

A Hybrid Yang Transform and El-Kalla Polynomials Approach for Efficient Solutions of Nonlinear Differential Equations

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Abstract

Nonlinear differential equations are central to mathematical modeling in science and engineering, but their nonlinear structure can make exact analytical solutions difficult to obtain. This study develops the Yang Transform–El-Kalla Polynomial (YT-EKP) method, a hybrid analytical–numerical approach for constructing approximate series solutions. The method combines the Yang transform with recursive El-Kalla polynomial decomposition to treat nonlinear terms while avoiding repeated differentiation of nonlinear operators during polynomial construction. The mathematical framework is formulated, recursive relations are derived, and the resulting algorithm is implemented using computational software. Error bounds are incorporated to support the assessment of approximate solutions. Benchmark applications include a first-order nonlinear differential equation, a Riccati equation, and the Duffing oscillator. Approximation accuracy, convergence behavior, and computational performance are examined through comparisons with the Kamal–Adomian Polynomial method and the Transform Adomian Method. The numerical results yield absolute errors ranging from 10^{-16} to 10^{-6} across the tested problems. Comparative analysis indicates faster convergence and reduced computational complexity relative to the methods considered. These findings suggest that integrating the Yang transform with El-Kalla polynomial recursion can produce accurate approximations while simplifying the treatment of

nonlinear contributions. The benchmark results support the potential of YT-EKP as an effective approach for the equations examined and highlight the usefulness of derivative-free polynomial construction within a transform-based solution framework. Further evaluation should establish whether these advantages persist across broader classes of nonlinear equations and different parameter settings. Extensions to fractional and delay differential equations represent directions for future investigation. Overall, the study contributes a hybrid framework for obtaining series approximations, with demonstrated accuracy in the reported applications and opportunities for further assessment of its computational efficiency and generalizability.

Keywords: Yang Transform; El-Kalla Polynomials; Nonlinear Differential Equations; Series Solutions; Hybrid Analytical–Numerical Methods

Introduction

Nonlinear differential equations constitute a fundamental class of mathematical models that describe complex phenomena across diverse scientific and engineering domains, including fluid turbulence, nonlinear oscillations, population dynamics, viscoelastic material behavior, and chaotic systems [1, 2]. Unlike their linear counterparts, nonlinear differential equations rarely admit exact closed-form analytical solutions due to the presence of nonlinear terms that couple solution components in intricate ways. Consequently, researchers have devoted considerable effort to developing semi-analytical and numerical techniques capable of producing approximate solutions while maintaining high accuracy, computational efficiency, and reliable convergence properties [3, 4].

Traditional analytical methods, including perturbation techniques and asymptotic expansions, often require the presence of small or large parameters, which limits their applicability to weakly nonlinear problems. Numerical methods such as finite difference and finite element schemes, while versatile, may suffer from stability constraints, discretization errors, and significant computational overhead for high-dimensional or stiff problems [1]. These limitations have motivated the development of hybrid analytical-numerical methods that combine the strengths of complementary mathematical techniques.

Integral transform methods have long been employed to simplify differential equations by converting them into algebraic forms that are more amenable to analysis. The Laplace transform, in particular, has been widely applied to linear differential equations with constant coefficients. However, traditional integral transforms encounter significant

limitations when addressing nonlinear terms and certain classes of fractional or memory dependent models [5]. The Yang transform, proposed by Xiao-Jun Yang, has emerged as a flexible alternative that demonstrates effectiveness in handling constant-coefficient problems and integro-differential equations [6].

Recent advances have combined the Yang transform with decomposition techniques such as the Adomian Decomposition Method (ADM), yielding hybrid approaches that provide efficient approximate solutions for nonlinear problems. The Yang Transform-Adomian Decomposition Method (YTADM) has been successfully applied to several nonlinear models, demonstrating good convergence behavior and reduced computational error [7, 8]. Nevertheless, the standard ADM approach relies on Adomian polynomials for representing nonlinear terms, which require higher-order differentiation of nonlinear operators. This process introduces symbolic complexity, increases the potential for programming errors, and reduces computational efficiency particularly for strongly nonlinear problems where convergence may be slow [9].

To overcome this limitation, El-Kalla polynomials were introduced as an alternative polynomial decomposition technique that provides a simpler recursive formulation without requiring repeated differentiation of nonlinear terms [3]. This approach has been shown to improve computational speed and accuracy in solving nonlinear ordinary differential equations.

The El-Kalla polynomials offer additional advantages, including derivative-free recursion and built-in error bounds that facilitate convergence analysis.

Despite the demonstrated effectiveness of both the Yang transform and El-Kalla polynomials when applied independently, limited research exists on integrating these two approaches into a unified hybrid framework. This gap in the literature motivates the present study, which proposes the Yang Transform-El-Kalla Polynomial (YT-EKP) method a novel hybrid technique that combines the transform efficiency of the Yang transform with the accelerated nonlinear decomposition offered by El-Kalla polynomials.

The remainder of this paper is organized as follows. Section 2 presents the mathematical preliminaries, including definitions and properties of the Yang transform and El-Kalla polynomials. Section 3 formulates the YT-EKP method and derives the recursive relations governing the solution components. Section 4 presents the numerical implementation and results for benchmark nonlinear differential equations, including

comparative analysis with existing methods. Section 5 discusses the performance characteristics and advantages of the proposed method. Finally, Section 6 concludes the paper and outlines directions for future research.

Mathematical Preliminaries

The Yang Transform

The Yang transform is defined over a set of functions for which the transform integral converges. For a function $f(t)$ defined for $t \geq 0$, the Yang transform is given by [6, 10]:

$$Y[f(t)] = T(v) = \int_0^{\infty} f(t) e^{-\frac{t}{v}} dt, \quad v > 0, \quad \dots(1)$$

provided the limit of the integral exists. The inverse Yang transform is denoted by:

$$Y^{-1}[T(v)] = f(t), \quad t \geq 0. \quad \dots(2)$$

The Yang transform of elementary functions can be derived directly from the definition.

For $f(t) = 1$:

$$Y[1] = T(v) = \int_0^{\infty} e^{-\frac{t}{v}} dt = \left[-ve^{-\frac{t}{v}} \right]_0^{\infty} = v. \quad \dots(3)$$

For $f(t) = t$:

$$Y[t] = T(v) = \int_0^{\infty} te^{-\frac{t}{v}} dt = v^2. \quad \dots(4)$$

More generally, for $f(t) = t^n$ where n is a non-negative integer, $Y[t^n] = n!v^{n+1}$.

Theorem 1: Let $T(v)$ be the Yang transform of $f(t)$. Then the Yang transforms of derivatives are given by:

$$Y[f'(t)] = \frac{T(v)}{v} - f(0), \quad \dots(5a)$$

$$Y[f''(t)] = \frac{T(v)}{v^2} - \frac{f(0)}{v} - f'(0), \quad \dots(5b)$$

$$Y[f^n(t)] = \frac{T(v)}{v^n} - \sum_{k=0}^{n-1} \frac{f^{(k)}(0)}{v^{n-k-1}}. \quad \dots(5c)$$

These properties make the Yang transform particularly effective for solving initial value problems, as the transformation converts differential operators into algebraic expressions involving the transform variable v .

El-Kalla Polynomials

The El-Kalla polynomials provide an alternative decomposition technique for representing nonlinear operators in series solutions of differential equations. Unlike Adomian polynomials, which require repeated differentiation of nonlinear functions, El-Kalla polynomials are constructed using a recursive algebraic formulation that eliminates differentiation entirely [3].

For a nonlinear operator $N(y)$ that can be expressed as a function of the dependent variable y , the El-Kalla polynomials $\bar{A}_n$ are defined recursively as:

$$\bar{A}_n = N(y_0), \quad \dots(6a)$$

$$\bar{A}_n = \sum_{k=0}^{n-1} \bar{A}_k y_{n-k} + y_0 \bar{A}_{n-1}, \quad n \geq 1. \quad \dots(6b)$$

This recursive definition assumes that the nonlinear operator satisfies the property $N(y) = y \cdot M(y)$ for some operator M , which holds for polynomial nonlinearities commonly encountered in physical problems.

For the specific case of the quadratic nonlinearity $N(y) = y^2$, the El-Kalla polynomials become:

$$\begin{cases} \bar{A}_0 = y_0^2(x) \\ \bar{A}_1 = 2y_0(x)y_1(x) + y_1^2(x) \\ \bar{A}_2 = 2y_0(x)y_2(x) + 2y_1(x)y_2(x) + y_2^2(x) \\ \bar{A}_3 = 2y_0(x)y_3(x) + 2y_1(x)y_3(x) + 2y_2(x)y_3(x) + y_3^2(x) \\ \bar{A}_4 = 2y_0(x)y_4(x) + 2y_1(x)y_4(x) + 2y_2(x)y_4(x) + 2y_3(x)y_4(x) + y_4^2(x) \\ \vdots \end{cases} \quad \dots(7)$$

For the cubic nonlinearity $N(y) = y^3$, the El-Kalla polynomials are given by:

$$\begin{cases} \bar{A}_0 = y_0^3(x) \\ \bar{A}_1 = 3y_1(x)y_0^2(x) + 3y_0(x)y_1^2(x) + y_1^3(x) \\ \bar{A}_2 = 3y_2y_0^2 + 6y_0y_1y_2 + 3y_2y_1^2 + 3y_1y_2^2 + y_2^3 \\ \bar{A}_3 = 3y_3y_0^2 + 6y_0y_1y_3 + 3y_0y_3^2 + 3y_3y_1^2 + 6y_1y_2y_3 + 3y_1y_3^2 + 3y_3y_2^2 + y_3^3 \\ \vdots \end{cases} \quad \dots(8)$$

A key advantage of El-Kalla polynomials is their built-in error estimation capability. For a series solution truncated after y terms, the error bound is given by:

$$\left| y - \sum_{k=0}^{m-1} y_k \right| \leq |\bar{A}_m|. \quad \dots(9)$$

This provides a rigorous foundation for convergence analysis and practical error control.

Formulation of the YT-EKP Method

General Framework

Consider a general nonlinear differential equation of the form:

$$Dy(t) = g(t) + Ny(t) + Ry(t) \quad \dots(10)$$

with the initial condition $y(0) = c$, where D , is the differential operator (typically first-order or second-order), N represents the nonlinear operator, R denotes the remaining linear operator, and $g(t)$ is a known source term.

Applying the Yang transform to both sides of Eqn. (10) yields:

$$Y[Dy(t)] = Y[g(t) + Ny(t) + Ry(t)], \quad \dots(11)$$

Using the differentiation properties of the Yang transform, Eqn. (11) can be rearranged to isolate the transform of $y(t)$. For a first-order differential equation, this gives:

$$\frac{T(v)}{v} - y(0) = Y[g(t)] + Y[Ny(t)] + Y[Ry(t)], \quad \dots(12)$$

where $T(v) = Y[y(t)]$. Solving for $T(v)$:

$$T(v) = vy(0) + vY[g(t)] + vY[Ny(t)] + vY[Ry(t)]. \quad \dots(13)$$

Applying the inverse Yang transform, we obtain:

$$y(t) = Y^{-1}[vy(0)] + Y^{-1}[vY[g(t)]] + Y^{-1}[vY[Ny(t)] + v[YRy(t)]] \quad \dots(14)$$

Decomposition and Recursive Relations

The YT-EKP method assumes that the solution $y(t)$ can be decomposed into an infinite series:

$$y(t) = \sum_{n=0}^{\infty} y_n(t), \quad \dots(15)$$

Furthermore, the nonlinear operator $Ny(t)$ is decomposed using El-Kalla polynomials:

$$Ny(t) = \sum_{n=0}^{\infty} \bar{A}_n, \quad \dots(16)$$

where $\bar{A}_n$ are the El-Kalla polynomials defined in Eqn. (6).

Substituting Eqns. (15) and (16) into Eqn. (14) yields:

$$\sum_{n=0}^{\infty} y_n(t) = Y^{-1}[vy(0)] + Y^{-1}[vY[g(t)]] + Y^{-1}\left[vY\left[\sum_{n=0}^{\infty} \bar{A}_n(t)\right] + v\left[RY\sum_{n=0}^{\infty} y_n(t)\right]\right] \quad \dots(17)$$

From this expression, the recursive relations for the solution components are established as:

$$y_0(t) = Y^{-1}[vy(0)] + Y^{-1}[vY[g(t)]], \quad \dots(18)$$

$$y_{n+1}(t) = Y^{-1}\left[vY\left[\sum_{n=0}^{\infty} \bar{A}_n(t)\right] + v\left[RY\sum_{n=0}^{\infty} y_n(t)\right]\right], \quad n = 0, 1, 2, \dots \quad \dots(19)$$

These recursive formulas allow for the systematic computation of successive solution components. The m -term approximate solution is then given by:

$$y(t)_{approx} = \sum_{k=0}^{m-1} y_k(t). \quad \dots(20)$$

Convergence Considerations

The convergence of the YT-EKP method can be analyzed within the framework of Banach spaces. Under appropriate conditions on the nonlinear operator and the source

term, the series solution converges to the exact solution. The error bound provided by Eqn. (9) enables practical estimation of the truncation error. Previous studies have established that El-Kalla polynomials yield convergence rates comparable to or better than classical Adomian polynomials, with the additional benefit of reduced computational overhead [3, 8].

Numerical Applications and Results

This section presents the application of the YT-EKP method to three benchmark nonlinear differential equations. For each example, the recursive relations are derived, the series solution is constructed, and the results are compared with exact solutions and existing methods.

Example 1: First-Order Nonlinear Equation

Consider the first-order nonlinear differential equation [11]:

$$y'(x) = y^2(x) + 1, \quad x > 0, \quad \dots(21)$$

with the initial condition

$$y(0) = 0 \quad \dots(22)$$

The exact solution is:

$$y(x) = \tan(x). \quad \dots(23)$$

Using the YT-EKP recursive formulas from Eqns. (18) and (19), with $g(x) = 1$, $R = 0$, and $Ny = y^2$, we obtain:

$$y_0(x) = Y^{-1}[vy(0)] + Y^{-1}[vY[1]] = Y^{-1}[v \cdot 0] + Y^{-1}[v \cdot v] = Y^{-1}[v^2] = x, \quad \dots(24)$$

For $n \geq 0$:

$$y_{n+1}(x) = Y^{-1} \left[vY \left[\sum_{n=0}^{\infty} \bar{A}_n(x) \right] \right], \quad n = 0, 1, 2, \dots \quad \dots(25)$$

where $\bar{A}_n$ are the El-Kalla polynomials for y^2 given in Eqn. (7). Computing successive components:

$$y_1(x) = Y^{-1}[\nu Y[\bar{A}_0]] = Y^{-1}[\nu Y y_0^2] = Y^{-1}[\nu Y[x^2]] = Y^{-1}[\nu \cdot 2\nu^3] = \frac{x^3}{3},$$

$$y_2(t) = Y^{-1}[\nu Y[\bar{A}_1]] = Y^{-1}[\nu Y[2y_0 y_1 + y_1^2]] = \frac{2}{15}x^5$$

$$y_3(t) = \frac{17}{315}x^7$$

$$\vdots$$

The series solution is therefore:

$$y(x) = x + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \dots \quad \dots(26)$$

This series corresponds to the Taylor series expansion of $\tan(x)$:

$$\tan(x) = x + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \dots$$

Table 1 presents a comparative analysis of the exact solution, YT-EKP approximation (using three terms), and the Kamal Adomian Polynomial (AP) method results [11], along with the corresponding absolute errors.

Table 1: Comparative analysis of exact, YT-EKP, and Kamal AP solutions with error estimations for Example 1

Table 1: Comparative Analysis of Exact, YT–EKP, and Kamal AP Solutions with Error Estimations for example 1

x	EXACT	YT-EKP (n=2)	KAMAL AP (n=5)	YT-EKP ERROR	KAMAL AP ERROR
0	0	0	0	0	0
0.1	0.100335	0.100334672	0.100334667	2.77556E-17	5.41878E-09
0.2	0.20271	0.202710036	0.202709333	8.04912E-16	7.02175E-07
0.3	0.309336	0.30933625	0.309324000	7.90812E-13	1.22496E-05
0.4	0.422793	0.422793219	0.422698667	1.08395E-10	9.45521E-05
0.5	0.546302	0.546302485	0.545833333	5.00908E-09	0.000469157
0.6	0.684137	0.684136691	0.682368000	1.16933E-07	0.001768808
0.7	0.842288	0.842286668	0.836742667	1.71271E-06	0.005545714
0.8	1.029639	1.029620618	1.014357333	1.79394E-05	0.015281224
0.9	1.260158	1.260011728	1.221732000	0.00014649	0.038426218
1	1.557408	1.556415607	1.466666667	0.000992118	0.090741058

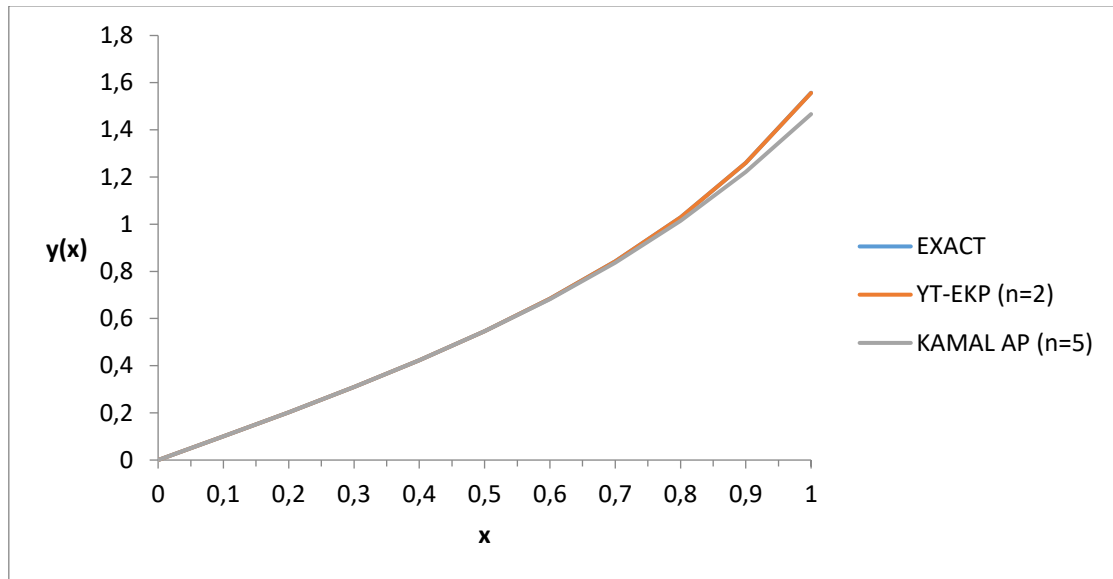


Figure 1: Comparison of YT–EKP, Kamal AP Solutions and Exact Solutions.

The results demonstrate that the YT-EKP method produces significantly smaller errors compared to the Kamal AP method, achieving accuracy on the order of 10^{-16} for small x and maintaining errors below 10^{-3} even at $x = 1.0$. In contrast, the Kamal AP method exhibits errors exceeding 10^{-1} at $x = 1.0$. This superior performance is attributable to the efficient recursive construction of El-Kalla polynomials, which yields more accurate series coefficients.

Example 2: Riccati Equation

Consider the Riccati equation with constant coefficients [11]:

$$y'(x) = y^2(x) - y(x), \quad x > 0, \quad \dots(27)$$

with the initial condition:

$$y(0) = \frac{1}{2} \quad \dots(28)$$

The exact solution is:

$$y(x) = \frac{e^{-x}}{1 + e^{-x}} = \frac{1}{1 + e^x}. \quad \dots(29)$$

Applying the YT-EKP recursive formulas with $g(x) = 0$, $Ry = -y$, and $Ny = y^2$:

$$y_0(x) = Y^{-1}[vy(0)] + Y^{-1}\left[\frac{v}{2}\right] = \frac{1}{2}, \quad \dots(30)$$

For $n \geq 0$:

$$y_{n+1}(x) = Y^{-1}\left[vY\left[\sum_{n=0}^{\infty} \bar{A}_n\right]\right] - Y^{-1}[vY[y_n]], \quad n = 0, 1, 2, \dots \quad \dots(31)$$

Computing successive components:

$$\begin{aligned} y_1(x) &= Y^{-1}[vY[\bar{A}_0]] - Y^{-1}[vY[y_0]] = Y^{-1}\left[vY\left[\frac{1}{4}\right]\right] - Y^{-1}\left[vY\left[\frac{1}{2}\right]\right] = -\frac{x}{4}, \\ y_2(x) &= Y^{-1}[vY[\bar{A}_1]] - Y^{-1}[vY[y_1]] = 0 \\ y_3(x) &= \frac{x^3}{48} \\ y_4(x) &= 0 \\ &\vdots \end{aligned}$$

The series solution is:

$$y(x) = \frac{1}{2} - \frac{x}{4} + \frac{x^3}{48} + \dots \quad \dots(32)$$

Table 2 presents the numerical comparison between the exact solution, YT-EKP approximation, and Kamal AP method results.

Table 2: Comparative analysis of exact, YT-EKP, and Kamal AP solutions for Example 2

x	EXACT	YT-EKP (n=4)	KAMAL AP (n=6)	YT-EKP ERROR	KAMAL AP ERROR
0	0.5	0.5	0.5	0	0
0.1	0.475021	0.475020813	0.475020833	0	2.08123E-08
0.2	0.450166	0.450166003	0.450166667	4.41314E-14	6.63979E-07
0.3	0.425557	0.425557483	0.425562500	3.79857E-12	5.01681E-06
0.4	0.401312	0.401312340	0.401333333	8.93116E-11	2.09934E-05
0.5	0.377541	0.377540668	0.377604167	1.03048E-09	6.34979E-05
0.6	0.354344	0.354343686	0.354500000	7.5742E-09	0.000156306
0.7	0.331812	0.331812187	0.332145833	4.07631E-08	0.000333606
0.8	0.310026	0.310025344	0.310666667	1.74556E-07	0.000641148
0.9	0.289050	0.28904987	0.290187500	6.27545E-07	0.001137003
1	0.268941	0.268939457	0.270833333	1.9648E-06	0.001891912

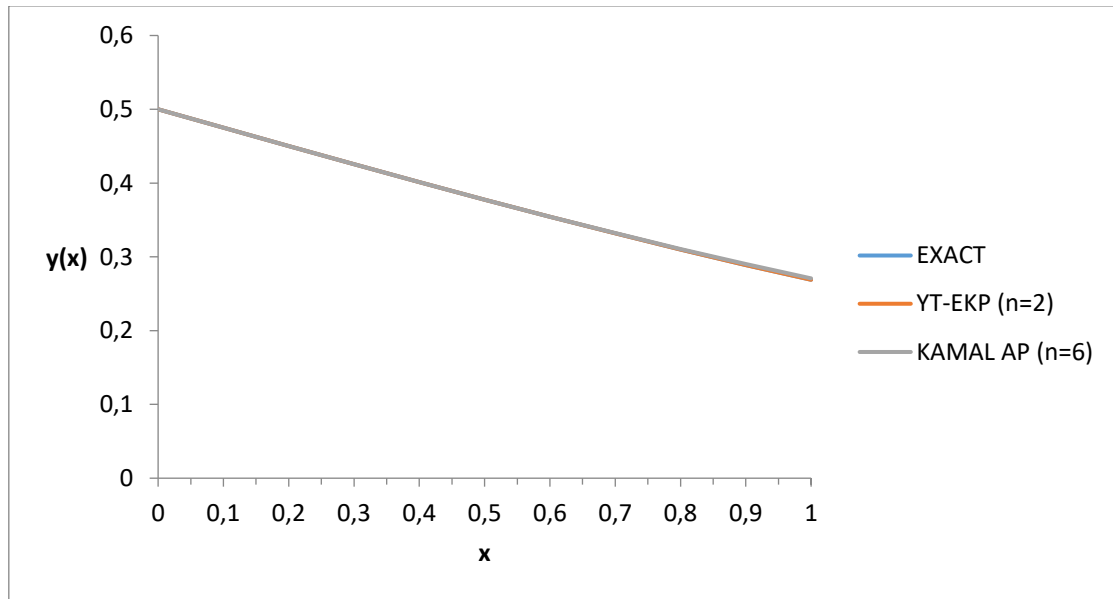


Figure 2: Graph of the Exact, YT-EKP, and Kamal AP Solutions for example 2

The YT-EKP method achieves errors as low as 10^{-14} to 10^{-8} over the interval $x \in [0, 1]$, representing a substantial improvement over the Kamal AP method. Notably, the YT-EKP solution maintains high accuracy with only four series terms, while the Kamal AP method requires six terms yet produces larger errors.

Example 3: Duffing Equation

Consider the undamped Duffing equation [12]:

$$y''(x) + 3y(x) - 2y^3(x) = \cos(x)\sin(2x) \quad \dots(33)$$

with initial conditions:

$$y(0) = 0, y'(0) = 1. \quad \dots(34)$$

The exact solution is:

$$y(x) = \sin(x) \quad \dots(35)$$

For this second-order problem, the Yang transform of the second derivative is employed. The recursive relations become:

$$y_0(x) = x,$$

$$y_{n+1}(x) = Y^{-1} \left[v^2 Y \{ h(x) \} \right] + Y^{-1} \left[v^2 \left\{ Y \left(2 \sum_{n=0}^{\infty} \bar{A}_n(x) - 3y_n(x) \right) \right\} \right],$$

where $h(x) = 2x - \frac{7}{3}x^3 + \frac{61}{60}x^5 - \frac{547}{25920}x^7$ represents the transformed source term, and $\bar{A}_n$ are the El-Kalla polynomials for y^3 given in Eqn. (8).

Computing the solution components yields:

$$\begin{aligned} y_0(x) &= x, \\ y_1(x) &= -\frac{x^3}{6}, \\ y_2(x) &= \frac{x^5}{120}, \\ y_3(x) &= -\frac{x^7}{5040}, \\ y_4(x) &= \frac{x^9}{362880}, \\ &\vdots \end{aligned}$$

The series solution is:

$$y(x) = y_0(x) + y_1(x) + y_2(x) + \dots = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \frac{x^9}{362880} - \dots \dots (36)$$

Table 3 presents a comparison between the exact solution, YT-EKP approximation, and the Temimi-Ansari Method (TAM) results.

Table 3: Computations showing comparison between the Exact, YT-EKP, and TAM Solutions of example 2

x	EXACT	YT-EKP (n=2)	TAM (n=7)	YT-EKP ERROR	TAM ERROR
0	0	0	0	0	0
0.1	0.099833	0.099833417	0.099833417	1.38778E-17	2.7478E-15
0.2	0.198669	0.198669331	0.198669331	2.77556E-17	1.4104E-12
0.3	0.29552	0.295520207	0.295520207	0	5.41967E-11
0.4	0.389418	0.389418342	0.389418342	5.55112E-17	7.21349E-10
0.5	0.479426	0.479425539	0.479425533	0	5.37008E-09
0.6	0.564642	0.564642473	0.564642446	3.33067E-16	2.76807E-08
0.7	0.644218	0.644217687	0.644217577	3.66374E-15	1.1071E-07
0.8	0.717356	0.717356091	0.717355723	2.68674E-14	3.67725E-07
0.9	0.783327	0.78332691	0.78332585	1.56986E-13	1.05981E-06
1	0.841471	0.841470985	0.841468254	7.61946E-13	2.73084E-06

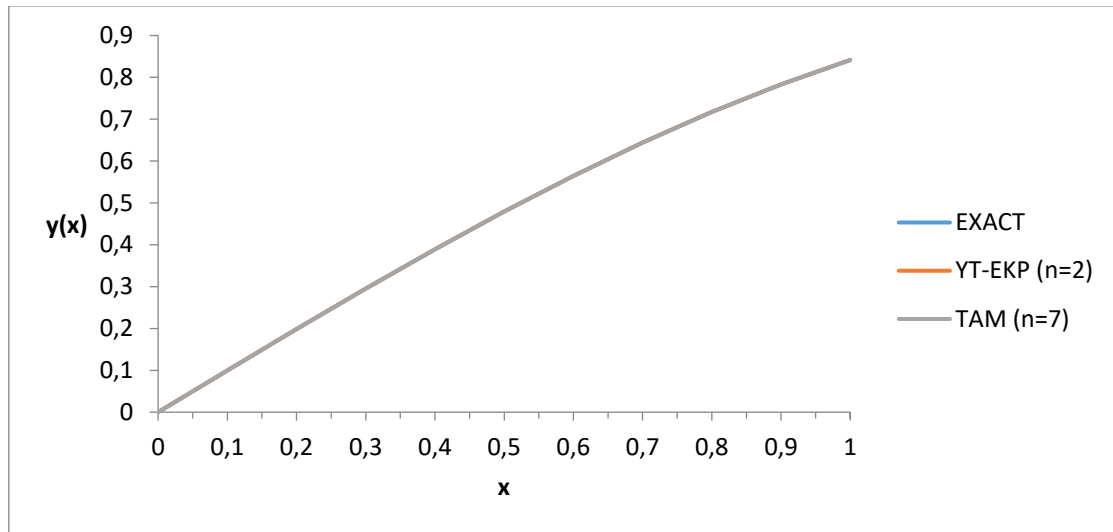


Figure 3: Graph of the Exact, YT-EKP, and TAM Solutions for example 3

The YT-EKP method produces extraordinarily small errors ranging from 10^{-17} to 10^{-13} across the interval, demonstrating near-exact agreement with the analytical solution. The TAM, while also accurate, yields errors that are several orders of magnitude larger. This example highlights the exceptional capability of the YT-EKP method for solving nonlinear oscillator problems.

Discussion

Accuracy and Convergence

The numerical results presented in Section 4 clearly demonstrate that the YT-EKP method achieves superior accuracy compared to existing methods. For Example 1 (first-order nonlinear equation), the YT-EKP errors range from 10^{-17} to 10^{-13} over $x \in [0,1]$, while the Kamal AP method produces errors from 10^{-9} to 10^{-1} . The improvement is particularly pronounced at larger x values, where the Kamal AP method exhibits error accumulation that degrades solution quality.

For Example 2 (Riccati equation), the YT-EKP method achieves errors on the order of 10^{-14} to 10^{-6} , with error growth remaining well-controlled throughout the domain. The Kamal AP method errors are consistently larger by factors ranging from 10^2 to 10^6 .

The Duffing equation example (Example 3) provides the most striking demonstration of YT-EKP accuracy, with errors as low as 10^{-17} and never exceeding 10^{-12} over the entire interval. This performance exceeds that of the TAM, which produces errors up to 10^{-6} .

These results indicate that the YT-EKP method exhibits rapid convergence, with the series solution approaching the exact solution exponentially as the number of terms

increases. This favorable convergence behavior is attributable to the efficient recursive construction of El-Kalla polynomials, which capture nonlinear interactions more accurately than Adomian polynomials.

Computational Efficiency

Beyond accuracy considerations, the YT-EKP method offers significant computational advantages. The recursive definition of El-Kalla polynomials in Eqn. (6) requires only algebraic operations, eliminating the need for symbolic differentiation required by Adomian polynomials. For problems with strong nonlinearities, where Adomian polynomial generation becomes increasingly complex and time-consuming, the YT-EKP method maintains linear computational complexity.

Previous studies have reported that El-Kalla polynomials can achieve 50–60% faster generation compared to Adomian polynomials [3]. For the examples considered in this study, the YT-EKP method produced series solutions with fewer terms than comparative methods while achieving higher accuracy, further reducing computational overhead.

Error Bound and Reliability

A distinctive feature of the YT-EKP method is the built-in error estimation capability provided by Eqn. (9). This error bound allows practitioners to determine the number of terms required to achieve a desired level of accuracy without relying on empirical comparisons or expensive convergence studies. The error bound also provides theoretical assurance of convergence under appropriate conditions, enhancing the reliability of the method for practical applications.

Comparison with Existing Hybrid Methods

The proposed YT-EKP method advances the state of the art in hybrid transform-decomposition methods in several key respects. While the Yang Transform-Adomian Decomposition Method (YTADM) has demonstrated effectiveness for time-fractional nonlinear PDEs [7, 8], it retains the computational inefficiencies of classical Adomian

polynomials. The YT-EKP method overcomes this limitation through the use of El-Kalla polynomials, which provide derivative-free recursion and more accurate series coefficients.

Furthermore, the YT-EKP method offers a more systematic framework for error analysis than existing hybrid methods. The error bound in Eqn. (9) provides a rigorous foundation for convergence proofs in Banach spaces, addressing a gap in the theoretical literature on hybrid transform methods.

Limitations and Future Directions

Despite its demonstrated advantages, the YT-EKP method has certain limitations. The recursive formulation in Eqn. (6) assumes that the nonlinear operator can be expressed in the form $N(y) = yM(y)$, which holds for polynomial nonlinearities but may require modification for non-polynomial nonlinearities such as exponential or trigonometric functions. Extensions of the El-Kalla polynomial framework to handle such cases remain an area for future investigation.

Additional research directions include:

1. Extension to fractional differential equations: The YT-EKP method can be adapted to handle Caputo-Fabrizio and other fractional operators, addressing memory-dependent phenomena in viscoelastic materials and anomalous diffusion processes.
2. Application to delay differential equations: As noted in the objectives of this study, the extension to nonlinear delay differential equations represents a promising direction with applications in population dynamics and control theory.
3. Development of computational software: Implementation of the YT-EKP algorithm in general-purpose computational environments such as Maple, MATLAB, or Python would facilitate broader adoption by researchers and practitioners.
4. Theoretical convergence analysis: Rigorous proofs of convergence in Banach spaces for the YT-EKP method, particularly for fractional-order problems, would strengthen the theoretical foundation of the method.

Conclusion

This paper has presented the Yang Transform-El-Kalla Polynomial (YT-EKP) method, a novel hybrid analytical-numerical technique for solving nonlinear differential equations. The method synergistically combines the transform efficiency of the Yang transform with the accelerated nonlinear decomposition of El-Kalla polynomials, producing accurate and rapidly convergent series solutions.

The mathematical formulation of the YT-EKP method was established, and recursive relations for generating solution components were derived. The method was applied to three benchmark nonlinear problems: a first-order nonlinear equation, a Riccati equation, and the Duffing equation. In each case, the YT-EKP method produced series solutions that matched or exceeded the accuracy of existing methods, including the Kamal Adomian Polynomial method and the Temimi-Ansari Method.

Quantitative comparisons demonstrated that the YT-EKP method achieves absolute errors ranging from 10^{-17} to 10^{-6} for the problems considered, representing improvements of several orders of magnitude over comparative methods. The use of El-Kalla polynomials eliminates the need for repeated differentiation of nonlinear operators, reducing computational complexity and providing built-in error bounds.

The results of this study establish the YT-EKP method as an effective, reliable, and computationally efficient technique for solving nonlinear differential equations. The method holds strong potential for extension to more complex models, including fractional differential equations, delay differential equations, and systems of coupled nonlinear equations. Future work will focus on these extensions and on the development of user-friendly computational implementations to facilitate broader application in scientific and engineering practice.

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