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A Poisson Quasi Suja Distribution

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Abstract

Two-parameter Poisson Quasi Suja distribution (PQSD) derived from the two-parameter quasi suja distribution is proposed for extremely positively count data. Its survival and hazard functions, first four raw moments' measures were expressed. The variance, coefficient of variation, index of dispersion, skewness and kurtosis were also obtained. The impacts of each parameter in the new distribution were assessed.

Keywords: Poisson, Quasi Suja distribution, Reliability measures, Moment Measures, Skewness and Kurtosis, Index of Dispersion

Introduction

It is well known that count data are model after Poisson distribution. Poisson distribution will fit the data well when the mean and variance of the count data are largely the same. Another very useful model for fitting count data is Negative Binomial distribution. This is very useful when the variance is really greater than the mean of the count data, a problem

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of over dispersion. Consider the situation when the count data is over dispersed and then skewed, a Negative Binomial distribution may not likely be able to handle such count data.

As noted by Aderoju et al, (2023), lot of models including Zhang et al, (2017), Aderoju (2020), Opone et al, (2021), Aryuyuen (2022), Jolayemi and Aderoju (2023), and Abiodun et al (2023) have been proposed to solve the problem of over dispersion common with count data.

In this study, it is desired to introduce another new model for dealing with skewed over dispersed count data.

Poisson Quasi Suja Distribution

Shanker et al, (2022) introduced Quasi Suja distribution (QSD), which is a two-parameter (θ, α) lifetime distribution. The probability density function, cumulative density function and survival functions, respectively, are given as

$$f(x; \theta, \alpha) = \frac{\theta^4}{\alpha\theta^3+24}(\alpha + \theta x^4)e^{-\theta x} \tag{1}$$

$$F(x; \theta, \alpha) = 1 - \left(1 + \frac{\theta^4x^4+4\theta^3x^3+12\theta^2x^2+24\theta x}{\alpha\theta^3+24}\right)e^{-\theta x} \tag{2}$$

$$S(x; \theta, \alpha) = \left(\frac{\theta^4x^4+4\theta^3x^3+12\theta^2x^2+24\theta x+(\alpha\theta^2+24)}{\alpha\theta^3+24}\right)e^{-\theta x} \tag{3}$$

$x > 0, \theta > 0, \alpha > 0$

Now, suppose that the parameter in the Poisson model, μ is assumed to follow the two-parameter,(θ, α), in QSD, then the probability mass function (pmf) of the new model which we shall name Poisson Quasi Suja distribution (PQSD) can be derived as

$$P(x; \theta, \alpha) = \int_0^\infty \frac{e^{-\mu}\mu^x}{x!}\frac{\theta^4}{\alpha\theta^3+24}(\alpha + \theta\mu^4)e^{-\theta\mu}\, d\mu \tag{4}$$

$$= \frac{\theta^4}{\alpha\theta^3+24}\left[\frac{\alpha(1+\theta)^4+\theta x^4+10\theta x^3+35\theta x^2+50\theta x+24\theta}{(1+\theta)^{x+5}}\right], \tag{5}$$

for $x \geq 0, \alpha > 0$ and $\theta > 0$.

Summing the new pmf over all the possible infinite outcomes of the random variable, X, gives $\sum_{x=0}^{\infty} P(x; \theta, \alpha) = \sum_{x=0}^{\infty} \frac{\theta^4}{\alpha\theta^3+24}\left[\frac{\alpha(1+\theta)^4+\theta x^4+10\theta x^3+35\theta x^2+50\theta x+24\theta}{(1+\theta)^{x+5}}\right] = 1,$ (6)

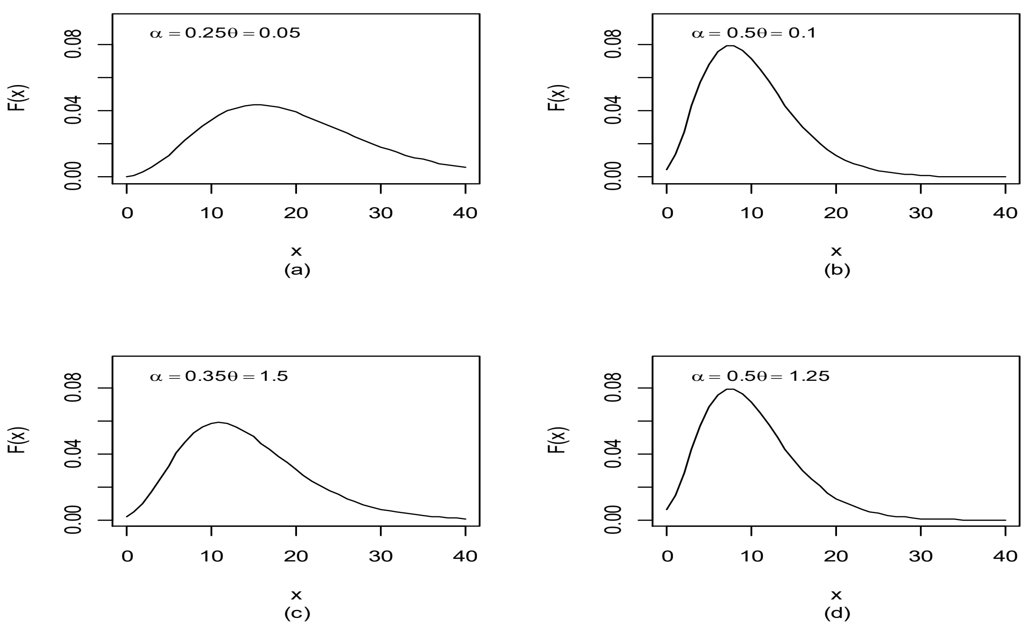
showing that the new pmf is a valid one.

The Cumulative density function (cdf) of the PQSD is derived as

$$F(x, \alpha, \theta) = 1 - \frac{1}{\alpha\theta^3+24}[(\alpha\theta^7 + \theta^4x^4 + 4\alpha\theta^6 + 14\theta^4x^3 + 6\alpha\theta^5 + 71\theta^4x^2 + 4\theta^4x^3 + 4\alpha\theta^4 + 154\theta^4x + 48\theta^3x^2 + \alpha\theta^3 + 120\theta^4 + 188\theta^3x + 12\theta^2x^2 + 240\theta^3 + 108\theta^2x + 240\theta^2 + 24\theta x + 120\theta + 24)(1+\theta)^{-x-5}], \quad (7)$$

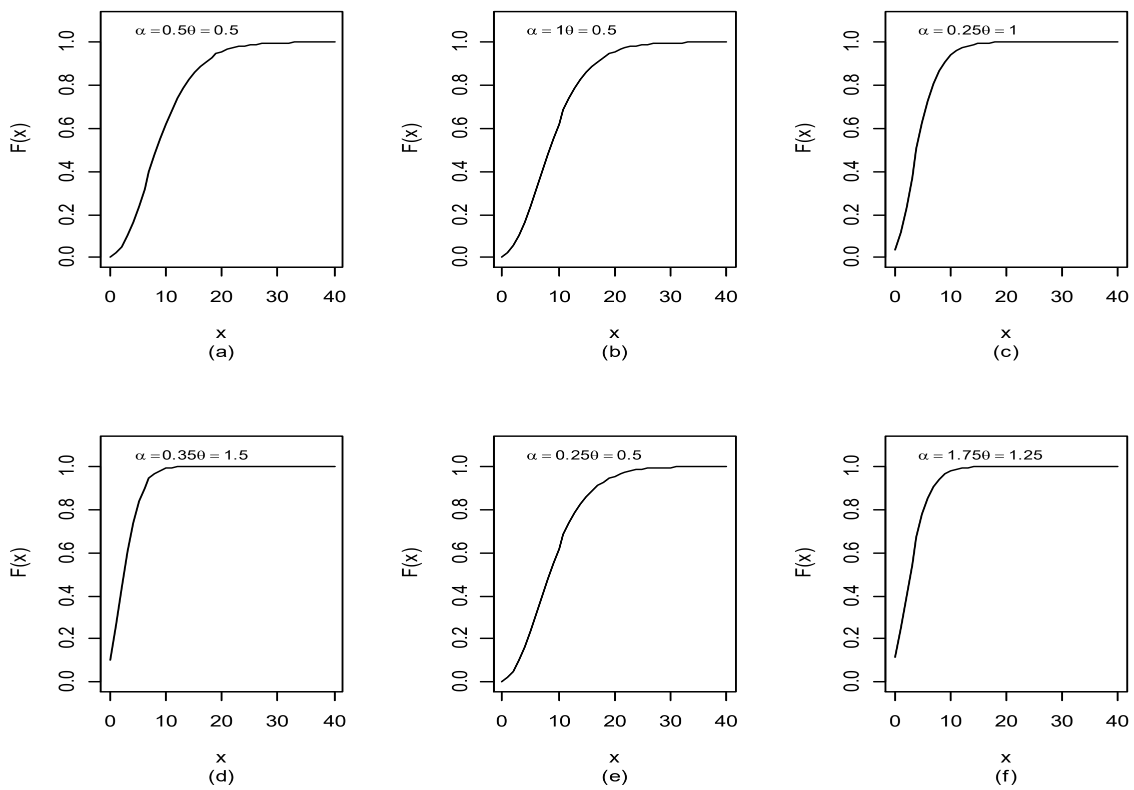
for $x \geq 0$, $\alpha > 0$ and $\theta > 0$.

Fig.1: pmf plot of Poisson Quasi Suja Distribution



Figures 1 and 2 showed the typical nature of the pmf and cdf at varying parameters values. The graphs of Figure 1 showed that PQSD is extreme positively skewed. And the graphs of Figure 2 showed that the new distribution is a valid distribution.

Fig.2: cdf plot of Two Parameters Poisson Quasi Suja Distribution



When the two parameters in the new distribution are equal, that is $\alpha = \theta$, then PQSD becomes one-parameter PQSD.

The Reliability Measures of the Poisson Quasi Suja Distribution

Survival function derived from PQSD is expressed as

$$S(x;\theta,\alpha) = \frac{1}{\alpha\theta^3+24}[(\alpha\theta^7 + \theta^4x^4 + 4\alpha\theta^6 + 14\theta^4x^3 + 6\alpha\theta^5 + 71\theta^4x^2 + 4\theta^3x^3 + 4\alpha\theta^4 + 154\theta^4x + 48\theta^3x^2 + \alpha\theta^3 + 120\theta^4 + 188\theta^3x + 12\theta^2x^2 + 240\theta^3 + 108\theta^2x + 240\theta^2 + 24\theta x + 120\theta + 24)(1+\theta)^{-x-5}] \quad (8)$$

The hazard function is also expressed as

$$H(x;\theta,\alpha) = A/B, \quad (9)$$

where

$$A = \theta^4(\alpha\theta^4 + \theta x^4 + 4\alpha\theta^3 + 10\theta x^3 + 6\alpha\theta^2 + 35\theta x^2 + 4\alpha\theta + 50\theta x + \alpha + 24\theta),$$

and

$B = \alpha\theta^7 + \theta^4x^4 + 4\alpha\theta^6 + 14\theta^4x^3 + 6\alpha\theta^5 + 71\theta^4x^2 + 4\theta^3x^3 + 4\alpha\theta^4 + 154\theta^4x + 48\theta^3x^2 + \alpha\theta^3 + 120\theta^4 + 188\theta^3x + 12\theta^2x^2 + 240\theta^3 + 108\theta^2x + 240\theta^2 + 24\theta x + 120\theta + 24$

Moments Measures of the Poisson Quasi Suja Distribution

The first four raw moments about origin μ_r' of PQSD are expressed as

$\mu_1' = \frac{\alpha\theta^3+120}{\theta(\alpha\theta^3+24)}$, $\mu_2' = \frac{\alpha\theta^4+2\alpha\theta^3+120\theta+720}{\theta^2(\alpha\theta^3+24)}$, $\mu_3' = \frac{\alpha\theta^5+6\alpha\theta^4+6\alpha\theta^3+120\theta^2+2160\theta+5040}{\theta^3(\alpha\theta^3+24)}$, and $\mu_4' = \frac{40320+\alpha\theta^6+14\alpha\theta^5+36\alpha\theta^4+(24\alpha+120)\theta^3+5040\theta^2+30240\theta}{\theta^4(\alpha\theta^3+24)}$.

The variance, σ^2, is expressed as $\sigma^2 = \frac{\alpha^2\theta^7+\alpha^2\theta^6+144\alpha\theta^4+528\alpha\theta^3+2880\theta+2880}{\theta^2(\alpha\theta^3+24)}$. The coefficient of variance, CV, is expressed as $CV = \frac{\sqrt{\alpha^2\theta^7+\alpha^2\theta^6+144\alpha\theta^4+528\alpha\theta^3+2880\theta+2880}}{\alpha\theta^3+120}$. The index of dispersion, γ, is expressed as $\gamma = \frac{\alpha^2\theta^7+\alpha^2\theta^6+144\alpha\theta^4+528\alpha\theta^3+2880\theta+2880}{\theta(\alpha\theta^3+120)(\alpha\theta^3+24)}$. The skewness, β_1, is expressed as $\beta_1 = \frac{(\alpha\theta^5+6\alpha\theta^4+6\alpha\theta^3+120\theta^2+2160\theta+5040)\theta(\alpha\theta^3+24)}{(\alpha\theta^4+2\alpha\theta^3+120\theta+720)^2}$. And the kurtosis, β_2, is expressed as $\beta_2 = \frac{(40320+\alpha\theta^6+14\alpha\theta^5+36\alpha\theta^4+(24\alpha+120)\theta^3+5040\theta^2+30240\theta)(\alpha\theta^3+24)}{(\alpha\theta^4+2\alpha\theta^3+120\theta+720)^2}$.

Descriptive Statistics for the Poisson Quasi Suja Distribution

Table 1 displays the descriptive statistics for the Poisson Quasi Suja distribution. The descriptive statistics of the PQSD are presented for various combinations of α and θ. These statistics include the mean, variance, skewness, kurtosis, index of dispersion (ID), and coefficient of variation (CV).

The Poisson Quasi Suja Distribution (PQSD) is a two-parameter discrete probability distribution derived by compounding the Poisson distribution with the Quasi Suja distribution. This distribution is particularly useful for modeling count data exhibiting over-dispersion, where the variance exceeds the mean. The parameters α (alpha) and θ (theta) govern the shape and dispersion of the distribution.

Table 1: Descriptive Statistics for Poisson Quasi Suja Distribution

Alpha (α)	Theta (θ)	Mean	Variance	Skewness	Kurtosis	ID	CV
0.25	0.05	100.000	12099.985	1.289	1.905	121.000	1.100
	0.55	9.078	108.092	1.395	2.271	11.907	1.145
	1.05	4.717	31.625	1.488	2.538	6.705	1.192
	1.55	3.129	15.181	1.551	2.521	4.851	1.245
	2.05	2.278	8.868	1.550	2.158	3.892	1.307
	2.55	1.730	5.709	1.463	1.676	3.301	1.381
	3.05	1.340	3.878	1.313	1.285	2.894	1.470
	3.55	1.050	2.724	1.144	1.024	2.594	1.572
	4.05	0.831	1.961	0.993	0.861	2.361	1.686
	4.55	0.664	1.443	0.870	0.760	2.174	1.810
Alpha (α)	Theta (θ)	Mean	Variance	Skewness	Kurtosis	ID	CV
0.5	0.05	100.000	12099.971	1.289	1.905	121.000	1.100
	0.55	9.066	107.920	1.397	2.278	11.904	1.146
	1.05	4.672	31.285	1.494	2.594	6.696	1.197
	1.55	3.040	14.688	1.545	2.696	4.832	1.261
	2.05	2.142	8.267	1.498	2.491	3.859	1.342
	2.55	1.558	5.066	1.354	2.142	3.252	1.445
	3.05	1.152	3.259	1.177	1.841	2.829	1.567
	3.55	0.865	2.174	1.019	1.640	2.513	1.705
	4.05	0.661	1.499	0.895	1.518	2.267	1.852
	4.55	0.517	1.070	0.801	1.444	2.071	2.002
Alpha (α)	Theta (θ)	Mean	Variance	Skewness	Kurtosis	ID	CV
1.5	0.05	99.999	12099.912	1.289	1.905	121.000	1.100
	0.55	9.016	107.237	1.405	2.308	11.894	1.149
	1.05	4.505	30.002	1.519	2.824	6.660	1.216
	1.55	2.739	13.025	1.535	3.444	4.756	1.318
	2.05	1.756	6.563	1.403	3.997	3.737	1.459
	2.55	1.162	3.585	1.222	4.380	3.084	1.629
	3.05	0.801	2.101	1.067	4.625	2.624	1.810
	3.55	0.579	1.322	0.947	4.757	2.286	1.988
	4.05	0.439	0.892	0.853	4.781	2.033	2.153
	4.55	0.347	0.640	0.776	4.706	1.843	2.303
Alpha (α)	Theta (θ)	Mean	Variance	Skewness	Kurtosis	ID	CV
2.0	0.05	99.999	12099.883	1.289	1.905	121.000	1.100
	0.55	8.991	106.899	1.409	2.323	11.889	1.150
	1.05	4.427	29.403	1.532	2.942	6.642	1.225
	1.55	2.615	12.341	1.535	3.844	4.720	1.344
	2.05	1.624	5.978	1.387	4.844	3.682	1.506
	2.55	1.051	3.166	1.208	5.681	3.013	1.693
	3.05	0.718	1.827	1.061	6.244	2.546	1.884
	3.55	0.520	1.149	0.946	6.524	2.209	2.061
	4.05	0.398	0.781	0.852	6.555	1.963	2.221
	4.55	0.319	0.569	0.774	6.400	1.782	2.363

Impact of Parameter θ:

For a fixed parameter α, increasing the parameter θ results in a decrease in both the mean and variance. This indicates that higher values of θ lead to distributions with lower central tendencies and reduced dispersion. For instance, with $\alpha = 0.25$ as θ increases from 0.05 to 4.55, the mean decreases from 100.000 to 0.664, and the variance decreases from 12099.985 to 1.443.

Additionally, skewness and kurtosis values generally decrease with increasing θ, suggesting that the distribution becomes more symmetric and less peaked. The index of dispersion (ID), defined as the ratio of variance to mean, also decreases with higher θ, indicating a transition from over-dispersion towards equi-dispersion or under-dispersion. Conversely, the coefficient of variation (CV), which measures relative variability, increases with θ, reflecting greater variability relative to the mean.

Impact of Parameter α:

For a fixed θ, increasing α generally leads to a decrease in both the mean and variance, though the rate of decrease is less pronounced compared to changes in θ. For example, with θ = 0.55, as α increases from 0.25 to 2.0, the mean decreases from 9.078 to 8.991, and the variance decreases from 108.092 to 106.899.

Skewness and kurtosis exhibit a decreasing trend with increasing α, indicating that the distribution's asymmetry and peakedness diminish. The index of dispersion decreases, while the coefficient of variation increases with higher α, reflecting changes in dispersion and relative variability.

General Observations:

Mean and Variance: Both parameters α and θ significantly influence the mean and variance of the PQSD. Higher values of θ have a more substantial impact in reducing these measures compared to α.

Skewness and Kurtosis: These shape parameters decrease with increasing α and θ, indicating that the distribution becomes more symmetric and less heavy-tailed.

Index of Dispersion (ID): A decreasing ID with higher α and θ suggests that the distribution moves from over-dispersion towards equi-dispersion or under-dispersion.

Coefficient of Variation (CV): An increasing CV with higher α and θ indicates greater relative variability in the distribution.

Conclusion

Two-parameter Poisson Quasi Suja distribution (PQSD) derived from the two-parameter quasi suja distribution is proposed for extremely positively count data. Its survival and hazard functions, first four raw moments' measures were expressed. Its statistical properties including the mean, variance, skewness, kurtosis, index of dispersion and coefficient of variation were used to assess the impact of each parameter in the new distribution.

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