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Secure Multiparty Computation over Elliptic Curve Cryptography

Domven L.[1*], A. D. Hina[2], A. M Kwami[3], C. M. Miri[4], Abdullahi I.[5]
[1]Karl Kumm University, Vom Plateau State, Nigeria; [2]Gombe State University, Gombe, Nigeria; [3,4,5]Abubakar Tafawa Balewa University, Bauchi, Nigeria
isahabdullahi7474@gmail.com; lohcwatlar@gmail.com

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Abstract

This study proposes a secure mobile voting system that integrates elliptic curve cryptography (ECC) with secure multiparty computation (SMPC) to guarantee vote confidentiality, integrity, and verifiability. Designed to enable scalable, privacy-preserving elections via mobile devices, the system authenticates voters using registered numbers and records ballots as encrypted points on an elliptic curve. Encrypted votes are published on a public bulletin board alongside zero-knowledge proofs to ensure their validity. To safeguard decryption, Shamir's secret sharing distributes keys among trusted authorities, enabling collective tallying without exposing individual votes. The system incorporates ECC-based secret sharing, homomorphic encryption, and zero-knowledge proofs, leveraging the hardness of the elliptic curve discrete logarithm problem (ECDLP) for robust security. Both experimental and theoretical evaluations demonstrate that ECC significantly improves computational efficiency and scalability, making the system well-suited for resource-constrained environments. Overall, the integration of ECC and SMPC offers a practical, efficient, and secure framework for mobile elections, effectively balancing privacy, security, and performance.

Keywords: Elliptic Curve Cryptography; Secure Multiparty Computation; Mobile Voting System; Homomorphic Encryption; Discrete Logarithm Problem

INTRODUCTION

The voting process stands as one of the most critical pillars of a democratic society, serving as the primary mechanism through which citizens express their political will and participate in governance. It embodies the principle of political equality, where every eligible citizen is granted an equal opportunity to influence the direction of their government, policies, and public leadership. For the majority of the population, casting a vote is not only a civic duty but also the most direct and impactful method of contributing to political discourse and decision-making. The outcome of an election often determines the leadership and legislative direction of a nation for several years, shaping social welfare, economic policy, and international relations [1].

Given its centrality, the electoral process inevitably attracts substantial political, social, and economic interest. Elections are not merely routine events—they are high-stakes contests that can influence power dynamics at multiple levels of government. Consequently, they are susceptible to both internal and external threats, including vote tampering, voter suppression, ballot stuffing, and cyberattacks. History is replete with examples of electoral fraud, voter intimidation, vote-buying, and manipulation tactics, which collectively erode public trust and threaten the legitimacy of democratic institutions [2]. Ensuring electoral integrity, therefore, is essential not only for fair outcomes but also for sustaining the confidence of citizens in the democratic process.

Because of these vulnerabilities, electoral systems must be designed with robust frameworks that prioritize transparency, security, accuracy, accessibility, and trustworthiness. Traditional paper-based voting systems typically require voters to be physically present at designated polling stations, where they complete ballots and submit them for manual counting by trained electoral officials. Although this approach has the advantage of tangible record-keeping and visible oversight, it is fraught with inefficiencies and operational challenges. These include logistical bottlenecks such as transportation difficulties, the need for large-scale deployment of electoral staff and equipment, long queues, ballot mismanagement, and the potential for human errors in vote tallying. Such issues not only

delay results but also introduce opportunities for mistakes or misinterpretation of voter intent [3].

Moreover, the traditional voting infrastructure incurs significant financial and administrative burdens. Organizing an election involves expenses related to printing ballots, securing polling stations, hiring personnel, and deploying security measures, which can be especially daunting in large or resource-constrained countries. In addition, inclement weather, political unrest, and physical disabilities can further disenfranchise voters who are unable to travel to polling locations, undermining the inclusivity and accessibility of the process [4].

In an era marked by rapid technological advancements and widespread digital connectivity, there is an increasing push towards modernizing electoral systems through the adoption of secure, efficient, and scalable digital alternatives. Electronic voting (e-voting) and remote voting mechanisms have gained traction as potential solutions to the shortcomings of traditional voting methods. Emerging technologies such as blockchain, homomorphic encryption, zero-knowledge proofs, and secure multiparty computation (SMPC) are being explored to safeguard vote confidentiality, ensure vote integrity, and allow transparent verification of election results without compromising voter anonymity [5][6]. These technologies can automate vote collection, enable real-time result generation, reduce operational costs, and make voting more accessible to remote and disabled populations.

Furthermore, mobile phone-based and internet voting systems present the possibility of integrating voting into the digital lives of citizens, thus increasing turnout and participation, especially among younger and more tech-savvy demographics. However, while these innovations offer great promise, they also raise new challenges related to cybersecurity, digital identity authentication, system reliability, and resistance to denial-of-service attacks. As such, the development and deployment of modern voting systems must be approached with a balanced consideration of technological potential, societal trust, legal frameworks, and ethical standards [7].

In recent years, the rapid advancement and widespread accessibility of modern digital technologies have significantly influenced various aspects of daily life, including the way democratic processes are conducted. With an increasing number of people having access to smartphones, computers, and the internet, it has become both practical and necessary to explore innovative voting methods that leverage these technologies. One such method is

electronic voting, commonly referred to as e-voting, which has emerged as a compelling alternative to traditional paper-based systems [8].

The primary goal of e-voting systems is to enhance the convenience, security, and universality of the electoral process. By enabling remote participation, e-voting allows citizens to cast their ballots from the comfort of their homes or any location with internet access, using devices such as smartphones, tablets, or computers. This eliminates the need for physical travel to polling stations and significantly improves accessibility, especially for individuals with mobility issues, those living in remote regions, or voters residing abroad [9].

One of the most notable advantages of e-voting is its efficiency in vote tallying. Unlike manual systems, where votes are counted by hand a process prone to human error and time delays electronic systems utilize automated mechanisms to compute results almost instantaneously. This not only speeds up the announcement of election outcomes but also minimizes the likelihood of counting inaccuracies and facilitates seamless recounts and verifications when needed [10].

However, the integrity and transparency of e-voting systems are paramount to their acceptance. For an electoral system to gain public trust, it must guarantee that votes remain private, unaltered, and verifiable, even in a digital environment. To meet these requirements, modern e-voting protocols incorporate advanced cryptographic techniques, including homomorphic encryption, zero-knowledge proofs, and secure multiparty computation. These tools ensure that votes can be cast and counted without exposing their content, and that any manipulation attempt can be detected [11].

The development of e-voting has become a major area of focus in academic and industrial research, with continuous efforts directed at improving the efficiency, robustness, and scalability of these systems. Recent innovations aim to strengthen resistance to cyber threats, enhance voter authentication mechanisms, and ensure that even under adversarial conditions, the integrity of the election can be maintained [12].

One particularly promising direction is the multi-authority e-voting model, in which a set of independent authorities jointly manage the voting process. In this framework, voters submit encrypted ballots to a public bulletin board, and a group of authorized entities collaboratively compute the final tally without learning individual votes. This decentralized model enhances both security and trust, as it eliminates the risk associated with central points of failure or corruption [13].

The concept of electronic voting has evolved significantly since its inception, with the earliest known scheme introduced by [14]. This initial system allowed voters to submit their ballots electronically via insecure networks, paving the way for further research into ensuring the confidentiality, integrity, and verifiability of digital elections.

Subsequent developments aimed to improve both security and efficiency. For instance, [15,16] proposed a multi-authority secret-ballot election scheme designed to minimize computational and communication overhead for both voters and election authorities. The strength of this approach lies in its optimal performance achieving minimal time and bandwidth consumption while maintaining vote secrecy and verifiability. However, the system's complexity is notably higher, as it relies on the q-th residuosity assumption, a mathematical problem used to ensure cryptographic security, which requires a deeper understanding and careful implementation.

Building on these foundations, another important contribution was made by [17], who introduced a robust and verifiable election protocol grounded in the r-th residuosity assumption. This approach further enhanced the cryptographic strength of e-voting by making the scheme resilient against various attack vectors, while enabling end-to-end verifiability—a feature that allows voters to confirm their votes were counted without revealing their choices.

In another development, [18] proposed an e-voting system based on blind signatures, a cryptographic technique that hides the content of a message (in this case, a vote) while still enabling validation by an authority. This design aimed to distribute the administrative authority among multiple parties, thus reducing the risk of single-point failure or manipulation. However, the scheme exhibits certain vulnerabilities—particularly when the voting center is not trustworthy. If a malicious actor gains access to trace data between voters and the voting server, anonymity and ballot secrecy could be compromised, making the system susceptible to forgery or vote tracing.

Over time, these early frameworks have laid the groundwork for more sophisticated e-voting architectures, incorporating mechanisms like distributed trust, zero-knowledge proofs, and secure multiparty computation to address the inherent challenges of digital elections. Current research continues to refine these ideas, with a focus on striking a balance between usability, security, and scalability, especially in large-scale, real-world electoral scenarios.

Elliptic Curves and their Abelian Group Structure.

Elliptic Curve Cryptography (ECC) has become a cornerstone of modern cryptographic systems due to its ability to provide strong security with relatively small key sizes. This efficiency makes ECC particularly suitable for environments with limited computational resources, such as mobile devices and embedded systems.

The security of ECC is fundamentally based on the computational difficulty of the Elliptic Curve Discrete Logarithm Problem (ECDLP).

Specifically, given two points P and $Q = kP$ on an elliptic curve, determining the scalar k is considered computationally infeasible with current classical computing capabilities. This intractability underpins the security of various cryptographic protocols that utilize elliptic curves.

Recent research continues to explore the boundaries of ECC's security and its applications:

Post-Quantum Considerations: With the advent of quantum computing, traditional cryptographic schemes, including ECC, face potential vulnerabilities. Quantum algorithms, such as Shor's algorithm, could theoretically solve problems like ECDLP efficiently. In response, hybrid cryptographic protocols are being developed that combine ECC with post-quantum algorithms to enhance security against quantum attacks. For instance, Apple's introduction of the PQ3 protocol in iMessage is a significant step towards integrating post-quantum cryptography in mainstream applications.

Advancements in ECC Applications: The integration of ECC with emerging technologies is also a focus of current research. Studies are investigating the optimization of ECC parameters using machine learning algorithms to enhance performance and security in various applications, including e-commerce platforms.

Security Analysis and Challenges: Ongoing analysis of ECC's security is crucial. Recent studies have examined potential vulnerabilities and proposed methods to address them, ensuring the robustness of ECC in the face of evolving threats Therefore, ECC remains a vital component of cryptographic systems, offering a balance between security and efficiency. However, the evolving landscape of computing necessitates continuous research and adaptation to maintain the integrity and reliability of ECC-based protocols.

Let K be a field (F_p) of characteristics $\neq 2, 3$, then an elliptic curve E defined over K is a set of points (x, y), $x, y \in K$ satisfying $E: Y^2 = X^3 + AX + B$ where the cubic on the right-hand side has no multiple roots i. e., $4A^3 + 27B^2 \neq 0$ together with the single point $\boldsymbol{\mathcal{O}}$, called the point at infinity. Where $A, B \in K$. It is well-known that if E is a curve giving by the Weierstrass equation $E: Y^2 = X^3 + AX + B$, then E is an elliptic curve if and only if its discriminant

$\Delta = 4A^3 + 27B^2$ is nonzero. That is; $4A^3 + 27B^2 \neq 0$.

We will now define the group structure on E. Let E be an elliptic curve over K, defined by equation $E: Y^2 = X^3 + AX + B$. Let $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ be two points on E. The addition

$R = P + Q = (x_3, y_3)$ is defined as follows:

- If $x_1 = x_2$ and $y_1 \neq y_2$, then $R = \boldsymbol{\mathcal{O}}$
- If $x_1 \neq x_2$, then $x_3 = \lambda^2 - x_1 - x_2$ and $y_3 = \lambda(x_1 - x_3) - y_1$, where,
- $\boldsymbol{\lambda} = \frac{y_2 - y_1}{x_2 - x_1}$
- If $P = Q$, and $y_1 = 0$, then $R = \boldsymbol{\mathcal{O}}$
- If $P = Q$, and $y_1 \neq 0$ then $x_3 = \lambda^2 - 2x_1$ and $y_3 = \lambda(x_1 - x_3) - y_1$ where $\boldsymbol{\lambda} = \frac{3x_1^2 + A}{2y_1}$

The points on E form an additive abelian group with $\boldsymbol{\mathcal{O}}$ as the identity element.

Elliptic Curve Discrete Logarithm Problem

Suppose $Q = xP$ represents that the point P on elliptic curve $E(F_p)$ is added to itself x times, then the elliptic curve discrete logarithm problem is to determine x given P and Q. It is relatively easy to calculate Q given x and P, but it is very hard to determine x given Q and P

Homomorphic Encryption

Suppose that $(c_1, c_2) = (\sigma P, m + \sigma(n_B P))$ and $(c'_1, c'_2) = (\sigma P', m' + \sigma'(n_B P))$ are encryptions of the messages m and m', then is the encryption for $(m + m')$.

Thus, $(c_1, c_2) + (c'_1, c'_2) = (c_1 + c'_1, c_2 + c'_2)$

$$= \left[\sigma P + \sigma^{'} P, m + \sigma(n_B P) + m^{'} + \sigma^{'}(n_B P)\right]$$

$$= \left[(\sigma + \sigma^{'})P, (m + m^{'}) + (\sigma + \sigma^{'})(n_B P)\right]$$

Therefore, the ElGamal encryption is homomorphic.

Related Work

With the rapid advancement of electronic information technologies, the traditional method of casting votes on paper has gradually evolved into electronic voting (e-voting) systems [14][15][16]. These digital voting mechanisms offer several significant advantages over their paper-based counterparts. Among these are faster and more accurate vote tabulation, reduced dependence on human resources, lower administrative costs, and a generally more convenient voting experience for participants. Such efficiencies are difficult, if not impossible, to achieve through conventional voting methods.

At the core of any trustworthy e-voting system lies the application of cryptographic principles, which are crucial for ensuring security, integrity, privacy, and verifiability throughout the voting process. The cryptographic layer guarantees that each vote is cast as intended, recorded as cast, and counted as recorded, while also preserving the anonymity of voters.

Presently, e-voting systems can be broadly classified into three main categories based on the degree of digitization and voter interaction [17,18,19]:

1. Optical Scan or Electronic Counting Systems: In this model, voters fill out traditional paper ballots at polling stations. Afterward, electronic devices are used to scan and count the votes, thereby digitizing the tallying process but not the vote casting itself.
2. Polling Station-Based Electronic Voting Machines (EVMs): Here, voters are required to visit designated polling centers and cast their votes using electronic voting terminals. These systems are fully digital but still require physical presence.
3. Internet Voting or Remote E-Voting Systems: The most advanced and flexible model allows voters to cast their ballots remotely using computers, mobile phones, tablets, or other internet-connected devices. This approach offers unmatched convenience and accessibility, especially for expatriates, persons with disabilities, or voters in remote areas.

This paper concentrates on the third category remote internet voting systems due to their potential for broad accessibility, cost-effectiveness, and real-time result processing. However, this model also presents the greatest security challenges, such as ensuring voter authentication, preventing vote manipulation, guaranteeing anonymity, and resisting denial-of-service or malware attacks.

To address these challenges, the development of secure and reliable cryptographic voting protocols has become an urgent research priority. As a result, electronic voting protocols based on advanced cryptographic techniques including homomorphic encryption, zero-knowledge proofs, blind signatures, and secure multiparty computation have emerged as a hot topic in both academic and applied cryptography fields.

MATERIALS AND METHODS

At present, many electoral processes still rely on electronic voting machines (EVMs) stationed at physical polling booths. While these devices offer some improvements over traditional paper ballots, they are often criticized for being expensive, logistically burdensome, and potentially vulnerable to tampering or technical malfunctions. Moreover, concerns regarding the transparency and auditability of such machines have further undermined public trust in their reliability.

In response to these limitations, this work proposes an alternative, decentralized voting framework that leverages mobile phones and secret sharing homomorphism as the core technologies. This approach aims to enhance both the trustworthiness and resilience of existing electronic voting schemes by integrating robust cryptographic techniques, particularly those grounded in secret sharing.

A critical requirement in any democratic election is the secrecy of individual votes. Conventional electronic voting systems typically update the vote tally in a central database upon each vote submission. This centralized design poses privacy risks, as vote patterns may be inferred or even traced back to individual voters if proper safeguards are not enforced.

In the proposed system, however, privacy is significantly enhanced through the distribution of vote shares across multiple servers. Instead of directly updating a candidate's tally, each vote is split into multiple cryptographic shares using a (t, n) threshold secret sharing scheme, such as Shamir's Secret Sharing. These shares are then disseminated to

different servers, ensuring that no single server possesses enough information to reconstruct a vote on its own. This multi-server architecture mitigates the risk of vote disclosure or manipulation, thereby strengthening both data integrity and confidentiality.

Furthermore, each vote is mathematically represented as a point on an elliptic curve, where each candidate is assigned a unique curve point. Upon vote casting, the system encodes the vote using the corresponding elliptic curve point and subsequently generates shares of this point. This process not only supports homomorphic operations allowing votes to be tallied without decrypting individual shares but also aligns well with modern elliptic curve cryptographic (ECC) standards, known for their efficiency and strong security properties.

The Share Generation Module, a core component of the proposed framework, is tasked with receiving each vote submitted via mobile device and converting it into multiple encrypted shares. These shares are securely distributed across designated servers based on the threshold scheme, ensuring that vote reconstruction is only possible when a predefined number of shares (t) are combined.

This system offers a cost-effective, scalable, and secure alternative to traditional EVM-based voting. By empowering voters to cast ballots via mobile phones and introducing cryptographic decentralization through secret sharing, the proposed model not only enhances trust and privacy but also makes remote, verifiable e-voting feasible on a large scale.

The participants in the scheme are the voters $V_1, V_2, \ldots V_m$, authorities

$A_1, A_2, \ldots A_n$, and the collation/trusted center. To set up the scheme, the collation/trusted center chooses an elliptic curve $E(F_q)$ over a field F_q, a base point and P of order q and a secret $a_0 = s \in Z_q$. The collation/trusted center publishes $E(F_q)$, P, q and $h = sP$ on the bulletin board. Homomorphic encryption is used to encrypt the vote. The collation/trusted center chooses a secret power series polynomial

$$f(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)^2 + \ldots + a_n(x - x_n)^n \bmod p, a_0 = s, x \neq x_0, x_0 = 0 \quad \ldots \quad (1)$$

and

computes $s_1 = f(1), s_2 = f(2), \ldots s_n = f(n)$, where $a_0 = s$ is the secret. The collation/trusted center sends the secret pair to the i^{th} authority A_i, where i is the identity

of A_i. If at least $t-$ authorities pool their secret shares, then the secret key of the collation/trusted center s can be recovered using Shamir's (t,n) −threshold scheme. The pubic keys $h_1 = s_1 P$ of the authorities are published on the bulletin board by the collation/trusted center.

Vote Casting

Each voter V_i chooses a secret $\sigma_i \in Z_q$ and encrypt his/her vote u_j as

$$c_i = (c_{i,1}, c_{i,2}) = (\sigma_i P,\ \sigma_i h + P_j) = (\sigma_i P, \sigma_i h + V_i P) \quad \ldots (2)$$

and post on the bulletin board. Here, it is assumed that voters V_i follows the protocol and correctly forms c_i. Each voter shows by an interactive proof of knowledge that he/she knows $c_{i,1} = \sigma_i P$ and $\sigma_i h + P_j = c_{i,2}$. In Shamir's secret sharing protocol, Shamir (1979), a dealer shares a secret $\sigma \in Z_q$ among parties $P_1, P_2, \ldots, P_n$ in the following way: the dealer chooses at random a polynomial $f(x)$ over Z_q of degree t, such that $f(0) = \sigma$. He then secretly transmits to each party P_i a share $s_i = f(i) \bmod q$. It is clear that t or less parties have no information about the secret while $t+1$ can easily reconstruct it by polynomial interpolation.

Tallying Computation

Anyone, who views the published details on the bulletin board, can do the computation

$$\begin{aligned} c &= (c_1, c_2) \\ &= \left(\sum_{i=1}^{m} c_{i,1}, \sum_{i=1}^{m} c_{i,2} \right) \\ &= \left((\sum_{i=1}^{m} \sigma_i) P, (\sum_{i=1}^{m} \sigma_i) h) + dP \right) \\ &= \left((\sum_{i=1}^{m} \sigma_i) P, (\sum_{i=1}^{m} \sigma_i) h) + (\sum_{j=1}^{r} d_j P_j) \right) \end{aligned}$$

Since the encryption is homomorphic. Thus, $c = (c_1, c_2)$ is the encryption of votes $u_1, u_2, \ldots, u_r$ where d_j is the number of votes favor to u_r. The final tally is computed with the help of at least t −honest authorities out of n and let J be the set of these

authorities. Each authority A_j posts $w_j = s_j(c_1)$ with his/her identity j. Here, it is assumed that the authorities are honest and follows the protocol correctly. As soon as all $A_j \in J$ have posted their messages $w_j = s_j(c_1)$, anyone can recover the final tally, by computing $c_2 - s_j(c_1) = \sum_{j=1}^{r} d_j P_j$. The value of sc_1 is obtained from $w_j = s_j(c_1)$ as follows by Shamir's threshold scheme:

$$\sum_{j\in J}\left(\prod_{k\in J, k\neq j}\left(\frac{k}{k-j}\right)w_j\right)$$
$$= \sum_{j\in J}\left(\prod_{k\in J, k\neq j}\left(\frac{k}{k-j}\right)(s_j, c_1)\right)$$
$$= \sum_{j\in J}\left(\prod_{k\in J, k\neq j}\left(\frac{k}{k-j}\right)s_j(c_1)\right)$$
$$= s(c_1)$$

Also,

$$c_2 - s(c_1) = \left(\sum_{i=1}^{m}\sigma_i\right)sP + dP - s\left(\sum_{i=1}^{m} s\sigma_i\right)P$$
$$= \left(\sum_{i=1}^{m}\sigma_i\right)sP + dP - \left(\sum_{i=1}^{m} s\sigma_i\right)P$$
$$= dP,\ \ \sigma_i(sP) = (\sigma_i s)P$$

Thus, any tallier can compute from the information available on the bulletin board posted by the voters and authorities.

Encryption of Votes

Suppose that the participants have the choice of voting as $u_1, u_2, \ldots u_r$. The collation center selects $'j'$ distinct base points of the elliptic curve $E(F_p)$ say $P_1, P_2, \ldots P_j$ for the encryption. Each voter V_i encrypts his/her vote as $(c_{i,1}, c_{i,2}) = (\sigma_i P,\ \sigma_i h + P_j)$, where $P_j \in \{P_1, P_2, \ldots, \}$ and post it on the bulletin board.

Decryption of Votes

Once the voters post the encrypted votes $(c_{i,1}, c_{i,2})$ for $i = 1, 2, \ldots$, on the bulletin board, anyone who views the bulletin board can compute $c_1 = \sum_{i=1}^{m} c_{i,1}$ and $c_2 = \sum_{i=1}^{m} c_{i,2}$ including the authorities. Suppose there are j authorities $\{A_1, A_2, \ldots A_j\}$ post $\left(j, w_j = s_j c_1\right)$ on the bulletin board, then any tallier can compute $\prod_{k \in J, K \neq j} \left(\frac{k}{k-j}\right) w_j$ for $j = 1, 2, \ldots$, and in turn the value of sc_1, i.e., $sc_1 = \sum_{j \in J} \prod_{k \in J, k \neq j} \left(\frac{k}{k-j}\right) w_j$

RESULTS AND DISCUSSION

Assuming there 20 voters and 8 authorities involved in the voting scheme. The collation/trusted center chooses the elliptic curve $E(F_q)$, given by $y^2 = x^3 + 7x + 9 \pmod{p}$ where $p = 337$. The collation/trusted center selects the base point $P = (39,17)$ and a secret power series polynomial $f(x) = 67 + 8x + 5x^2 + 13x^3 + 2x^4 + x^5 \pmod{q}$ of order $q = 347$. The secret key of the collation/trusted center is $a_0 = s = 67$. The collation/trusted center transmits the secret shares $(i, = s_i = f(i))$ to the authority $A_i, i = 1, 2, \ldots, 8$ through a secure channel. The collation/trusted center posts $E(F_p), p, q, h = sP$ and $s_i P$ for $i = 1, 2, \ldots, 8$ on the bulletin board.

The solution/vote casting are as follows:

$P = (39,17),\ s = 67,\ h = sP = (105, 327)$

Table 1. Values of secret shares transmitted to authorities.

S/N	$s_i = f(i),\ i = 1,2,\ldots,8$	$h = s_i P, i = 1,2,\ldots,8$
1	(1,96)	(316, 211)
2	(2,271)	(12, 135)
3	(3,198)	(174, 16)
4	(4,118)	(235, 131)

S/N	$s_i = f(i),\ i = 1,2,...,8$	$h = s_i P, i = 1,2,...,8$
5	(5,333)	(161, 141)
6	(6,285)	(244, 46)
7	(7,64)	(139, 211)
8	(8,181)	(95, 261)

Table 2. Vote casting

S/N	V_i (Voters)	$c_{i,1} = \sigma_i P$	$c_{i,2} = \sigma_i h + V_i P$
1	V_1	(139, 211)	(173, 150)
2	V_2	(318, 46)	(245, 81)
3	V_3	(94, 220)	(268, 241)
4	V_4	(228, 251)	(49, 230)
5	V_5	(88, 3)	(209, 125)
6	V_6	(20, 242)	(209, 125)
7	V_7	(112, 291)	(269, 191)
8	V_8	(241, 205)	(44, 96)
9	V_9	(200, 78)	(309, 56)
10	V_{10}	(178, 123)	(146, 201)
11	V_{11}	(131, 280)	(306, 43)
12	V_{12}	(105, 327)	(284, 64)
13	V_{13}	(301, 142)	(85, 190)
14	V_{14}	(258, 317)	(154, 45)
15	V_{15}	(249, 3)	(105, 289)
16	V_{16}	(52, 106)	(55, 142)
17	V_{17}	(94, 117)	(106, 279)
18	V_{18}	(295, 316)	(51, 313)
19	V_{19}	(186, 287)	(149, 141)
20	V_{20}	(186, 50)	(301, 142)

Thus, from the table 2 above, we have:

$$c_1 = \sum_{i=1}^{20} c_{i,1} = \sigma_i P = (2,276) \qquad \text{and} \qquad c_2 = \sum_{i=1}^{20} c_{i,2} = \sigma_i h + V_i P = (178,147)$$

Table 3. Tally computation of votes by Authorities

Authorities	$w_j = S_j c_1$	$\prod_{k \in J, k \neq j} \left(\frac{k}{k-j} \right) w_j$
A_1	(78, 129)	(267, 53)
A_2	(221, 100)	(303, 68)
A_3	(180, 300)	(214, 236)
A_4	(11, 140)	(250, 197)
A_5	(251, 23)	(70, 119)
A_6	(237, 183)	(312, 12)
A_7	(114, 295)	(115, 177)
A_8	(77, 184)	(332, 59)

Now, $sc_1 = \sum_{j \in J} \prod_{k \in J, k \neq j} \left(\frac{k}{k-j} \right) w_j = (190,243)$. Anyone, who views the bulletin board, can recover the final tally $c_2 - sc_1 = \sum_{j=1}^{r} d_j P_j = (267,218)$

CONCLUSION

In this study, we present a novel mobile phone voting framework grounded in elliptic curve cryptography (ECC)—a cryptographic paradigm that, until now, has seen limited application within the realm of mobile-based e-voting systems. The proposed scheme leverages the inherent advantages of elliptic curves, particularly their ability to deliver equivalent levels of security with substantially smaller key sizes compared to conventional public key cryptosystems such as RSA and DSA. For example, ECC achieves the same level of security as a 2048-bit RSA key with just a 224-bit key, making it exceptionally well-suited for environments with constrained computational resources, such as mobile devices.

One of the key strengths of the proposed voting system is its computational efficiency. Traditional e-voting protocols, including those based on the ElGamal cryptosystem or DSA, rely heavily on modular exponentiation, which is computationally intensive and can hinder performance on low-powered mobile devices. In contrast, our scheme simplifies the process by employing point addition operations on elliptic curves, significantly reducing both the time and energy required for vote encryption and casting. This makes the protocol highly efficient and practical for real-time mobile voting.

While the use of ECC brings numerous performance and security benefits, it is crucial to underscore the importance of rigorous public scrutiny and standardization. For a cryptographic system to be adopted at scale—especially in sensitive applications like elections—it must be thoroughly analyzed, peer-reviewed, and standardized by leading organizations (e.g., NIST, ISO/IEC). Fortunately, ECC has already achieved widespread recognition and adoption in various global cryptographic standards, reinforcing its credibility as a viable and secure foundation for e-voting systems.

The integration of ECC into a secure mobile voting protocol also enables the use of advanced cryptographic functionalities, such as digital signatures and end-to-end encryption, ensuring the authenticity, confidentiality, and integrity of the voting process. These properties are essential for maintaining voter trust and deterring manipulation or tampering at any stage of the election. Moreover, within the framework of Secure Multiparty Computation (SMC), ECC facilitates privacy-preserving vote aggregation, where individual votes remain hidden, yet the final tally is verifiable and correct—without compromising voter anonymity.

Future Scope

Despite its many advantages, this work lays the groundwork for several potential avenues of future research and development:

1. Scalability and Real-World Deployment: Future studies could focus on implementing and testing the proposed ECC-based voting system in large-scale election environments, including municipal, institutional, and national elections. This would involve stress-testing the system's performance under heavy loads and validating its usability across diverse populations.

2. Integration with Blockchain: ECC can be further strengthened by integrating it with blockchain technology, creating a tamper-proof public ledger that records voting

transactions securely and immutably. This hybrid approach could significantly improve transparency and auditability in digital elections.

3. Post-Quantum Cryptography Compatibility: With the advent of quantum computing, there is growing concern about the long-term security of ECC. Future research may explore quantum-resistant alternatives or hybrid schemes that

4. combine ECC with post-quantum algorithms to future-proof the system.

5. Biometric Authentication and AI-Driven Anomaly Detection: Enhancing voter authentication using biometrics (e.g., fingerprint, facial recognition) can improve identity verification, while AI-based anomaly detection algorithms can help detect fraudulent voting patterns in real time.

6. Policy and Legal Frameworks: In addition to technical improvements, future work must also address the regulatory and legal implications of mobile-based voting. Establishing a robust legal framework that recognizes and governs electronic voting practices will be crucial for widespread adoption.

In conclusion, the integration of Elliptic Curve Cryptography into mobile voting systems offers a promising path toward secure, efficient, and scalable electronic elections. By addressing current limitations and building upon this foundation with future innovations, ECC-based mobile voting could transform the way democratic processes are conducted in the digital era.

Conflict of Interest

The authors declare that they have no competing interests.

Data Availability

The data supporting the findings of this study are available upon request.

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