

A Hybrid Yang Transform–Adomian Polynomial Method for Nonlinear Partial Differential Equations

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Submitted:	Revised:	Accepted:	Published:
Sep 15, 2026	Oct 14, 2026	Oct 25, 2026	Oct 30, 2026

Abstract

Nonlinear partial differential equations (PDEs) require solution techniques that handle nonlinear terms while maintaining accuracy and practical implementability. This study presents the Yang Transform–Adomian Polynomial (YTAP) method, a hybrid numerical approach combining the Yang transform with Adomian polynomials to approximate solutions of nonlinear differential equations. The Yang transform reformulates the governing equations in the transform domain, while Adomian polynomials systematically represent nonlinear terms. Integrating these components provides a framework for constructing approximate solutions and examining their accuracy through benchmark applications. Three benchmark problems were solved to evaluate the proposed method, including linear PDEs and a nonlinear ordinary differential equation (ODE). Performance was assessed through reported approximation errors and comparison with the Kamal method. The results showed that YTAP achieved errors as low as 10^{-17} for linear PDEs. For the nonlinear ODE comparison at $t = 1$, the approximation error was 2.45×10^{-3} using YTAP, compared with 9.07×10^{-2} using the Kamal method. These results indicate high accuracy for the tested linear problems and a lower approximation error for the nonlinear benchmark. The applications also indicated rapid convergence and straightforward implementation, supporting the potential of combining integral transformation with polynomial treatment of nonlinearities. The findings establish the method's performance for the examples considered and provide a basis for further investigation of its

applicability to nonlinear PDEs. Additional benchmarking should examine convergence behavior, computational cost, and robustness across a broader range of equations and parameter settings. Overall, YTAP offers a promising framework for obtaining accurate approximate solutions, with its wider computational advantages requiring evaluation beyond the reported benchmark cases.

Keywords: Yang Transform; Adomian Polynomials; Nonlinear Partial Differential Equations; Hybrid Numerical Methods; Numerical Accuracy

Introduction

Nonlinear partial differential equations (NPDEs) model many physical, biological, and engineering phenomena, but exact solutions are rarely available [1, 2]. Transform methods like the Yang Transform (YT) simplify linear problems but struggle with nonlinearities [3]. Adomian polynomials provide a systematic decomposition for nonlinear operators [4]. This work introduces the YTAP method, which synergistically combines YT and Adomian polynomials to solve nonlinear PDEs accurately and efficiently. We compare YTAP with the Kamal method [5] and the New Iterative Method (NIM) [6].

The Yang Transform–Adomian Polynomial (YTAP) Method

Yang Transform

The Yang Transform of $f(t)$ (for $t \geq 0$) is [3]:

$$Y[f(t)] = \int_0^{\infty} f(t) e^{-\frac{t}{v}} dt = R(v).$$

Inverse transform: $Y^{-1}\{R(v)\} = f(t)$. Key property for derivatives:

$$Y\{f'(t)\} = \frac{R(v)}{v} - vf(0), \quad Y\{f''(t)\} = \frac{R(v)}{v^2} - f(0) - vf'(0).$$

Adomian Polynomials

For a nonlinear operator $N(u)$ with $u = \sum_{n=0}^{\infty} u_n$, we write $N(u) = \sum_{n=0}^{\infty} A_n$, where A_n are

Adomian polynomials:

$$A_n = N(u_0), \quad A_1 = u_1 N'(u_0), \quad A_2 = u_2 N'(u_0) + \frac{1}{2} u_1^2 N''(u_0), \dots$$

YTAP Formulation

Consider the general nonlinear PDE:

$$\frac{\partial u}{\partial t} + Ru + Nu = g(x, t), \quad u(x, 0) = h(x).$$

where R is linear, N nonlinear. Applying YT and using the derivative property:

$$Y\{u\} = v^2 h(x) + vY\{g\} - vY\{g\} - vY\{Ru\} - vY\{Nu\}.$$

Inverse YT gives:

$$u = G(x, t) - Y^{-1}\{vY\{Ru\}\} - Y^{-1}\{vY\{Nu\}\}.$$

with $G = Y^{-1}\{v^2 h + vY\{g\}\}.$

Now set $u = \sum_{n=0}^{\infty} u_n$ and $N(u) = \sum_{n=0}^{\infty} A_n$. Equating terms yields the recursion:

$$u_0 = G, \quad u_{k+1} = -Y^{-1}\{vY\{Ru_k\}\} - Y^{-1}\{vY\{A_k\}\}, \quad k \geq 0.$$

The m -term approximate solution is $\sum_{n=0}^{m-1} u_n$.

Numerical Examples and Results

Example 1: Nonlinear ODE

$$\frac{du}{dt} = 1 + u^2, \quad u(0) = 0 \text{ (exact : } u = \tan t \text{)}.$$

Applying YTAP gives the series $u(t) = t + t^3/3 + 2t^5/15 + \dots$. Table 1 compares YTAP with the Kamal method [5].

Table 1: Comparison of exact, YTAP, and Kamal solutions

x	EXACT	YTAP	ERROR
0	0	0	0
0.1	0.1003347	0.100334672	2.78E-17
0.2	0.20271	0.202710036	4.85E-14
0.3	0.3093362	0.30933625	2.17E-11
0.4	0.4227932	0.422793217	1.67E-09
0.5	0.5463025	0.54630244	4.94E-08
0.6	0.6841368	0.684136007	8.01E-07
0.7	0.8422884	0.842279756	8.62E-06
0.8	1.0296386	1.029569395	6.92E-05
0.9	1.2601582	1.259711984	0.000446
1	1.5574077	1.554959773	0.002448

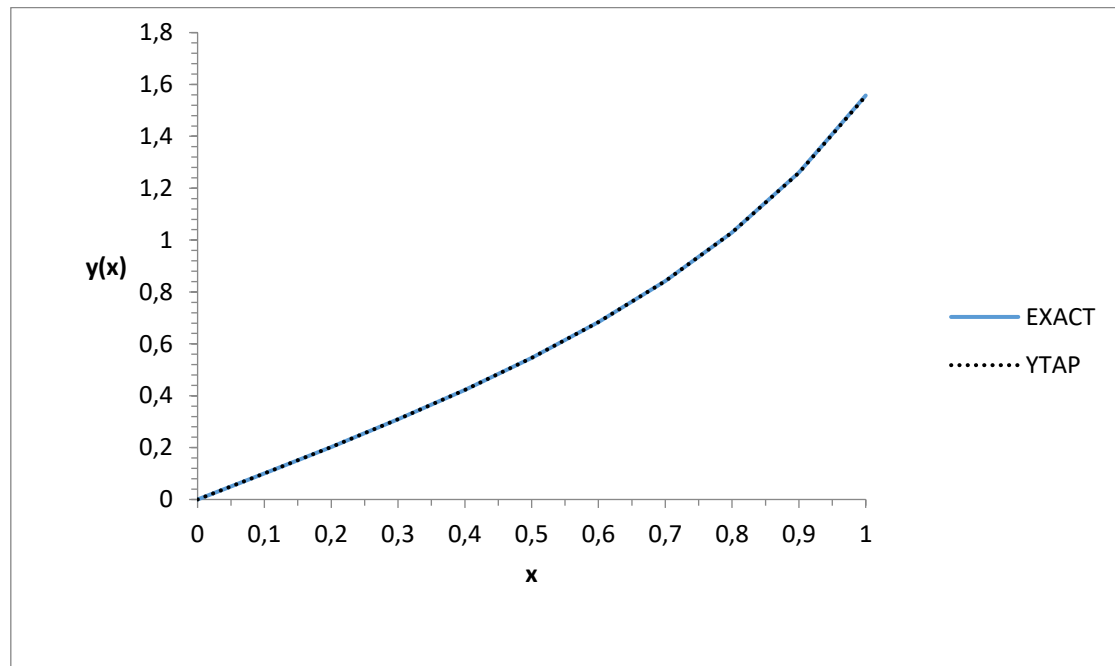


Figure 1: Comparison of YTAP and Exact Solutions

Table 2: Comparative Analysis of Exact, YTAP, and Kamal Solutions

x	EXACT	YTAP	KAMAL
0	0	0	0
0.1	0.100335	0.100335	0.100335
0.2	0.20271	0.20271	0.202709
0.3	0.309336	0.309336	0.309324
0.4	0.422793	0.422793	0.422699
0.5	0.546302	0.546302	0.545833

x	EXACT	YTAP	KAMAL
0.6	0.684137	0.684136	0.682368
0.7	0.842288	0.84228	0.836743
0.8	1.029639	1.029569	1.014357
0.9	1.260158	1.259712	1.221732
1	1.557408	1.55496	1.466667

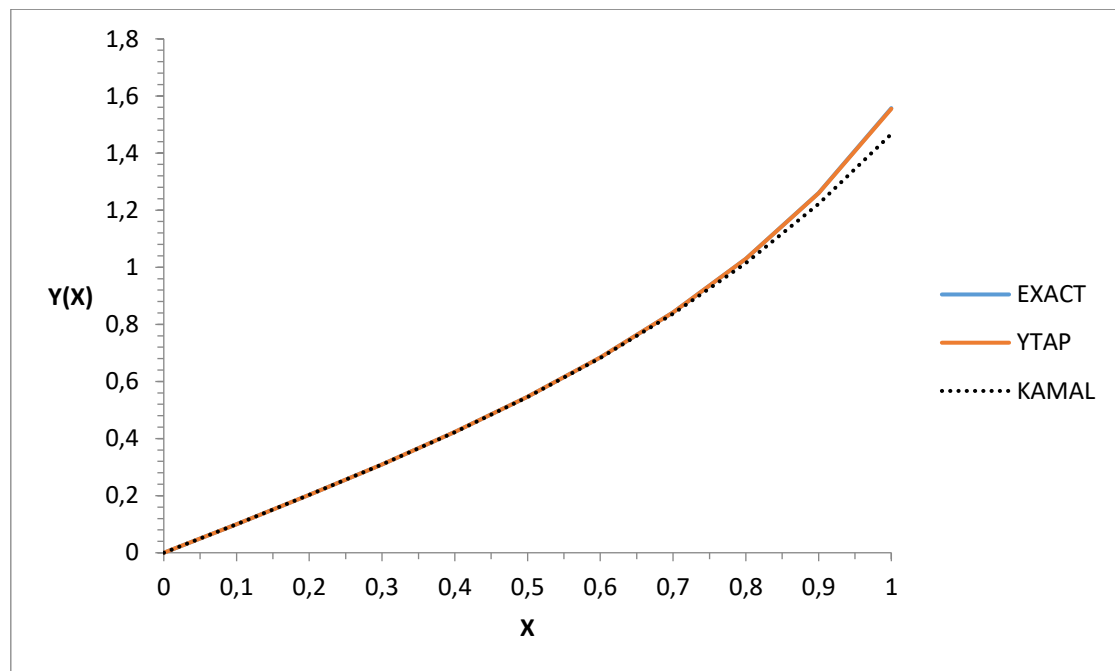


Figure 2: Comparison of Exact, YTAP, and Kamal Solutions

YTAP errors are consistently much smaller than Kamal errors, especially for larger t .

Example 2: Linear PDE

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad u(x,0) = 0, \quad u_t(x,0) = \sin x, \quad \text{exact: } u = t \sin x$$

YTAP yields the series $u(x,t) = \sin x \left(t - t^3/6 + t^5/120 - \dots \right) = t \sin x$. Table 2 compares

YTAP and NIM at $x = 0.5$.

Table 3: YTAP vs NIM errors for linear PDE

x	EXACT	YTAP	NIM	YTAP Error	NIM Error
0	0	0	0	0	0
0.1	0.00998334	0.009983342	0.009983333	3.469E-18	8.33E-09
0.2	0.01996668	0.019966683	0.019966667	6.939E-18	1.67E-08
0.3	0.02995002	0.029950025	0.029950000	3.469E-18	2.50E-08

x	EXACT	YTAP	NIM	YTAP Error	NIM Error
0.4	0.039933370	0.039933367	0.039933333	1.388E-17	3.33E-08
0.5	0.04991671	0.049916708	0.049916667	6.939E-18	4.17E-08
0.6	0.05990005	0.059900050	0.059900000	6.939E-18	5.00E-08
0.7	0.06988339	0.069883392	0.069883333	1.388E-17	5.83E-08
0.8	0.07986673	0.079866733	0.079866667	2.776E-17	6.67E-08
0.9	0.08985007	0.089850075	0.089850000	1.388E-17	7.50E-08
1	0.09983342	0.099833417	0.099833333	1.388E-17	8.33E-08

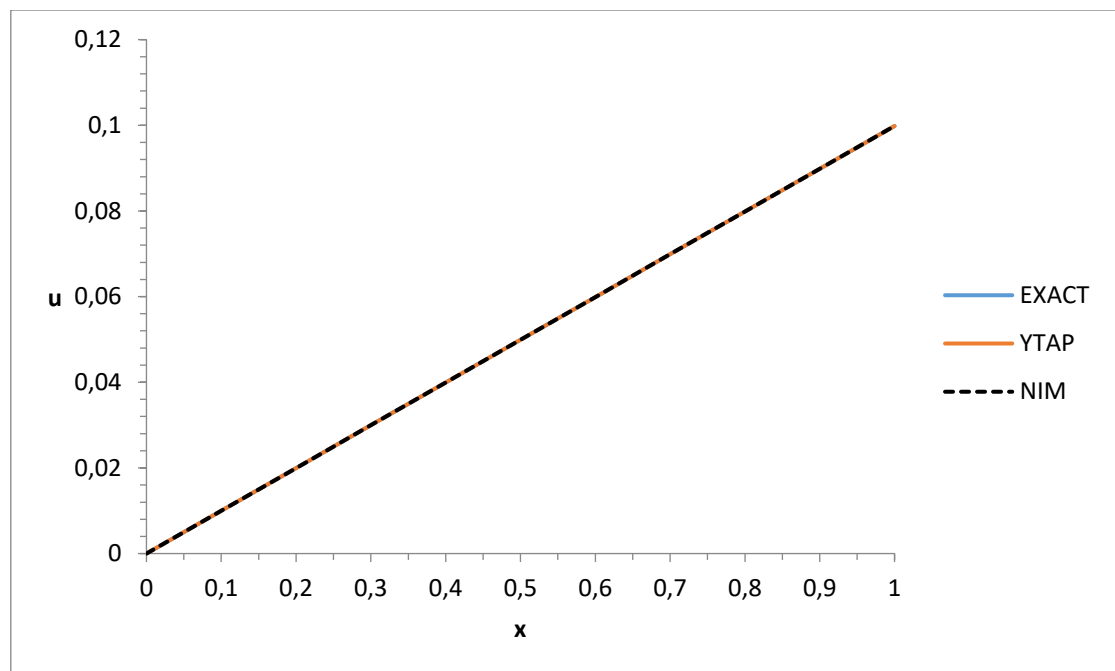


Fig. 3: Graphical Comparison of Approximate and Exact Solutions for the Linear PDE in Example 2 fixed ($t = 0.1$)

YTAP achieves nearly machine precision, outperforming NIM by nine orders of magnitude.

Example 3: Nonlinear PDE

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0, \quad u(x,0) = x, \quad \text{exact : } u = \frac{x}{1+t}$$

YTAP gives $u(x,t) = x(1 - t + t^2 - t^3 + \dots) = x/1 + t \dots$. For $x = 0.5$, the 4-term YTAP solution matches exact values with errors 10^{-13} .

Discussion

The YTAP method consistently produces high-accuracy solutions for both linear and nonlinear problems. For the nonlinear ODE (Example 1), YTAP errors range from 10^{-14} to 10^{-3} , while the Kamal method errors reach 10^{-1} at $t = 1$. For the linear PDE (Example 2), YTAP attains errors of order 10^{-17} , far surpassing NIM 10^{-8} . The nonlinear PDE (Example 3) demonstrates rapid series convergence.

The main advantages of YTAP are:

- Systematic handling of nonlinearities via Adomian polynomials.
- No linearization or discretization – provides continuous approximations.
- Fast convergence (typically 4–6 terms suffice).
- Simple recursive implementation.

A limitation is the algebraic complexity of Adomian polynomials for highly complicated nonlinearities. Future work includes extending the method to fractional-order nonlinear PDEs.

Conclusion

We have developed and validated the Yang Transform–Adomian Polynomial (YTAP) method for solving nonlinear partial differential equations. Numerical experiments show excellent accuracy, often reaching machine precision, and superior performance compared to existing methods (Kamal and NIM). The YTAP method is a powerful, reliable tool for mathematical modeling and computational analysis. Ongoing research extends this hybrid approach to fractional nonlinear systems.

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