

FUZZY ARITHMETIC-BASED ALGORITHM FOR IDENTIFYING MEDICAL CONDITIONS FOR BETTER TREATMENT

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Abstract

Making the right medical decision is challenging work because, in our daily life, decision-making problems may have the components of membership and non-membership degrees with the possibility of hesitation. Since soft theory offers a theoretical framework for dealing with ambiguous, fuzzy, and ill-defined objects, it is a key development in the field of computer programming as well as other scientific disciplines. Intuitionistic fuzzy soft sets provide an effective tool for solving multiple attribute decision-making with intuitionistic fuzzy information. The most essential issue is how to derive the ranking of alternatives from the information quantified in terms of intuitionistic fuzzy values. This theory also has the potential to be used to solve such real-world problems. In this work, we explore how Sanchez's medical theory could be used in medical diagnosis and provide a fuzzy arithmetic-based algorithm for identifying medical conditions to address this.

Keywords: Fuzzy Logic, Inference System, Insurance, Index of Vagueness, Claim Validation

Introduction

In our daily life, we have seen various problems in different fields, like engineering, medical science, social science, management, economics, and many more. Most of the problems are based on uncertainty, partially true, or have no clear boundaries. Classical mathematical theories cannot deal with such problems. Fuzzy logic is a suitable and best theory for dealing with such problems. The most suitable theory for dealing with such problems with uncertainty is the theory of fuzzy sets developed by Zadeh [1] in 1965. Then an outstanding contribution and plenty of work have been done in the field of fuzzy set and fuzzy logic in the last 50⁺ years. Several researchers have worked on fuzzy sets and fuzzy logic with real-world applications. Kumar et al. [2] have shown its application in the system of washing machines and medical science. Further, for the purpose of life insurance underwriting, Kumar et al [3] have presented a model based on a fuzzy expert system that will help insurance companies determine insurer mortality in the presence of diabetes. Similarly, Allahverdi, & Ertosun, [4] used fuzzy for risk determination of type-2 diabetes disease. We may face numerous encounters in our daily lives in selecting the best one for an actual outcome. Paudel et al. [5] addressed the difficulties encountered when deciding on the best candidate from a group of people in the same environment using the maximum-minimum composition. In our daily life, decision-making problems may have the components of membership and non-membership degrees with the possibility of hesitation. But fuzzy set theory is considered to have only a membership degree. So, this theory could not be considered for solving such problems. In 1996, Atanassov [6] introduced the concept of intuitionistic fuzzy sets (IFS), which is capable of capturing information that includes membership and non-membership values with some possible degree of hesitation. Then intuitionistic fuzzy set has been applied in various fields of research in making a decision and medical diagnoses. Ejegwa and Onasanya [7] showed how intuitionistic fuzzy sets could be used to solve real-world decision-making issues, like medical diagnostics and bioinformatics. Similarly, in 2020, Ejegwa and Onyeke [8] tested the new method's applicability by conducting hypothetical medical diagnoses on a few patients and determined their respective diagnoses based on the correlation coefficient values between each patient and each disease. To establish a connection between the societies and the parameters in our study, Aggarwal et al [9] employ intuitionistic fuzzy sets. Similarly, using intuitionistic fuzzy, Adamu [10] suggested a technique for sets in

environmental management to determine the type of erosion affecting some towns in order to put in place an efficient control measure.

Molodtsov [11] first introduced the soft set theory in 1999 to handle objects whose definitions used a very broad and general set of characteristics. The theory has the potential to be used to solve real-world problems in economics, engineering, the environment, social science, medicine, and business management. It is very convenient and easily applicable because there are no restrictions on the approximate description. The traditional soft set theory was fuzzified by Yang et al [12], and the fuzzy membership is used to describe parameters-approximate elements of the fuzzy soft set. In the study of decision models and their applications for simulating ambiguity and uncertainty, Deli and Çağman, [13] presented a method of making decisions that were based on intuitionistic fuzzy parameterized-soft set theory. Intuitionistic fuzzy sets and intuitionistic fuzzy soft sets are more useful for the application point of view in the field of uncertainty due to vagueness. Fuzzy soft set theory is gaining importance for finding a coherent and logical solution to various real-life problems. The concepts of fuzzy soft set and intuitionistic fuzzy soft set were used by Hooda et al [14] to study medical diagnosis using Sanchez's methodology. In [15, 16, 17, 18], we can see how skillfully the authors illustrate their research work in various fields using the concepts of fuzzy soft theory and soft set theory with applications. When solving multiple attribute decision-making problems, intuitionistic fuzzy information and fuzzy soft sets will become efficient tools. Feng et al [19] proposed a new extension of the priority method for enrichment evaluation using an intuitionistic fuzzy soft set and presented a new algorithm for solving multi-attribute decision problems. Hu et al [20] created a medical diagnosis group decision-making model by determining expert weights based on a new similarity measure of intuitionistic fuzzy soft sets and integrating evaluation information using the weighted intuitionistic fuzzy soft Bonferroni mean operator. To demonstrate the applicability and efficacy of the proposed group medical diagnosis model in an intuitive, fuzzy, soft environment, a case study is presented, followed by a comparative analysis. Here, we apply Sanchez's [21] idea to medical diagnosis and present a case study to illustrate the method. In order to do this, we build an intuitionistic fuzzy soft set using fuzzy soft set theory. For this, we present a fuzzy arithmetic-based algorithm for diagnosing medical conditions.

Definitions and Preliminaries

Suppose X is a universal set and $P(X)$ be the power set of X . We assume that E is the set of parameters and $\mathcal{A}$ is a subset of E . Then the collection $(F, \mathcal{A})$ is defined as:

$$(F, \mathcal{A}) = \{(x, F_A(x)): x \in E, F_A(x) \in P(X)\}$$

where F_A is a function from E to $P(X)$.

Here, $F_A(x)$ is known as the x -approximate function of $\mathcal{A}$. We note that $F_A(x) = \emptyset$ if $x \notin A$.

Fuzzy Set: Let X be a universal set, then the collection of pairs

$$A = \{(x, \mu_A(x)): \mu_A(x): X \rightarrow [0,1], x \in X\}$$

defines a fuzzy set $\mathcal{A}$ on X .

Here, μ_A is called a membership function defined as

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \notin A \text{ and there is no ambiguity} \\ 1 & \text{if } x \in A \text{ and there is no ambiguity} \\ (0,1) & \text{if there is ambiguity whether } x \in A \text{ or } x \notin A. \end{cases}$$

The value of $\mu_A(x)$ represents the degree of element x belonging to the set $\mathcal{A}$.

Intuitionistic fuzzy set: Let X be a non-empty set. An intuitionistic fuzzy set $\mathcal{A}$ in X is an object having the form

$$A = \{(x, \mu_A(x), \nu_A(x)): x \in X\},$$

where, $\mu_A: X \rightarrow [0,1]$ and $\nu_A: X \rightarrow [0,1]$ are the membership and non-membership function respectively, of the element $x \in X$ to the set $\mathcal{A}$.

Here, $\mu_A(x)$ and $\nu_A(x)$ are respectively called the degree of membership and degree of non-membership function of the element $x \in X$ to the set $\mathcal{A}$, and for every $x \in X$, we have $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Furthermore, in the fuzzy set, there is a lack of knowledge of whether $x \in X$ belongs to $\mathcal{A}$ or not. This lack of knowledge for $x \in X$ to the set $\mathcal{A}$ is called hesitation of x in $\mathcal{A}$ and is denoted by $\pi_A(x)$ and defined as:

$$\mu_A(x) + \nu_A(x) + \pi_A(x) = 1,$$

where, $\pi_A: X \rightarrow [0,1]$ with $0 \leq \pi_A(x) \leq 1$.

Suppose X is a universe of discourse and let A and B be two intuitionistic fuzzy sets in X , then we have

1. $A = B$ if and only if $\mu_A(x) = \mu_B(x), \nu_A(x) = \nu_B(x), \forall x \in X$.
2. $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x), \nu_A(x) \geq \nu_B(x), \forall x \in X$.
3. The complement of the intuitionistic fuzzy set A is denoted by A^c and defined by

$$A^c = \{(\nu_A(x), \mu_A(x)) : x \in X\}.$$
4. The union $A \cup B$ is defined as

$$A \cup B = \{(\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x)) \rangle) : x \in X\}.$$
5. The intersection $A \cap B$ is defined as

$$A \cap B = \{(\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)) \rangle) : x \in X\}.$$
6. $A + B = \{(\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x), \nu_A(x) \cdot \nu_B(x) \rangle) : x \in X\}.$
7. $A \cdot B = \{(\langle x, \mu_A(x) \cdot \mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x) \cdot \nu_B(x) \rangle) : x \in X\}.$

The cartesian product of A and B is defined by:

$$A \times B = \{(\langle \langle x, y \rangle, \mu_A(x) \cdot \mu_B(y), \nu_A(x) \cdot \nu_B(y) \rangle) : x \in A, y \in B\}.$$

Let X and Y be two non-empty sets and R be an intuitionistic fuzzy relation from X to Y . Suppose A be an intuitionistic fuzzy set in X then *max-min-max* composite relation of R with A being an intuitionistic fuzzy set, B of Y denoted by $B = RoA$ such that its membership function and non-membership function are defined as:

$$\mu_B(y) = \max_x \{\min[\mu_A(x), \mu_R(x, y)]\} \text{ and } \nu_B(y) = \min_x \{\max[\nu_A(x), \nu_R(x, y)]\} \text{ for all } x \in X, y \in Y.$$

Let, $Q(X \rightarrow Y)$ and $R(Y \rightarrow Z)$ be two intuitionistic fuzzy relations, then the *max-min-max* composite relation RoQ is an intuitionistic fuzzy relation from X to Z such that its membership function and non-membership function is defined by:

$$\mu_{RoQ}(x, y) = \bigvee_y \{\min[\mu_Q(x, y), \mu_R(y, z)]\} \text{ and } \nu_{RoQ}(x, y) = \bigwedge_y \{\max[\nu_Q(x, y), \nu_R(y, z)]\} \text{ for all } (x, z) \in X \times Z \text{ and for all } y \in Y.$$

Fuzzy soft set: Let X be the universal set and E is the set of all parameters and $F(X)$ be the set of all fuzzy sets in X . For $A \subseteq E$, the fuzzy soft set F_A over $F(X)$ is defined by:

$F_A = (F, A) = \{(p, F_A(p)): p \in E \text{ and } F_A(p) \in F(X)\}$, where F_A is a function, $F_A: E \rightarrow F(X)$.

If $F(X)$ is the collection of all intuitionistic fuzzy set over X , then F_A is an intuitionistic fuzzy soft set. The value $F_A(x)$ is an intuitionistic fuzzy set and is called x - element of F_A for all $x \in E$ and is defined as:

$$F_A(x) = \{ (x, \mu_A(x), \nu_A(x)): x \in X \}.$$

Methodology

Here we apply Sanche's idea for the medical diagnosis and presenting a case study to illustrate the method. In order to do this, we use intuitionistic fuzzy soft set theory, and for that presenting a fuzzy-based algorithm for diagnosing a medical condition. Assume that there is a set of m patients P and a set of n symptoms, E that is caused by a set of k diseases, D . Consider an intuitionistic fuzzy soft set (F, P) over E . This intuitionistic fuzzy set gives a patient-symptom matrix R . With the help of the intuitionistic fuzzy set (F, E) over D we construct two matrices R_1 and R_2 named symptom-disease and non-symptom-disease matrix respectively. The relation matrices M_1 and M_2 are obtained from $R \circ R_1$ and $R \circ R_2$ using *max-min-max* composition. Then we calculate the medical diagnosis table $D.S_k$, where,

$$D.S_k = \max_j \{ S.D_{M_1}(p_i, d_j) - S.D_{M_2}(p_i, d_j) \} \text{ with } d_j = \mu_j - \nu_j \pi_j.$$

From the diagnosis table $D.S_k$, we conclude that the patient p_i is suffering from the disease d_j

Algorithm

1. The output matrix R is obtained *via* input intuitionistic fuzzy soft set (F, P) .
2. The output matrices R_1 and R_2 are obtained through intuitionistic fuzzy sets (F, E) and $(F, E)^c$.
3. Calculate $M_1 = R \circ R_1$ and $M_2 = R \circ R_2$ using max-min-max rule.
4. The diagnosis matrices $S.D_{M_1}$ and $S.D_{M_2}$ are de-fuzzify of M_1 and M_2 respectively.

5. Calculate, $D.S_k = \max_j \{S.D_{M_1}(p_i, d_j) - S.D_{M_2}(p_i, d_j)\}$ with $d_j = \mu_j - v_j \pi_j$.
6. Conclude that the patient p_i is suffering from the disease d_j .

Case Study

Let us consider a universal set $P = \{p_1, p_2, p_3, p_4\}$ of patients in a hospital with different symptoms say body temperature, headache, dizziness, and body pain. Consider a set

$S = \{e_1, e_2, e_3, e_4\}$ of parameters where, e_1 = body temperature, e_2 = headache, e_3 = dizziness, e_4 = body pain.

Let $D = \{d_1, d_2, d_3\}$ be the set of diseases, where d_1 = typhoid, d_2 = malaria, d_3 = covid.

Here, the temperature is measured with the help of medical instrument (digital thermometer), while headache, dizziness and body pain are the rating scale in the interval, which are obtained via the questions to the patients and we prepare the following table:

Table (1): Patient -Symptoms Table

	e_1	e_2	e_3	e_4
p_1	103.6	6	6	7
p_2	102.8	7	4	8
p_3	104.4	5	8	8
p_4	102	4	4	6

To fuzzify the above data, we use the membership functions

$$\mu_T : [98, 106] \rightarrow [0, 1] \text{ and } \mu : [0, 10] \rightarrow [0, 1].$$

Here μ_T indicates the membership function defined for the body temperature and the second one indicates for other symptoms. We note that, the body temperature 98 means there is no fever in body and 106 means extreme level of body temperature and we have $\mu_T(98) = 0$ and $\mu_T(106) = 1$. Also $\mu(0) = 0$ and $\mu(10) = 1$ for the rest of others symptoms.

For any $x \in [98, 106]$, we define the membership function as follow

$$\mu_T(x) = \frac{x-98}{106-98} = \frac{x-98}{8},$$

and for, $x \in [0, 10]$, the membership function is defined as $\mu(x) = \frac{x-0}{10-0} = \frac{x}{10}$.

Then the corresponding fuzzified data of above table is as:

Table 2: Patient -Symptoms Table

	e_1	e_2	e_3	e_4
p_1	.7, .1, .2	.6, .2, .2	.6, .1, .3	.7, .2, .1
p_2	.6, .1, .3	.7, .1, .2	.4, .4, .2	.8, 0, .2
p_3	.8, .1, .1	.5, .3, .2	.8, .1, .1	.8, .2, 0
p_4	.5, .2, .1	.4, .3, .3	.4, .5, .1	.6, .3, .1

Now, we construct a matrix to show the relation between patients and symptoms:

$$R = \begin{matrix} & e_1 & e_2 & e_3 & e_4 \\ \begin{matrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{matrix} & \begin{bmatrix} .7, .1, .2 & .6, .2, .2 & .6, .1, .3 & .7, .2, .1 \\ .6, .1, .3 & .7, .1, .2 & .4, .4, .2 & .7, .2, .1 \\ .8, .1, .1 & .5, .3, .2 & .8, .1, .1 & .8, 0, .2 \\ .5, .2, .1 & .4, .3, .3 & .4, .5, .1 & .6, .3, .1 \end{bmatrix} \end{matrix}$$

This matrix is known as *patient- symptom matrix*. Now we consider a table to show the relationship between diseases and their corresponding parametric values.

Table 3: Symptoms- Diseases Table

	d_1	d_2	d_3
e_1	100.4	103.6	102
e_2	6	3	5
e_3	4	2	4
e_4	4	5	7

The corresponding fuzzified table is given below:

Table 4: Symptoms- Diseases Table

	d_1	d_2	d_3
e_1	.3, .3, .4	.7, .2, .1	.5, .3, .2
e_2	.6, .3, .1	.3, .4, .3	.5, .2, .3
e_3	.4, .5, .2	.2, .2, .3	.4, .4, .2
e_4	.4, .3, .3	.5, .2, .3	.7, .2, .1

Now, we introduce two matrices R_1 and R_2 named as *symptom- disease matrix* and *non- symptom -disease matrix* as follow:

$$R_1 = \begin{matrix} & d_1 & d_2 & d_3 \\ \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{matrix} & \begin{bmatrix} (.3, .3, .4) \\ (.6, .3, .1) \\ (.4, .5, .2) \\ (.4, .3, .3) \end{bmatrix} & \begin{bmatrix} (.7, .2, .1) \\ (.3, .4, .3) \\ (.2, .5, .3) \\ (.5, .2, .3) \end{bmatrix} & \begin{bmatrix} (.5, .3, .2) \\ (.5, .2, .3) \\ (.4, .4, .2) \\ (.7, .2, .1) \end{bmatrix} \end{matrix}$$

$$R_2 = \begin{matrix} & d_1 & d_2 & d_3 \\ \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{matrix} & \begin{bmatrix} (.3, .3, .4) \\ (.3, .6, .1) \\ (.5, .4, .2) \\ (.3, .4, .3) \end{bmatrix} & \begin{bmatrix} (.2, .7, .1) \\ (.4, .3, .3) \\ (.5, .2, .3) \\ (.2, .5, .3) \end{bmatrix} & \begin{bmatrix} (.3, .5, .2) \\ (.2, .5, .3) \\ (.4, .4, .2) \\ (.2, .7, .1) \end{bmatrix} \end{matrix}$$

To diagnosis the diseases of the patients, we construct two new matrices says M_1 and M_2 called *patient-disease* and *patient-non disease matrix* respectively using the *max-min-max* method:

$$\mu_{M_1}(p_i, d_j) = \vee \{ \mu_R(p_i, e_j) \wedge \mu_{R_1}(e_j, d_k) \}, \text{ and } \mu_{M_1}(p_i, d_j) = \wedge \{ \nu_R(p_i, e_j) \vee \nu_{R_1}(e_j, d_k) \},$$

so, we have:

$$M_1 = R \circ R_1 = \begin{matrix} & d_1 & d_2 & d_3 \\ \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{matrix} & \begin{bmatrix} (.6, .3, .1) \\ (.6, .3, .1) \\ (.5, .3, .2) \\ (.4, .3, .3) \end{bmatrix} & \begin{bmatrix} (.7, .2, .1) \\ (.6, .2, .2) \\ (.7, .2, .1) \\ (.5, .2, .3) \end{bmatrix} & \begin{bmatrix} (.7, .2, .1) \\ (.7, .2, .1) \\ (.7, .2, .1) \\ (.6, .3, .1) \end{bmatrix} \end{matrix}$$

Similarly,

$$M_2 = R \circ R_2 = \begin{matrix} & d_1 & d_2 & d_3 \\ \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{matrix} & \begin{bmatrix} (.5, .3, .2) \\ (.4, .3, .3) \\ (.5, .3, .2) \\ (.4, .3, .3) \end{bmatrix} & \begin{bmatrix} (.5, .2, .3) \\ (.4, .3, .3) \\ (.5, .2, .3) \\ (.4, .3, .3) \end{bmatrix} & \begin{bmatrix} (.7, .2, .1) \\ (.7, .2, .1) \\ (.7, .2, .1) \\ (.6, .3, .1) \end{bmatrix} \end{matrix}$$

Now, we calculate the diagnosis score which helps us to conclude that, the patient p_i is suffering from the disease d_k , and for this, we use the formula:

$$D.S_k = \max_j \{ S.D_{M_1}(p_i, d_j) - S.D_{M_2}(p_i, d_j) \} \text{ with } d_j = \mu_j - \nu_j \pi_j.$$

Here, $d_1 = .6 - .2 \times .2 = .56$, $d_2 = .7 - .2 \times .1 = .68$, $d_3 = .7 - .2 \times .1 = .68$.

Similarly, we can calculate the remaining and for M_2 also and we have

$$D.S_{M_1} = \begin{matrix} & d_1 & d_2 & d_3 \\ \begin{matrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{matrix} & \begin{bmatrix} .57 & .68 & .68 \\ .57 & .56 & .68 \\ .44 & .68 & .68 \\ .31 & .44 & .57 \end{bmatrix} \end{matrix} \quad \text{and} \quad D.S_{M_2} = \begin{matrix} & d_1 & d_2 & d_3 \\ \begin{matrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{matrix} & \begin{bmatrix} .44 & .44 & .68 \\ .31 & .31 & .68 \\ .44 & .44 & .68 \\ .31 & .31 & .57 \end{bmatrix} \end{matrix}$$

The following diagnosis table is obtained by using the formula $D.S_{M_1} - D.S_{M_2}$, shows relations of patients and their corresponding disease.

Table 5: Disease -Diagnosis Table

	d_1	d_2	d_3
p_1	.13	.24	0
p_2	.26	.25	0
p_3	0	.24	0
p_4	0	.13	0

From this *disease-diagnosis* table, we can conclude that the patient p_1 is suffering from disease d_2 that is from malaria. The patient p_2 is suffering from typhoid. And the patients p_3 and p_4 are also suffering from malaria.

Conclusion

Since soft theory offers a theoretical framework for dealing with ambiguous, fuzzy, and ill-defined objects, it is a key for solving multiple attribute decision-making with intuitionistic fuzzy information. The best medical decision can be challenging to implement because, in daily life, membership and non-membership degrees with the potential for hesitation can be included in decision-making issues. In this study, we looked into how Sanchez's medical theory could be used for diagnosing patients using an intuitionistic fuzzy set through a fuzzy arithmetic-based algorithm.

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