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Equilibrium Solutions of Stackelberg Competition Strategy

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Abstract

Strategic interactions between public and private firms have important implications for competition policy and privatization decisions. This study examines equilibrium outcomes in Stackelberg duopoly and oligopoly markets, focusing on how a public firm's strategic position shapes the welfare consequences of privatization. A game-theoretic equilibrium framework is used to compare market outcomes before and after privatization under alternative leadership and follower configurations, with Cournot competition also examined. The results indicate that privatization does not necessarily improve social welfare under Cournot competition when the number of private firms is relatively small. Under Stackelberg competition, privatization reduces social welfare when the initially public firm retains its leadership position after privatization, irrespective of the number of private competitors. When the firm instead retains its follower position, privatization also reduces social welfare in markets with relatively few private competitors. These findings demonstrate that the welfare consequences of privatization depend on both market concentration and firms' strategic roles. The study contributes to the industrial organization literature by clarifying how market structure and leadership arrangements condition privatization outcomes. Its policy implications emphasize the need to consider the number of private competitors and the strategic position of the firm undergoing privatization when assessing potential welfare gains.

Keywords: Stackelberg Competition; Cournot Competition; Mixed Oligopoly; Privatization; Social Welfare

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INTRODUCTION

Competition is a strategy where multiple businesses compete for scarce resources by altering the four components of the marketing mix: location, price, product, and promotion. According to traditional economic theory, competition forces companies to create new goods, services, and technologies, giving customers more options and better products. Product prices are often lower than they would be under an oligopoly or during a monopoly when there are more options available (Abedian et al, 2022).

In economics, the term "Stackelberg" refers to a strategic game model in which a firm (the leader) chooses a quantity first, and the other firms (the followers) then successively choose their quantities after seeing the leader's choice. Backward induction is used to solve the subgame-perfect equilibrium because of its sequential character, which enables the leader to predict the follower's reaction. In contrast to the simultaneous-move Cournot model, this kind of duopoly involves firms competing on quantity (Kalashnikov et al, 2010). So, a Stackelberg game is defined as a sequential game where players are divided into leaders and followers. Leaders make the first move to maximize their profits, and followers respond accordingly to maximize their own profits (Demacarty, 2009). It is commonly used in communication and networking for various applications such as resource allocation and power allocation.

Unlike other non-cooperative games in which players act together, the Stackelberg game is a sequential game to model sequential multi-agent decision-making processes (Zhang, 2023). Players are divided into two groups, namely, leaders and followers. The leaders make the first move to maximize their profits. Followers then make corresponding moves and maximize their profits given the strategies of leaders. The whole process can be framed into a two-stage interplay, which is desirable in wireless network scenarios with less transmission overhead. In a Stackelberg game, the Nash equilibrium can be solved with backward induction when the best response strategy of leaders is calculated with the anticipation for the best responses from followers (Sinha et al, 2014).

The Stackelberg game is one of the most widely used game models in communication and networking, e.g., for resource allocation in cognitive radio networks, anti-jamming, proactive caching in mobile edge networks and power allocation in vehicle-to-everything communications. In the typical use case of the

Stackelberg game for satellite communications, usually satellites are leaders, and terrestrial users are followers (Zhou et al, 2021). The number of enterprises, entry barriers, information, and the availability/accessibility of resources are only a few of the many variables that influence a market's level of competition. Because every consumer has a propensity to pay, the number of consumers in the market influences both the level of competition and the total demand for the product (Schmidt et al, 2015).

A firm's sub-sector, or a country's ability and performance to sell and supply goods and services in a given market is referred to as its competitiveness when compared to the ability and performance of other firms, sub-sectors, or countries in the same market. One firm is attempting to determine ways to deprive another company of market share. The Latin word "competere," which describes the competition that exists between entities in markets and industries, is where the word "competitiveness" originates. It is frequently used in management discourse when discussing comparisons of regional and global economic performance (Muhammad & Idris, 2025).

Stackelberg Competition

The Stackelberg Competition Model is an important theory in microeconomics that explains how firms compete in an oligopoly, a market structure where only a few firms dominate the industry. The model was introduced by the German economist Heinrich von Stackelberg in 1934.

The unique feature of this model is that firms do not make their production decisions at the same time. Instead, one firm, called the leader, decides how much to produce first. The second firm, called the follower, observes the leader's decision and then decides its own output. Because the leader moves first, it can influence the follower's behavior and usually earns a higher profit.

A strategic game in economics known as the Stackelberg leadership model has the leader firm moving first, followed by the followers businesses in that order. It bears the name of the German economist Heinrich Freiherr von Stackelberg, who described the concept in his 1934 book Marktform und Gleichgewicht (Market Structure and Equilibrium) (Von Stackelberg, 1953). According to game theory, there are two participants in this game: a leader and a follower. They compete on

quantity. Sometimes the Market Leader is used to refer to the Stackelberg leader (Simaan et al, 1973).

There are several extra limits upon the maintenance of a Stackelberg equilibrium. The leader must be aware before acting that the follower is watching. The leader must be aware that the follower lacks the ability to commit to a future non-Stackelberg leader's activity. The optimal course of action for the "leader" would be to play a Stackelberg follower action if the "follower" had the ability to commit to a Stackelberg leader action and the "leader" was aware of this.

Stackelberg competition may occur between firms if one has a competitive edge that allows it to act early. Generally speaking, the leader needs to have commitment power. Moving observably first is the most clear way to demonstrate commitment since, once the leader has moved, it cannot go back on its decision because it has committed to it. If the leader is the current industry monopoly and the following is a newcomer, it might be conceivable to move first. Another method of commitment is to allow future changes of strategy.

Literature Review

The Stackelberg model is one of the foundational theories in industrial organization and oligopoly economics. Introduced by Heinrich von Stackelberg in Market Structure and Equilibrium, the model explains market competition under conditions where firms make production decisions sequentially rather than simultaneously (von Stackelberg, 2010). The model assumes that one firm acts as the market leader by committing to an output level first, while the competing firm, known as the follower, observes this decision before determining its own output. This sequential decision-making process creates a first-mover advantage, enabling the leader to influence market outcomes and generally earn higher profits than the follower.

The Stackelberg model was developed as an extension of Cournot's (1897) oligopoly theory (Cournot, 1897). While the Cournot model assumes that firms choose their output simultaneously without knowledge of competi-tors' decisions, the Stackelberg model introduces sequential decision-making into the analysis. This distinction significantly alters market equilibrium. Under the Stackelberg framework, the leader strategically considers the follower's reaction when determining its optimal

output, resulting in higher total market output, lower market prices, and greater profits for the leader compared with the Cournot equilibrium (Von Stackelberg, 2010).

According to von Stackelberg (2010), strategic commitment is central to the leader's advantage. Since the leader moves first, it can anticipate the follower's optimal response and make decisions that maximize its own profit. This concept of strategic commitment has become one of the most influential ideas in game theory and has shaped subsequent research on strategic interactions among competing firms.

Building upon Stackelberg's original work, several scholars have extended the model to account for more realistic market conditions. Spence (1977) demonstrated that firms can use investment decisions, such as expanding productive capacity, as strategic commitments to deter potential competitors (Spencer, 1977). His study showed that early investment strengthens a firm's leadership position and influences the behaviour of rival firms. Similarly, Dixit (1980) argued that investment decisions can serve as barriers to entry, reinforcing the strategic advantage enjoyed by incumbent firms in oligopolistic markets (Dixit, 1980).

The theoretical foundations of the Stackelberg model were further strengthened by Fudenberg and Tirole (1991), who integrated the model into modern non-cooperative game theory. They emphasized that leadership and commitment are fundamental elements of strategic interaction and demonstrated that the Stackelberg framework can explain decision-making in a broad range of economic environments. Their contribution expanded the model beyond traditional industrial organization and established it as one of the core concepts in game-theoretic analysis.

Over time, the application of the Stackelberg model has expanded considerably. Researchers have suc-cessfully applied the framework to energy markets, transportation systems, telecommunications, environmental economics, healthcare, and supply chain management. In supply chain analysis, manufacturers often assume the role of leaders by determining wholesale prices or production quantities, while retailers respond by select-ing order quantities or retail prices. Such leader–follower relationships closely resemble the assumptions of the Stackelberg model and demonstrate its practical relevance in modern business environments (Fudenberg & Tirole, 1991).

The model has also gained prominence outside economics. In engineering and computer science, Stackelberg games are widely employed in wireless communication networks, cybersecurity, artificial intelligence, smart grids, cloud computing, and resource allocation problems. In these applications, one decision-maker commits to a strategy before other agents respond, making the model particularly suitable for analysing hierarchical optimization and decentralized decision-making.

Despite its extensive application, the Stackelberg model has attracted several criticisms. One limitation is its assumption of perfect information, whereby the follower is assumed to observe the leader's decision com-pletely and accurately. In practice, firms often operate under conditions of uncertainty regarding competitors' production costs, market demand, and strategic intentions. Additionally, the model assumes that the leader's commitment is both credible and irreversible, assumptions that may not hold in dynamic markets characterized by rapid technological change and strategic flexibility. Furthermore, many real-world industries involve multiple firms and differentiated products, making competition more complex than the simple two-firm leader–follower structure proposed in the original model.

Nevertheless, the Stackelberg model remains one of the most influential frameworks for analysing strategic competition in oligopolistic markets. Its emphasis on sequential decision-making, strategic commitment, and first-mover advantage continues to provide valuable insights into firm behaviour, market pricing, production decisions, and profitability. Consequently, the model remains widely used in economics, management, operations research, and engineering to explain competitive behaviour and guide strategic decision-making.

Recent Empirical and Theoretical Developments

Recent studies have significantly expanded the application of the Stackelberg model beyond its traditional use in industrial organization and oligopoly theory. Contemporary research demonstrates that the leader–follower framework has become an essential analytical tool in energy systems, supply chain management, manufacturing, artificial intelligence, transportation, and communication networks.

For instance, Zhang et al. (2025) developed a Stackelberg game framework for mixed stochastic control problems involving mean-field jump-diffusion systems with

Markovian switching (Zhang et al, 2025). Their findings demonstrate that leader–follower strategies can effectively optimize dynamic decision-making under uncertainty, highlighting the increasing importance of Stackelberg games in engineering and optimal control theory.

Similarly, Li et al. (2025) applied the Stackelberg model to cloud energy storage systems using time-based pricing strategies. Their study showed that the leader–follower approach improves energy allocation efficiency while balancing the economic interests of operators and consumers, illustrating the model's relevance in modern electricity markets (Li et al., 2025).

In sustainable logistics, Zhao et al. (2025) proposed a two-stage scheduling framework that integrates a Stackelberg game with energy management in seaport operations (Zhao et al., 2025). The authors found that treating port authorities as leaders and shipowners as followers enhances operational efficiency, reduces energy costs, and improves the overall economic performance of the logistics system.

The application of the Stackelberg model has also expanded into advanced manufacturing systems. Yu-wono, Schwung, and Schwung (2025) introduced an automated mechanism for selecting leaders and followers in modular manufacturing environments. Their research demonstrated that data-driven leader selection improves coordination among production modules and enhances manufacturing flexibility, thereby extending the practical applicability of Stackelberg games in Industry 4.0 environments (Yuwono et al., 2025).

In environmental and sustainable supply chain management, Zhang et al. (2020) developed a Stackelberg game model integrating production planning, pricing, retailer selection, and emission control. Their findings indicate that sequential decision-making enables firms to improve profitability while simultaneously reducing environmental impacts, supporting the growing application of Stackelberg models in sustainable operations management (Zhang et al., 2020).

These recent contributions demonstrate that the Stackelberg model has evolved from a classical duopoly framework into a versatile analytical tool capable of addressing complex hierarchical decision-making problems across diverse disciplines. The continued integration of Stackelberg games with optimization methods, artificial intelligence, sustainable energy systems, and supply chain management confirms its

enduring relevance in both theoretical and applied research.

MODELLING

In this section, we will demonstrate how the Stackelberg competition model has been applied to solve specific problems, particularly in the context of the duopoly model. We will also apply the context of oligopoly and monopoly to some problems. The Stackelberg competition is a leadership model in economics in which the leader firm moves first and then the follower firm moves sequentially. Here, the firms compete on the amount of output they will produce (Von Stackelberg, 1953).

Real-World Examples of Stackelberg Competition

1 Microsoft's hegemony in the software markets may be a Stackelberg leadership example. Despite Mi-crosoft's ability to decide first, other smaller businesses will follow.

2 Another practical example of this is diamond production, where De Beers (currently, almost 29.5% of the raw diamond production in the world is sold by the De Beers Group) is the pioneer in the production of diamond.

3 Automobile Industry: A large car manufacturer increases production before smaller firms decide on their production levels.

4 Airline Industry: A major airline introduces new routes or expands capacity, and smaller airlines respond.

5 Telecommunications: A dominant telecom company launches a new pricing plan before competitors adjust theirs.

6 Oil Industry: Large oil-producing firms increase or reduce production, influencing the decisions of smaller producers.

Note:

• The leader earn a higher profit for two reasons:

i the leader knows that by increasing q_1 the follower will reduce q_2 (i.e; strategic substitutes)

ii the decision is irreversible (otherwise the leader would undo its choice and we would end up in Cournot again).

• The sequential move game (Stackelberg) leads to a more competitive equilibrium than the simultaneous move game (Cournot).

Stackelberg Duopoly Model

Here the firms moves sequentially

In a duopoly model, the Stackelberg competition will be a competition between two firms: firm 1 and 2

Let the price function P(Q) be:

$$P(Q) = a - bQ,$$

where P(Q) is the inverse demand function (i.e; P is the price)

Q is the total quantity demand, $Q = \sum_i^n q_i$, $i = 1, ..., n$

but here $Q = q_1 + q_2$

a and b are constants

Let the total cost be:

$$TC_i = C_i(q) = Cq_i$$

$$\Rightarrow \quad MC_i = \frac{\partial TC_i}{\partial q_i} = c$$

then, $MC_i = c$, i.e the marginal cost of firm i

$$also \quad AC_i = \frac{TC_i}{q_i}$$
$$= \frac{cq_i}{q_i} = c$$

then, $AC_i = c$, i.e; the Average cost

$$Thus \quad MC_i = AC_i = c$$

Therefore:

$$P(Q) = a - bQ = a - bq_1 - bq_2$$

Note: the profit of a given firm is equal to the revenue minus cost. Revenue is the product of price and quantity of the firm and cost is given by the firm's cost structure, so profit is:

Each firm wants to maximize profit.

$$\begin{aligned}\pi_i &= TR_i - TC_i \\ &= P(Q)q_i - cq_i \\ &= (a - bQ)q_i - cq_i \\ &= (a - bQ - c)q_i \\ &= (a - bq_1 - bq_2 - c)q_i, \quad (i.e; \quad the \quad profit \quad function \quad for \quad firm \quad i)\end{aligned}$$

Note that: in Stackelberg competition, firm 1, (i.e; the leader), will move first and then firm 2, (i.e; the follower) will then move

Backward induction:
Followers output choice

Here q_1 is given and q_2 is not given

Note: in Stackelberg competition, we will first find the Best Response Function (BRF) of the follower firm (i.e; firm 2), since the output of firm 1 is given but the output of firm 2 is not given

Therefore;

The profit function for firm 2 is:

$$\begin{aligned}\pi_2 &= P(Q)q_2 - cq_2 \\ &= (a - bQ)q_2 - cq_2 \\ &= (a - bQ - c)q_2 \\ &= (a - bq_1 - bq_2 - c)q_2 \\ &= aq_2 - bq_1q_2 - bq_2^2 - cq_2\end{aligned}$$

Recall that $\frac{\partial \pi_2}{\partial q_2} = 0$

$$\Rightarrow \quad a - bq_1 - 2bq_2 - c = 0$$

$$then \quad 2bq_2 = a - bq_1 - c$$

$$thus \quad q_2 = \frac{a - bq_1 - c}{2b}$$

(i.e; the best response function (BRF) of firm 2)

The profit function of firm 1:

$$\begin{aligned}\pi_1 &= (a - bq_1 - bq_2 - c)q_1 \\ &= aq_1 - bq_1^2 - bq_1q_2 - cq_1\end{aligned}$$

Recall that $q_2 = \frac{a-bq_1-c}{2b}$

Note, in Stackelberg competition, we substitute the BRF of firm 2 into the profit function of firm 1

Therefore;

$$\pi_1 = aq_1 - bq_1^2 - bq_1(\frac{a - bq_1 - c}{2b}) - cq_1$$

$$also \quad \frac{\partial \pi_1}{\partial q_1} = 0$$

$$\Rightarrow \quad a - 2bq_1 - q_1(\frac{-b}{2}) - (\frac{a - bq_1 - c}{2})(1) - c = 0$$

$$and \quad a - 2bq_1 + b(\frac{q_1}{2}) - \frac{a}{2} + b(\frac{q_1}{2}) + \frac{c}{2} - c = 0$$

$$then \quad \frac{a}{2} - bq_1 - \frac{c}{2} = 0$$

$$Thus \quad q_1^* = \frac{a - c}{2b},$$

(i.e; the best response function (BRF) of the leader firm (firm 1), in terms of a, b and c)
Substituting $q_1^* = \frac{a-c}{2b}$ in q_2, we have:

$$\begin{aligned} q_2^* &= \frac{a - bq_1 - c}{2b} \\ &= \frac{a - (\frac{a-c}{2}) - c}{2b} \\ &= \frac{2a - a + c - 2c}{4b} \\ &= \frac{a-c}{4b}, \end{aligned}$$

(i.e; the BRF of firm 2 in terms of a, b and c)
Therefore:

$$\begin{aligned} Q^* &= q_1^* + q_2^* \\ &= \frac{a-c}{2b} + \frac{a-c}{4b} \\ &= \frac{3(a-c)}{4b}, \end{aligned}$$

(i.e; the total quantity in terms of a, b and c)

$$\begin{aligned} \Rightarrow \quad P(Q) &= a - bQ \\ &= a - \frac{3(a-c)}{4} \\ &= \frac{4a - 3a + 3c}{4} \\ &= \frac{a + 3c}{4} \end{aligned}$$

$$Thus \quad P^* = \frac{a+3c}{4}$$

Note: in Stackelberg competition, q_1 and q_2 are not equal, therefore π_1 and π_2 are also not equal

$$\begin{aligned} Thus \quad \pi_1 &= P^* q_1^* - c q_1^* \\ &= (P^* - c) q_1^* \\ &= (a - bQ^* - c) q_1^* \\ &= (a - \frac{3(a-c)}{3} - c)(\frac{a-c}{2b}) \\ &= (\frac{4a - 3a = 3c - 4c}{4})(\frac{a-c}{2b}) \\ &= (\frac{a-c}{4})(\frac{a-c}{2b}) \\ &= \frac{(a-c)^2}{8b}, \end{aligned}$$

(i.e; the profit function of firm 1)

$$
\begin{aligned}
\textit{and} \quad \pi_2 &= P^* q_2^* - c q_2^* \\
&= (P^* - c) q_2^* \\
&= (a - bQ^* - c) q_2^* \\
&= (a - \frac{3(a-c)}{3} - c)(\frac{a-c}{4b}) \\
&= (\frac{4a - 3a = 3c - 4c}{4})(\frac{a-c}{4b}) \\
&= (\frac{a-c}{4})(\frac{a-c}{4b}) \\
&= \frac{(a-c)^2}{16b},
\end{aligned}
$$

(i.e; the profit function of firm 2)
'herefore, the total profit for both firms is:

$$
\begin{aligned}
\pi^* &= \pi_1^* + \pi_2^* \\
&= \frac{(a-c)^2}{8b} + \frac{(a-c)^2}{16b} \\
&= \frac{3(a-c)^2}{16b},
\end{aligned}
$$

we can see that here, $\pi^* = 3\pi_2$

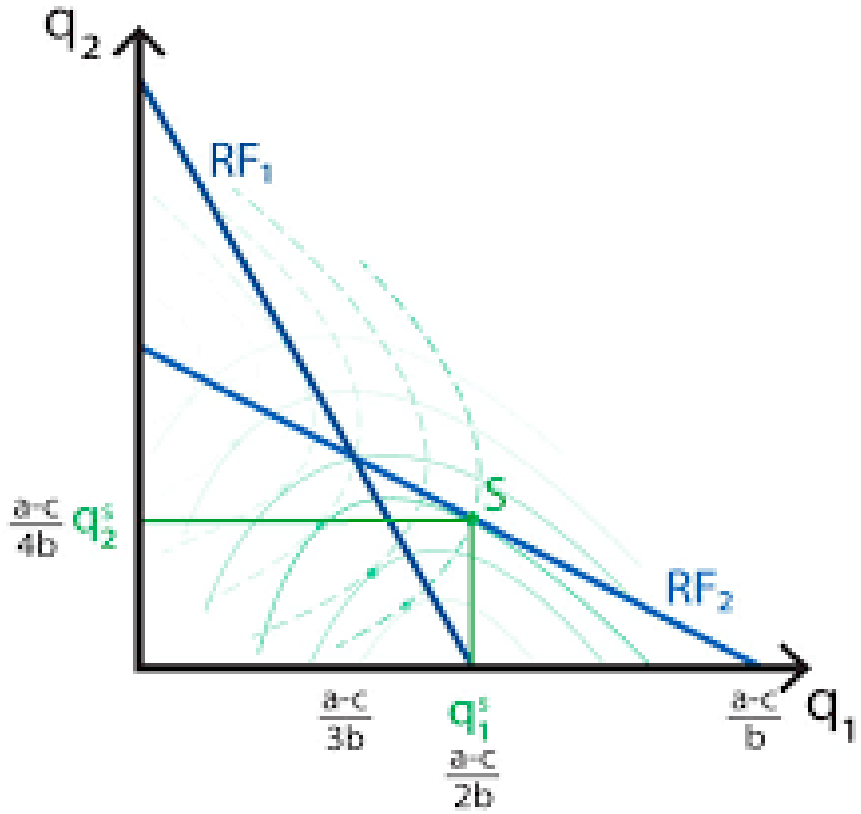


Figure 1: Stackelberg Duopoly (from policonomics.com)

Nash Equilibrium (NE) for Stackelberg competition

The Stackelberg model can be solved to determine the subgame perfect Nash equilibrium (SPNE), which is the optimal strategy profile for each player given the other player's strategy and requires that each player play in a Nash equilibrium in each subgame.

In the broadest sense, let P represent the price function for the (duopoly) industry. Since price is merely a function of total output, so is $P(Q)$. Assume company i has cost structure $c_i q_i$. By using backward induction, the model is solved. The leader takes into account the follower's best response, or how it will react after observing the leader's quantity. The leader then chooses an amount that maximises its reward while predicting the follower's anticipated reaction. The follower actually notices this and, in a stable state, chooses the anticipated amount as a response.

The best response functions of the follower must first be calculated before the SPNE can be calculated (calculation moves "backwards" due to backward induction).

So, the Nash equilibrium for Stackelberg model are q^*, q^* i.e; when:

$$q_1^* = \frac{a-c}{2b},$$

$$q_2^* = \frac{a-c}{4b}$$

$$and \quad Q = q_1 + q_2$$

Note: In Stackelberg competition $q_1 \neq q_2$, therefore $\pi_1 \neq \pi_2$

Stackelberg Oligopoly Model (n-firms model)

For an oligopoly Stackelberg model with n-firms: Let the inverse demand function be:

$$P(Q) = a - bQ,$$

where $P(Q)$ is the price function,
Q is the total quantity, $Q = \sum_{i=1}^{n} q_i$,
a and b are constants
Let $TC_i = C_i(q) = Cq_i$ i.e the total cost

$$\Rightarrow \quad MC_i = \frac{\partial TC_i}{\partial q_i} = c$$

So the marginal cost is:

$$MC_i = C$$

$$also \quad AC_i = \frac{TC_i}{q_i} = \frac{Cq_i}{q_i} = c$$

$$then \quad AC_i = C, \quad i.e; \quad the \quad Average \quad cost$$

$$Thus \quad MC_i = AC_i = c$$

This implies the marginal cost and the corresponding average cost of a given firm are equal

$$\begin{aligned} Then \quad P(Q) &= a - bQ \\ &= a - b(q_1 + q_2 + ... + q_n) \\ &= a - bq_1 - bq_2 - ... - bq_n \end{aligned}$$

Note: the profit of a given firm is equal to the revenue minus cost. Revenue is the product of price and quantity of the firm and cost is given by the firm's cost structure.

Backward induction

Therefore we will first find the BRF of the follower firms, so the profits are:
Profit function for firm 2:

$$\begin{aligned} \pi_2 &= P(Q)q_2 - cq_2 \\ &= (a - bQ)q_2 - cq_2 \\ &= (a - bQ - c)q_2 \\ &= (a - bq_1 - bq_2 - ... - bq_n - c)q_2 \\ &= aq_2 - bq_1q_2 - bq_2^2 - ... - bq_2q_n - cq_2 \end{aligned}$$

Recall that First order condition (FOC) is:

$$\frac{\partial \pi_2}{\partial q_2} = 0$$

$$\Rightarrow \quad a - c - bq_1 - 2bq_2 - ...bq_n = 0$$

$$then \quad 2bq_2 = a - c - bq_1 - ... - bq_n$$

$$thus \quad q_2 = \frac{a - c - bq_1 - ... - bq_n}{2b}, \quad i.e; \quad the \quad BRF_2$$

Profit function for firm n:

$$\begin{aligned} \pi_n &= P(Q)q_n - cq_n \\ &= (a - bQ)q_n - cq_n \\ &= (a - bQ - c)q_n \\ &= (a - bq_1 - bq_2 - ... - bq_n - c)q_n \\ &= aq_n - bq_1q_n - bq_2q_n - ... - bq_n^2 - cq_n \end{aligned}$$

Recall that First order condition (FOC) is:

$$\frac{\partial \pi_1}{\partial q_1} = 0$$

$$\Rightarrow \quad a - c - 2bq_1 - bq_2 - ...bq_n = 0$$

$$then \quad 2bq_1 = a - c - b\left[\frac{a - c - bq_1 - ... - bq_n}{2b}\right] - ... - b\left[\frac{a - c - q_1 - bq_2 - ...}{2b}\right]$$

$$\begin{aligned} 2bq_1 &= a - c - \frac{a}{2} + \frac{c}{2} + \frac{bq_1}{2} + ... + \frac{bq_n}{2} - ... - \frac{a}{2} + \frac{c}{2} + \frac{bq_1}{2} + \frac{bq_2}{2} + ... \\ &= a - \frac{a(n-1)}{2} - c + \frac{c(n-1)}{2} + \frac{bq_1(n-1)}{2} + \frac{bq_2(n-2)}{2} + ... + \frac{bq_n(n-2)}{2} \\ &= \frac{a(3-n)}{2} - \frac{c(3-n)}{2} + \frac{bq_1(n-1)}{2} + \frac{bq_2(n-2)}{2} + ... + \frac{bq_n(n-2)}{2} \end{aligned}$$

$$then \quad 2bq_1 - \frac{bq_1(n-1)}{2} = \frac{a(3-n)}{2} - \frac{c(3-n)}{2} + \frac{bq_2(n-2)}{2} + ... + \frac{bq_n(n-2)}{2}$$

$$and \quad q_1\left[\frac{b(5-n)}{2}\right] = \frac{a(3-n)}{2} - \frac{c(3-n)}{2} + \frac{bq_2(n-2)}{2} + ... + \frac{bq_n(n-2)}{2}$$

$$Thus \quad q_1 = \frac{a(3-n)}{b(5-n)} - \frac{c(3-n)}{b(5-n)} - q_2\frac{(2-n)}{(5-n)} - ... - q_n\frac{(2-n)}{(5-n)}$$

Substituting q_1 into q_2, we have:

$$\begin{aligned} q_2 &= \frac{a - c - bq_1 - ... - bq_n}{2b} \\ &= \frac{a - c - b\left[\frac{a(3-n)}{b(5-n)} - \frac{c(3-n)}{b(5-n)} - q_2\frac{(2-n)}{(5-n)} - ... - q_n\frac{(2-n)}{(5-n)}\right] - ... - bq_n}{2b} \\ &= \frac{a}{2b} - \frac{c}{2b} - \frac{a(3-n)}{2b(5-n)} + \frac{c(3-n)}{2b(5-n)} - q_2(\frac{n-2}{2(5-n)}) - ... - q_n(\frac{n-2}{2(5-n)}) - ... - \frac{bq_n}{2b} \\ &= \frac{a}{b(5-n)} - \frac{c}{b(5-n)} - q_2(\frac{n-2}{2(5-n)}) - ... - q_n(\frac{n-2}{2(5-n)}) - ... - \frac{q_n}{2} \end{aligned}$$

$$\Rightarrow \quad q_2 + q_2(\frac{n-2}{2(5-n)}) = \frac{a}{b(5-n)} - \frac{c}{b(5-n)} - ... - q_n(\frac{5-n+n-2}{2(5-n)})$$

$$then \quad q_2(\frac{8-n}{2(5-n)}) = \frac{a}{b(5-n)} - \frac{c}{b(5-n)} - ... - q_n(\frac{3}{2(5-n)})$$

$$Thus \quad q_2 = \frac{2a}{b(8-n)} - \frac{2c}{b(8-n)} - q_3(\frac{3}{8-n}) - ... - q_n(\frac{3}{8-n})$$

Also substituting q_1 into q_n, we have:

$$
\begin{aligned}
q_n &= \frac{a-c-bq_1-bq_2-\ldots}{2b} \\
&= \frac{a-c-b\left[\frac{a(3-n)}{b(5-n)}-\frac{c(3-n)}{b(5-n)}-q_2\frac{(2-n)}{(5-n)}-\ldots-q_n\frac{(2-n)}{(5-n)}\right]-bq_2-\ldots}{2b} \\
&= \frac{a}{2b}-\frac{c}{2b}-\frac{a(3-n)}{2b(5-n)}+\frac{c(3-n)}{2b(5-n)}-q_2(\frac{n-2}{2(5-n)})-\ldots-q_n(\frac{n-2}{2(5-n)})-\frac{bq_2-\ldots}{2b} \\
&= \frac{a}{b(5-n)}-\frac{c}{b(5-n)}-q_2(\frac{n-2}{2(5-n)})-\ldots-q_n(\frac{n-2}{2(5-n)})-\frac{q_2}{2}-\ldots
\end{aligned}
$$

$$
\Rightarrow \quad q_n+q_n(\frac{n-2}{2(5-n)}) = \frac{a}{b(5-n)}-\frac{c}{b(5-n)}-q_2(\frac{5-n+n-2}{2(5-n)})-\ldots
$$

$$
\textit{then} \quad q_n(\frac{8-n}{2(5-n)}) = \frac{a}{b(5-n)}-\frac{c}{b(5-n)}-q_2(\frac{3}{2(5-n)})-\ldots
$$

$$
\textit{Thus} \quad q_n = \frac{2a}{b(8-n)}-\frac{2c}{b(8-n)}-q_2(\frac{3}{8-n})-\ldots-q_{n-1}(\frac{3}{8-n})
$$

Hence we have the following system of equations:

$$
q_1 = \frac{a(3-n)}{b(5-n)}-\frac{c(3-n)}{b(5-n)}-q_2(\frac{2-n}{5-n})-\ldots-q_n(\frac{2-n}{5-n}) \tag{1}
$$

$$
q_2 = \frac{2a}{b(8-n)}-\frac{2c}{b(8-n)}-q_3(\frac{3}{8-n})-\ldots-q_n(\frac{3}{8-n}) \tag{2}
$$

$$
\ldots
$$

$$
q_n = \frac{2a}{b(8-n)}-\frac{2c}{b(8-n)}-q_2(\frac{3}{8-n})-\ldots-q_{n-1}(\frac{3}{8-n}) \tag{3}
$$

Note: We have seen in the Stackelberg competition that the product quantities of firms 1 and 2 are not equal; therefore, their corresponding profits are also not equal.

RESULTS

This section illustrates how the Stackelberg model can be applied to analyze competition among firms in a simplified manner. It also compares the equilibrium prices and quantities obtained under the Stackelberg, Cournot, Bertrand, and Cartel models.

If we use Stackelberg model in the case where the price function $P(Q) = 100 - 2Q$ and the marginal cost $MC_1 = MC_2 = 20$, the total quantity will be $Q = 45$, market price will be $P = 55$ and the total profit will be $\pi = \pi_1 + \pi_2 = 1575$, while in the Cournot model with price function $P(Q) = 100 - 2Q$ and the marginal cost $MC_1 = MC_2 = 20$, the total quantity will be $Q = 40$, market price will be $P = 60$ and the total profit will be $\pi = \pi_1 + \pi_2 = 1600$. Showing that the Stackelberg price is lower than that of Cournot, so also the profit.

Note: The Stackelberg price is lower than the Cournot price.but greater than the

Bertrand price.

Lets consider a given price function $P(Q) = 180-2Q$, with a marginal cost $MC = 20$ (i.e; $MC_1 = MC_2 = 20$) and relate between Stackelberg, Cournot, Bertrand and Cartel competition modelling. Note the following differences in the Table 1, below:

Table 1: Comparing results from different competition models.

Competition Model	Decision Rule	Market Price (P)	Market Quantity (Q)	Total Profit (π)
Stackelberg	Leader–Follower (Sequential)	64	58	2552
Cournot	Simultaneous Output Choice	70	55	2750
Bertrand	Firms compete on price	20	80	0
Cartel	Joint Profit Maximization	100	40	3200

We can interpret Table 1 as follows:

i The Bertrand model results in the lowest market price because firms compete aggressively on price until price equals marginal cost.

ii The Stackelberg model generates a lower price and higher output than the Cournot model because the leader commits to its production before the follower.

iii The Cournot model produces intermediate outcomes, with higher prices and lower output than Stackelberg.

iv The Cartel model maximizes joint profits by restricting output, leading to the highest price and the lowest market quantity.

Let us also consider another price function given as $P(Q) = 500 - 2Q$, with marginal cost $MC_1 = MC_2 = 20$. We can also show the results in Table 2, as follows:

Table 2: Comparing results from different competition models.

Competition Model	Decision Rule	Market Price (P)	Market Quantity (Q)	Total Profit (π)
Stackelberg	Leader–Follower (Sequential)	148	176	22528
Cournot	Simultaneous Output Choice	170	165	24750
Bertrand	Firms compete on price	20	240	0
Cartel	Joint Profit Maximization	260	120	28800

The results in Table 2 are consistent with economic theory. The Bertrand model produces the most compet-itive outcome, characterized by the lowest price and the highest market output. The Stackelberg model follows, reflecting the leader's first-mover advantage and producing a lower price than the Cournot model. The Cournot model yields moderate outcomes because firms choose quantities simultaneously without leadership. Finally, the Cartel model achieves the highest market price and profit by coordinating production and limiting market output.

Table 3: Some comparisons between different competition models

Competition Model	Market Price	Market Output	Industry Profit	Degree of Competition
Stackelberg	Low	High	Moderate	High
Cournot	Moderate	Moderate	High	Medium
Bertrand	Lowest	Highest	Lowest	Very High
Cartel	Highest	Lowest	Highest	Very Low

From the Stackelberg duopoly model we derive above, we have the following results: The quantity of product of firm 1 is:

$$q_1^* = \frac{a-c}{2b},$$

the quantity of product of firm 2 is:

$$q_2^* = \frac{a-c}{4b},$$

The total quantity is:

$$Q^* = q_1^* + q_2^* = \frac{3(a-c)}{4b},$$

The price function is:

$$P^* = a - bQ^* = \frac{a+3c}{4},$$

The profit of firm 1 is:

$$\pi_1^* = \frac{(a-c)^2}{8b},$$

The profit of firm 1 is:

$$\pi_2^* = \frac{(a-c)^2}{16b},$$

$$\begin{aligned} Thus \quad \pi^* &= \pi_1^* + \pi_2^* \\ &= \frac{3(a-c)^2}{16b} \\ &= 3\pi_2, \quad since \quad 3\pi_2 \end{aligned}$$

Recall that, from the previous example in the Stackelberg model, we have:
$P(Q) = 500 - 2Q$, i.e; the price function
and $MC_1 = MC_2 = 20$, i.e; the marginal costs of firms 1 and 2
Here, $a = 500$, $b = 2$ and $c = 20$
Therefore:

$$\begin{aligned} q_1^* &= \frac{a-c}{2b} \\ &= \frac{500-20}{2(2)} \\ &= \frac{480}{4} = 120, \end{aligned}$$

(i.e; the quantity of product of firm 1)

$$\begin{aligned} q_2^* &= \frac{a-c}{4b} \\ &= \frac{500-20}{4(2)} \\ &= \frac{480}{8} = 60, \end{aligned}$$

(i.e; the quantity of product of firm 2)

$$\begin{aligned} thus \quad Q^* &= \frac{3(a-c)}{4b} \\ &= 3(60) = 180, \quad the \quad total \quad quantity \quad Q^* = 3q_2^* \end{aligned}$$

$$\begin{aligned} P^* &= \frac{a+3c}{4} \\ &= \frac{500+3(20)}{4} \\ &= \frac{500+60}{4} \\ &= \frac{560}{4} = 140, \quad the \quad price \end{aligned}$$

$$\pi_1^* = \frac{(a-c)^2}{8b}$$
$$= \frac{(500-20)^2}{8(2)}$$
$$= \frac{(480)^2}{16} = 14400, \quad the \quad profit \quad of \quad firm1$$

$$\pi_2^* = \frac{(a-c)^2}{16b}$$
$$= \frac{(500-20)^2}{16(2)}$$
$$= \frac{(480)^2}{32} = 7200, \quad the \quad profit \quad of \quad firm2$$

$$Thus \quad \pi^* = \pi_1^* + \pi_2^*$$
$$= \frac{3(a-c)^2}{16b}$$
$$= 3(7200) = 21600, \quad the \quad total \quad profit, \quad \pi^* = 3(\pi_2^*)$$

Also, from the previous example, we have the following:
$P(Q) = 26 - 2Q$, i.e; the price function
and $MC_1 = MC_2 = 2$, i.e; the marginal costs of firms 1 and 2
Here, $a = 26$, $b = 2$ and $c = 2$
Therefore:

$$\Rightarrow \quad q_1^* = \frac{a-c}{2b}$$
$$= \frac{26-2}{2(2)}$$
$$= \frac{24}{4} = 6, \quad the \quad product \quad quantity \quad of \quad firm1$$

$$and \quad q_2^* = \frac{a-c}{4b}$$
$$= \frac{26-2}{4(2)}$$
$$= \frac{24}{8} = 3, \quad the \quad product \quad quantity \quad of \quad firm2$$

$$Q^* = \frac{3(a-c)}{4b} = 3(3) = 9, \quad the \quad total \quad quantity, \quad Q^* = 3q_2^*$$

$$\begin{aligned} P^* &= \frac{a+3c}{4} \\ &= \frac{26+3(2)}{4} \\ &= \frac{26+6}{4} \\ &= \frac{32}{4} = 8, \quad the \quad price \end{aligned}$$

$$\begin{aligned} \pi_1^* &= \frac{(a-c)^2}{8b} \\ &= \frac{(26-2)}{8(2)} \\ &= \frac{(24)^2}{16} = 36, \quad the \quad profit \quad of firm1 \end{aligned}$$

$$\begin{aligned} \pi_2^* &= \frac{(a-c)^2}{16b} \\ &= \frac{(26-2)}{16(2)} \\ &= \frac{(480)^2}{32} = 18, \quad the \quad profit \quad of \quad firm2 \end{aligned}$$

$$\begin{aligned} Thus \quad \pi^* &= \pi_1^* + \pi_2^* \\ &= \frac{3(a-c)^2}{16b} \\ &= 3(18) = 54, \quad since \quad \pi^* = 3(\pi_2^*), \quad the \quad total \quad profit, \quad \pi^* = 3(\pi_2^*) \end{aligned}$$

Note: we can see that the results here and the previous ones are equal. Thus, we can apply the models to solve competition problems in a simplified approach.

CONCLUSION

Competition causes commercial companies to develop new products, services, and technologies. It also helps to encourage companies (industries) to improve the quality and quantity of their products, so consumers can have the choice of which company will best suit their demand in the purchase of goods and services. We have seen how competition models differ in terms of quantities, prices, and profits despite having unique price functions and marginal costs.

Competition is mainly categorized into two parts, perfect and imperfect competition, where perfect compe-tition refers to a market situation where many

small companies sell individual products. Because no company is large enough to control the price, each simply accepts the market price (Nock, 2020). The price is determined by supply and demand. However, imperfect competition refers to a situation in which the characteristics of an economic market do not meet all the necessary conditions of a perfectly competitive market (Liberto, 2020). Since perfect competition does not occur in real life, we have considered imperfect competition which encom-passes various models, some of which we discussed. We explained the different forms of competition, where there may be only one competitor as a monopolistic competition, in the case where two companies are competing, that will be referred to as duopolistic competition (it is also a form of oligopoly), and in the case where there are few (i.e; two and above) companies competing, it is also known as oligopolistic competition.

The Stackelberg Competition Model is a key theory for understanding competition in oligopolistic markets. Unlike the Cournot model, firms do not make decisions at the same time. Instead, the leader chooses its output first, while the follower responds after observing that choice. This sequential process gives the leader a first-mover advantage, allowing it to earn higher profits and influence the market. The model is widely applied in economics, business strategy, and industrial organization to explain how dominant firms shape market behavior. The Stackelberg model solution in this paper can be applied in finding the price, quantity and profit to a given problem in a simplified approach.

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