erlang and obtained a service rate of 5,5 minutes/customer. To determine the optimum number of servers, it is necessary to have a description of the characteristics of a tray, namely performance measures. Performance measures were obtained over six days with a total of 252 customers, time between arrivals of 0,35 customers/minute and a service speed of 5,5 minutes/customer. Get a score of L_s = 0,0660 minutes, W_s = 0,0028 customers, L_q = 0,1886 minutes and W_q = 0,0081 customers. The front system in Cafe X is optimal because it has a steady-state value of less than 1 which is 0,06 so it does not require adding servers.

CONCLUSION

Based on the results and analysis, it was concluded that the cable system in Cafe X was rated optimal because the steady state value of 0,06 was less than 1. This indicates that cable systems in Café X have a low server utilization rate and the cable rarely occurs, so there is no need for the addition of a server.

REFERENCES

Arif, D., & Ekasari, R. (2020). Pengaruh Kualitas Pelayanan, Harga, dan Suasana Cafe Terhadap Kepuasan Konsumen. *Jurnal Ecopreneur*, 139-146.

Aseha, P., Rahmi, D., Yuniati, S., & Kurniati, A. (2023). Analisis Sistem Antrian Untuk Meningkatkan Efektifitas Pelayanan Pada Rumah Sakit Pekanbaru. *Matematika Sains* , 35-43.

Bahar, M. S., Mananohas, M. L., & Montolalu, C. E. (2018). Model Sistem Antrian dengan Menggunakan Pola Kedatangan dan Pola Pelayanan Pemohon SIM di Satuan Penyelenggaraan Adminstrasi SIM Resort Kepolisian Manado. *Jurnal Matematika dan Aplikasi*, 15-21.

Hillier, F. S., & Liberman, G. J. (1980). *Introduction to Operation Research Third Edition.* USA: Holden-Day.

Holland, N. (1981). *Introducing Queieng Theory Second Edition.* England: Elsevier North Holland.

Kakiay, T. J. (2004). *Dasar Teori untuk Kehidupan Nyata* . Yogyakarta: Penerbit ANDI.

Masnikafah, S., Rohmaniah, S. A., & Pradan, M. S. (2021). Analisis Sistem Antrian Pasien Rawat Jalan Menggunakan Distribusi Poisson dan Distribusi Erlang. *Jurnal UJMC*, 39-48.

Masriya, S. S., & Chaerudin. (2022). Pengaruh Kualitas Pelaynan dan Suasana Cafe terhadap Keputusan Pembelian Rai Raka Cafe di Kabupaten Karawag. *Jurnal Ilmiah Manajemen*, 233-245.

Melinda , I. D., Marpaung, S. T., & Liquiddanu, E. (2018). Analsis Sistem Antrian Restoran Cepat Saji McDonald's dengan Menggunakan SImulasi Area. *Seminar dan Konfersi Nasional IDEC.*

MZ, H., Pratiwi, I., Tamalika, T., & Husin, I. (2019). Analisis Sistem Antrian dengan Metode Simulasi. *Jurnal Desiminasi Teknologi*, 51-59.

Purnomo, B. H., Suryadharma, B., & Ekasari, N. Y. (2021). Model Sistem Antrian Pada Pelayanan Restoran Cepat Saji. *Jurnal Agroteknologi*, 40-58.

Rizal, M. T., Widiarti, Wamiliana, & Ruswandi, R. (2018). Penggunaan Teori Antrian Multi-Server Dengan Distribusi Erlang. *Seminar Nasional Metode Kuantitatif II* (hal. 228-236). Bandar Lampung: Jurusan Matematika FMIPA Universitas Lampung.

Sholihah, I. A. (2020). Suasana Cafe dan Kualitas Layanan Terhadap Keputusan Konsumen di Merdeka Cafe Ngajuk. *Jurnal Pendidikan Tata Niaga*, 750-756.

Sihombing , M. M., Arifin, H. M., & Maryono. (2021). Pengaruh Varian Menu, Harga, dan Suasana Cafe Terhadap Kepuasan Konsumen Cafe Miltie Garden Mulawarman Banjarmasin. *Smart Business Journal*, 26-33.

Silaen, F., Gea, A., Simanullang, H. G., Larosa, F. G., & Sarkis, I. M. (2022). Implementasi Algoritma Erlang pada Kinerja Aliran Traffic Data di Coffe Dante Medan. *Methotika : Jurnal Ilmiah Teknik Informatika*, 55-60.

Simamora, R. J. (2016). Simulasi Antrian dengan Model M[X]/EM/C. *Jurnal Methodika*, 131-138.

Wahyudi, D., Yundari, & Perdana, H. (2022). Analisis Sistem Antrian Pada Pola Kedatangan Berkelompok di Kafe. *Buletin Ilmiah Matj. Stat. dan Terapannya*, 833-840.

Warella, R. Y., Wattimanela, H. J., & Ilwaru, V. Y. (2021). Sifat-Sifat dan Kejadian Khusus Distribusi Gamma. *BAREKENG: Jurnal Ilmu Matematika dan Terapan*, 47-58.