

SOME REAL LIFE APPLICATION OF DERIVATIVES IN ECONOMICS

Sarswati Kumari Sah¹, Suresh Kumar Sahani^{*2},
Kameshwar Sahani³, Dhiraj Kumar Sharma⁴

¹M.I.T. Campus, T.U., Nepal; ²Rajarshi Janak University, Janakpurdham, Nepal;

³Kathmandu University, Kathmandu, Nepal; ⁴R.R.M. Campus, Jankpurdham, Nepal
saraswatisah1234575@gmail.com; sureshsahani@rju.edu.np

Article Info:

Submitted:	Revised:	Accepted:	Published:
Apr 27, 2024	May 6, 2024	May 9, 2024	May 12, 2024

Abstract

In this assignment, we addressed the usage of derivatives in several areas of economics, such as demand elasticity, cost minimization, and profit maximization. In addition, derivatives are used to calculate marginal revenue, marginal cost, and marginal product. This work is motivated by the work of [1-24].

Keywords: Total Cost, Total Revenue, Marginal Cost, Marginal Revenue, Marginal Profit

Introduction

According to Mahommed Iftekher Hossain's article "Uses of the derivatives in economics"

Marginal functions

The change in the total function caused by a one unit change in the independent variable is known as the marginal function in economics. The marginal function can be produced by differentiating the total function with respect to the relevant independent variable if the total function is a continuous and differentiable function. The difference in the overall cost resulting from producing an extra unit is known as the marginal cost. The matching marginal cost function is the first-order differentiation of the total cost function, since the total cost function, $TC=TC(q)$, is a continuous and differentiable function:

If $TC=TC(q)$, then $dTC/dq=MC(q)$

The difference in total revenue that results from selling one additional unit of output is known as marginal revenue. The corresponding marginal revenue function is the first-order differentiation of the total revenue function, since the total revenue function $TR=TR(q)$ is a continuous and differentiable function:

If $TR=TR(q)$, then $dTR/dq=MR(q)$

If $TP=TP(L)$ is the total product as a function of labour (L), then dTP/dL is the marginal product of labour.

If $TP=TP(L)$, then $dTP/dL=MP(L)$

When a consumer's total utility is determined by the quantity of a good she consumes, as represented by the continuous and differentiable function $TU=TU(x)$, her marginal utility from the product is equal to dTU/dx .

If $TU=TU(x)$, then $dTU/dx=MU(x)$

Optimizing economic functions

If there are any optimum points for economic functions, we can locate them with the use of derivatives. Derivatives can be used, for instance, to determine the output level at which total revenue is maximum, profit is maximum or minimum, marginal costs and average costs are minimum, and so on. Using optimization techniques, a producer can determine the ideal output level and profit when faced with a continuous and differentiable profit function, represented as $\pi=TR(x) - TC(x)$. By using optimization techniques, a consumer with a continuous and differentiable utility function, $u=f(x)$, can determine the maximum

amount of consumption that maximizes her utility. According to M Johansyah, J Nahar, E Djauhari, H Napitupulu, and J Saputra's article "Determining the price elasticity of demand with and without memory effects using fractional order derivatives: A numerical simulation approach"

Demand elasticity is the sensitivity of changes in the number of goods demanded by consumers due to changes in the price of goods. This paper compares the price elasticity of demand with and without memory effect using fractional-order derivatives. This study is designed using the development theory of fractional derivatives for the economic field in determining the price elasticity of demand. The result of numerical simulation using the value of α and p indicated that the price elasticity of demand with memory effect is more accurate than without the memory effect. Furthermore, this study concluded that the price elasticity of demand does not only depend on the latest price (current price) but changes in all prices from a specific time interval. The findings of this study suggest future studies can examine the phenomenon of market equilibrium using fractional-order derivatives.

According to Oscar Lange's article "Theoretical derivation of elasticities of demand and supply: the direct method"

The Usual Method of deriving theoretically the elasticities of demand and supply consists in finding first the partial derivatives. These are subsequently multiplied by the ratio of the independent to the dependent variable and the elasticity is obtained. Thus in order to arrive at the elasticity of the quantity x_8 with respect to the price p_r the derivative $\frac{\partial x_8}{\partial p_r}$ is found and multiplied by $\frac{p_r}{x_8}$. The result is the elasticity $E_{x_8/p_r} = (\frac{\partial x_8}{\partial p_r})(\frac{p_r}{x_8})$. The results appear in rather complicated and awkward algebraic form and the expression obtained is simplified by means of the concept of the partial elasticity of substitution (usually denoted by σ_{r8}). This concept itself is represented by a rather complicated algebraic expression. The whole procedure, however, can be shortened considerably and the results put in much simpler and more elegant form by a direct method of derivation which makes use of the algorithm of the calculus of elasticities. This is the purpose of the present paper. According to R Marsitin's article "Analysis of differential calculus in economics" The differential is one of the mathematical material in calculus which is loaded with counts. Differential counts can be applied in economics for profit optimization. This study aims to analyse differential calculus in economics. This research is a descriptive qualitative study. This research analyses the profit optimization in the entrepreneurial world

with the second differential formula in calculus. Data analysis is to describe the results of the analysis of the second differential formula with economics in optimizing profits. Calculation of calculus uses the second differential of the mathematical model with the provision that the second differential result is negative which is smaller zero. The results of the analysis show that the second differential calculus smaller than zero, so it can be concluded that to obtain profit optimization in the economy it can be applied using the second differential calculus.

According to Valentina V Tarasova, Vasily E Tarasov's article "Elasticity for economic processes with memory: Fractional differential calculus approach" Derivatives of non-integer orders are applied to generalize notion of elasticity in framework of economic dynamics with memory. Elasticity of Y with respect to X is defined for the case of a finite-interval fading memory of changes of X and Y. We define generalizations of point price elasticity of demand to the case of processes with memory. In these generalizations we take into account dependence of demand not only from current price (price at current time), but also all changes of prices for some time interval. For simplification, we will assume that there is one parameter, which characterizes a degree of damping memory over time. The properties of the suggested fractional elasticities and examples of calculations of these elasticities of demand are suggested.

According to Vasily E Tarasov's article "On history of mathematical economics: Application of fractional calculus"

Modern economics was born in the Marginal revolution and the Keynesian revolution. These revolutions led to the emergence of fundamental concepts and methods in economic theory, which allow the use of differential and integral calculus to describe economic phenomena, effects, and processes. At the present moment the new revolution, which can be called "Memory revolution", is actually taking place in modern economics. This revolution is intended to "cure amnesia" of modern economic theory, which is caused by the use of differential and integral operators of integer orders. In economics, the description of economic processes should take into account that the behaviour of economic agents may depend on the history of previous changes in economy. The main mathematical tool designed to "cure amnesia" in economics is fractional calculus that is a theory of integrals, derivatives, sums, and differences of non-integer orders. This paper contains a brief review of the history of applications of fractional calculus in modern

mathematical economics and economic theory. The first stage of the Memory Revolution in economics is associated with the works published in 1966 and 1980 by Clive W. J. Granger, who received the Nobel Memorial Prize in Economic Sciences in 2003. We divide the history of the application of fractional calculus in economics into the following five stages of development (approaches): ARFIMA; fractional Brownian motion; econophysics; deterministic chaos; mathematical economics. The modern stage (mathematical economics) of the Memory revolution is Intended to include in the modern economic theory new economic concepts and notions that allow us to take into account the presence of memory in economic processes. The current stage actually absorbs the Granger approach based on ARFIMA models that used only the Granger–Joyeux–Hosking fractional differencing and integrating, which really are the well-known Grunwald–Letnikov fractional differences. The modern stage can also absorb other approaches by formulation of new economic notions, concepts, effects, phenomena, and principles. Some comments on possible future directions for development of the fractional mathematical economics are proposed.

According to S Thirunavukkarasu, T Lakshmi Pradha's "Differential Calculus and its Application in Economics-A Study with Reference to Consumer Demand Theory" Calculus is one of the important components of mathematical tools used in economics. This enables understanding, improving and problem-solving tools for economic variables. The mathematical analysis contains differential calculus and integral calculus. Calculus is mostly expressed in functions and derivatives. The two-consumer demand theories are cardinal and ordinal. The former is the marginal utility approach and the latter is indifference curve analysis popularised by authors like Gossen (1854), William Jevons (1871), Leon Walras (1874), Carl Marshall (1890), Menger (1950), Hicks (1956), Pareto (1909), PA Samuelson (1949), and Robbins (1984) etc. These models analysed the relationship between the price of a commodity and the quantity demanded of the same commodity for deriving individual and market demand curves. The coefficient of price, income and cross elasticities and price, income and substitution effects are also part of these theories.

Definitions

Derivatives: A function's rate of change with regard to an independent variable is referred to as a derivative.

Marginal Cost: The extra expense incurred when producing a single extra unit of an item or service is known as the marginal cost.

Marginal Revenue: The extra money received from the sale of one more unit of an item or service is known as marginal revenue.

Marginal Profit : This is the extra money made from the production and sale of one more unit of goods or services.

Marginal Utility : Marginal utility is the extra satisfaction a customer gets from purchasing one more unit of an item or service.

Discussion

Problem:

1. Lack of proper understanding of derivatives in economics students :

Because it is difficult for students to interpret calculated derivative values in an economic context and because using derivatives in marginal analysis requires students to interpret values in an economic context, very few economics students have a profound understanding of the rate of change, or marginal change, as derivatives. Parallel to this, the economic interpretation of the derivative is further complicated by the fact that it does not match up with any of the standard mathematical representations of derivatives. Consequently, it makes sense that economics students would struggle much more to comprehend derivative interpretations.

2. Complexity of Economic Systems:

Economic systems are by nature complicated, involving a number of variables that interact in nonlinear ways. It may be challenging to use derivatives to model and analyse economics because of this complexity. For example, according to economic theory, increasing the minimum wage may result in job losses because businesses may decide not to hire as many people to offset the higher labour expenses. The real connection between the minimum wage and employment, however, is nuanced and depends on a number of variables, including the structure of the business, the state of the labour market, and the demand for wages. These little differences may

be difficult for derivative models to incorporate, which could result in predictions that are off.

3. Assumptions and Simplifications:

Economic models frequently include simplifications and assumptions that might not always hold true in practical situations. Using derivatives can result in erroneous predictions because of this. For example, modeling the relationship between inflation and unemployment using the Phillips Curve. Policymakers can aim for low unemployment or low inflation, but not both at the same time, according to the Phillips Curve, which makes the steady trade-off between the two variables assumed. Nevertheless, empirical data has demonstrated that this link is not always constant and is subject to the influence of a variety of variables, including expectations, supply shocks, and wage negotiating power. It's possible that derivative models built on oversimplified Phillips Curve assumptions will miss these subtleties and provide incorrect policy recommendations.

4. Data Limitations:

The availability, completeness, and updating of economic statistics are frequently issues. When utilizing derivatives to analyse economic phenomena, this can come with inaccuracies and uncertainties. Calculating the effect of public spending on economic expansion. For parameter estimation and hypothesis testing, economic models frequently rely on historical data. Yet, there can be uncertainties in derivative models due to measurement errors, revisions, and lags in economic data. Changes to government spending estimates or GDP growth data, for example, could drastically affect how fiscal policy is predicted to affect economic growth and result in suggestions for policy that are not true.

5. External Shocks:

Economic dynamics can be disrupted by external events like natural disasters, geopolitical conflicts, or financial crises, making it challenging to forecast their effects with derivative models. Take the COVID-19 epidemic and how it affected international supply chains, for example. There was a shortage of supplies as a result of the pandemic's global disruption of manufacturing and delivery networks.

The abrupt and unparalleled character of these external shocks may be difficult for derivative models to account for, which could result in inaccurate forecasts and risk assessments.

Solution:

1. Calculus course should included in the syllabus of economics students :

In calculus, derivatives are a crucial tool that help economists in marginal analysis make the best decisions. Due to the fact that derivatives are a mathematical concept that are closely related to marginal analysis and are used to quantify the impact of modest changes on income or costs. Since the derivative is used in marginal analysis, economics students should be well knowledgeable about it. To ensure that economics students have a thorough understanding of marginal analysis, calculus should be incorporated in their course curriculum.

2. Apply more sophisticated modeling techniques:

Applying more complex modeling methods to better represent the non-linear interactions and feedback loops in economic systems, such as agent-based modeling or machine learning algorithms. To model the behaviour of individual agents (such as consumers or corporations) and their interactions in a dynamic economic environment, economists might employ agent-based modeling as an alternative to only depending on classic equilibrium models. Agent-based models, for example, may account for investors' diverse opinions and trading styles while investigating the formation of financial market bubbles, producing more accurate market dynamics.

3. Use more flexible modeling frameworks.

It is recommended to employ modeling frameworks that are more adaptable and accommodate diverse agent behaviours, dynamic regime change, and adaptive expectations. Dynamic Stochastic General Equilibrium (DSGE) models allow for time-varying parameters and shocks to the economy, and are therefore a useful tool for economists instead of assuming a constant trade-off between inflation and unemployment as in the Phillips Curve. An improved way to reflect the effect of shifting economic conditions on inflation dynamics is to incorporate adaptive

expectations into DSGE models. This would enable more precise policy evaluations.

4. Invest in data collection efforts:

Invest in data gathering initiatives to raise the standard of economic data. Provide econometric methods for managing time series data that contains structural breaks, missing data, and measurement mistakes. The quality of economic statistics can be enhanced by governments and international organizations through the use of administrative records, satellite imaging, and surveys. For example, estimating agricultural output using satellite data can give fast and accurate information for evaluating the effect of weather shocks on food prices, allowing policymakers to more successfully carry out targeted actions.

5. Build robustness into economic models:

To increase the robustness of economic models, consider using scenario-based analysis and stress testing to evaluate how resilient economic systems are to outside shocks. To lessen the impact of such systemic hazards, create early warning systems and backup measures. Financial regulators and central banks can evaluate the ability of financial institutions to withstand external shocks like financial crises or recessions by using macroeconomic stress testing. To identify potential vulnerabilities in the banking sector and inform policy responses to avoid systemic risks, for example, modeling the effects of an abrupt increase in interest rates or a crash in asset prices might be helpful.

Conclusion

Elasticity of supply and demand, profit maximization, cost minimization, and price determination are just a few of the common economic tasks that heavily include derivatives. Our project has covered the topics of price elasticity of demand, marginal profitability, marginal revenue, and marginal cost. According to this report's conclusion, total cost and total revenue both rise as the number of manufacturing units increases while the commodity price stays the same. Due to profit maximization, this results in an increase in overall profit.

References

- Jha, K.K.(1978). Agriculture Financing in Nepal. New Delhi: Heritage Publishers.
- Joshi, P.R. (2001). Research Methodology. Kathmandu: Buddha Academy.
- Rose, Peter S.(2001). Commercial Bank Management. (Fifth Edition), New York McGraw-Hill Publication.
- Singh, Hriday Bir. (2005). Banking and Insurance, Kathmandu: Asia Publication.
- Annual Reports of ADBL,(2057/58-2063/64), Head office Kathmandu. MIS Division.
- Sahani, S.K.2023.The Effective Theory of Producer and Consumer Surplus; Positive Demand, Negative Supply, Central Asian Journal of Mathematics Theory and Computer Sciences,Vol.4,Issue 8,70-76.
- Sahani, S.K. et al.2023.The International Pricing of Risk; An Empirical Investigation of the World Capital Market Structure, American Journal of Science and Learning Development,Vol,2,No.7,49-65.
- Sahani, S.K. and Prasad, K.S.2023.Study and Analysis of Input| Output Analysis to Non-Linear Science,ACCST Research Journal,Vol. XXL,No.1,22-30.
- Sahani, S.K.2023.Youth on Employment and Job Creation in African Countries; Opportunities, Challenges, Characteristics Determinants and Performance of Self-Employment Among Them the Youth in African Countries, Central Asian Journal of Mathematics Theory and Computer Sciences, Vol.04,No.7,27-41.
- Sahani, S.K. et al.2023.Economic Insights Unveiled: A Journey Through Input-Output Analysis in Non-Linear Mathematics, MIKAILALSYS Journal of Multidisciplinary Science, Vol.1,Issue 1,240-259.
- Sahani, S.K. et al.2023.Unraveling the Interdependence of Input and Output Analysis in Business Sector; A Case Study, International Journal of Education, Management, and Technology, Vol.1,Issue 1, 27-45.
- Sharma, S., Sahani, S., Chaudhary, B., Sahani, K., & Sah, N. (2023). Asan, The Traditional Market of Kathmandu Valley: Analyzing Market Trend and Consumer Buying Behavior of Such Market.EDUMALSYS Journal of Research in Education Management, 1(2), 215-226. <https://doi.org/10.58578/edumalsys.v1i2.2048>
- Sah, P., Sah, R., Sahani, S. K., & Sahani, K. (2024). Study and Analysis of Education System in Nepal and Its Challenges Along with Its Solution. *Mikailalsys Journal of Advanced Engineering International*, 1(1), 33-42. <https://doi.org/10.58578/mjaei.v1i1.2792>
- Karn, P., Sahani, S. K., & Sahani, K. (2024). Study and Analysis of Some Practical Life Uses and Applications of Exponential Function. *Mikailalsys Journal of Advanced Engineering International*, 1(1), 43-56. <https://doi.org/10.58578/mjaei.v1i1.2793>
- Sah, K. K., Sahani, S. K., Sahani, K., & Sah, B. K. (2024). A Study and Examined of Exponential Function: A Journey of Its Applications in Real Life. *Mikailalsys Journal of Advanced Engineering International*, 1(1), 23-32. <https://doi.org/10.58578/mjaei.v1i1.2791>
- Sahani, S., & Karna, S. K. (2024). Analytical Study of Cooperative Organizations Based on Domestic Area of Asian Countries: A Classical Approach of Village. *International Journal of Education, Management, and Technology*, 2(1), 40-62. <https://doi.org/10.58578/ijemt.v2i1.2850>

- Mahato, S. K., Sahani, S., Sahani, K., Sah, B. K., & Karna, S. K. (2024). Study and Analysis of Some Real Life Applications of Exponential Function. *EDUMALSYS Journal of Research in Education Management*, 2(1), 13-26. <https://doi.org/10.58578/edumalsys.v2i1.2845>
- Sah, B. K., & Sahani, S. (2024). Poisson-New Quadratic-Exponential Distribution. *Mikailalsys Journal of Mathematics and Statistics*, 2(2), 27-45. <https://doi.org/10.58578/mjms.v2i2.2862>
- Chaurasiya, P., Sharma, N., Chaudhary, S., Sah, S., Sahani, S., & Sahani, K. (2024). Study and Analysis of Some Real Life Applications of Exponential Function Based on Population Growth Rate of Nepal. *Asian Journal of Science, Technology, Engineering, and Art*, 2(3), 352-361. <https://doi.org/10.58578/ajstea.v2i3.2863>
- Pandit, J. K., Sahani, S., & Sahani, K. (2024). Analysis of Some Practical Life Uses and Applications of Exponential Function. *Journal of Multidisciplinary Science: MIKAILALSYS*, 2(2), 179-192. <https://doi.org/10.58578/mikailalsys.v2i2.2864>
- Sah, B. K., & Sahani, S. K. (2024). Premium Linear-Exponential Mixture of Poisson Distribution. *Communications on Applied Nonlinear Analysis*, 31(1), 187-199.
- Sah, B. K., & Sahani, S. K. (2022). Polynomial-exponential distribution. *Mathematical Statistician and Engineering Applications*, 71(4), 2474-2486.
- Sah, B. K., & Sahani, S. K. (2022). New Quadratic-Exponential Distribution. *Journal of Pharmaceutical Negative Results*, 2338-2351.
- Sah, B. K., & Sahani, S. K. (2023). Polynomial-Exponential Mixture of Poisson distribution. *Turkish Journal of Computer and Mathematics Education (TURCOMAT)*, 14(03), 505-516.
- Sahan, B. K. and Sahani.S.K. (2022). Modified Linear-Exponential Distribution. *NeuroQuantology*, 20(5), 3520.
- Sah, B. K., & Sahani, S. K. Generalised Poisson-Modified Mishra distribution. *Power System Technology*, 48(1), 157-167. <https://doi.org/10.52783/pst.261>
- Jha, S., Sahani, S.K., & Sahani, K. (2024). Research Paper Unveiling the Power of Exponential Functions: Applications in Our Daily Lives. *ARZUSIN*, 4(3), 402-409. <https://doi.org/10.58578/arzusin.v4i3.2861>
- Karna, S. K., & Sahani, S. (2024). An Analytical Study of Deposit Collection and Fund Mobilization in Developing Countries : A Financial Approach to Nepalese Commercial Banks in Nonlinear Analysis. *PALAPA*, 12(1), 40-61. <https://doi.org/10.36088/palapa.v12i1.4649>
- Karna, S. K. ., & Sahani, S. K. . (2024). A Empirical Study of Micro Finance Services and Participatory Self-development Expedition Accelerating in Developing Countries: An Approach to Rural Perspective of Nepal. *EUROPEAN JOURNAL OF BUSINESS STARTUPS AND OPEN SOCIETY*, 4(4), 119–134. Retrieved from <https://inovatus.es/index.php/ejbsos/article/view/2938>
- Karna, S.K. and Sahani, S.K., (2024)A Statistical Analysis of Discrimination and Exploitation based on Ethnicity, Caste, Sex, Religion and Class in Developing Countries: A Graphical Study of Nepalese Societies, Vol.2, No.4,174-197,

- Sah, B., Mandal, N., Sahani, S., & Sahani, K. (2024). Applications of Exponential Function in Our Real Lives. *International Journal of Humanities, Education, and Social Sciences*, 2(2), 144-153. <https://doi.org/10.58578/ijhess.v2i2.2872>.
- Johansyaha, M., Nahar, J., Djauhari, E., Napitupulu, H., & Saputra, J.,(2022), Determining the price elasticity of demand with and without memory effects using fractional order derivatives: A numerical simulation approach, *Decision Science Letters* 11(3), 311-322.
- Oscar, Lange(1942), Theoretically derivation of elasticities of demand and supply : the direct method, *Econometrica, Journal of the Econometrica Society*, 193-214
- Marsitin, R.,(2019), Analysis of differential calculus in economics, *Journal of Physics: Conference series* 1381 (1), 012003
- Tarasova, Valentina V, & Tarasova, Vasily E (2016), Elasticity for economic processes with memory: Fractional differential calculus approach, *Fractional Differential Calculus* 6 (2), 219-232,
- Tarasova, Vasily E (2019), On history of mathematical economics: Application of fractional calculus, *Mathematics* 7 (6), 509
- Thirunavukkarasu, S. & Pradha, T Lakshmi(2023), Differential Calculus and its Application in Economics-A Study with Reference to Consumer Demand Theory, *Centre for Development Economic* 10 (17), 30-37