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INVESTIGATION OF INTEGRAL TRANSFORMATION ASSOCIATED WITH EXTENDED GENERALIZED SRIVASTAVA'S HYPERGEOMETRIC MULTI VARIABLE SPECIAL FUNCTION

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Abstract

In recent year study on multivariate special functions and Integral transformation have been booming. In this work, we have focused on Srivastava hypergeometric function $H_A(\cdot)$, $H_B(\cdot)$, and $H_C(\cdot)$ with triple variable. We have discussed the literature study and motivation from the recent works on the extension of Srivastava's multivariable hypergeometric function $H_A(\cdot)$, $H_B(\cdot)$, and $H_C(\cdot)$. In this paper, the extension of $H_A(\cdot)$, $H_B(\cdot)$, and $H_C(\cdot)$ is studied based on the generalized beta (p, q) function $\mathfrak{B}_{p,q}(\alpha, \beta)$ and the generalized Pochhammer's symbol $(\mathcal{P}; p, q)_v$. Furthermore, the Mellin integral transformation and Inverse Mellin integral transformation have been studied for the $\mathfrak{B}_{p,q}(\alpha, \beta)$ based extension of the functions $H_A(\cdot)$, $H_B(\cdot)$, and $H_C(\cdot)$. A few of the most recent uses of these transformations in various scientific and engineering fields are also highlighted in this paper. In general, this work seeks to offer a thorough overview of recent breakthroughs in the importance and applications of several integral transforms of Multivariable functions.

Keywords: Srivastava's triple hypergeometric function, Mellin integral transformation, Beta and Gamma functions, Extended hypergeometric function, Multivariable special function

INTRODUCTION

The development of integral transforms associated multivariable special functions, which have various uses in a variety of scientific domains, has made major strides in recent years. Several researchers have explored the features of integral transforms and special functions in many different aspects [Agarwal, Praveen(2012), Khan,et al. (2016), Gradshteyn, I. S., and I. M. Ryzhik(1994), Li, Chun-Fang(2007)].
Special functions have unique properties often utilised in various applications in mathematics, physics, and engineering difficulties. Due primarily to their extraordinary characteristics and successful applications in basic sciences and technical sectors, many generalisations for special functions have been discovered and constructed in contemporary literature. Kummer's confluent hypergeometric function, The gamma function, beta function, Mittag-Leffler function, Gauss' hypergeometric function, Bessel function, Whittaker function, and others are prominent examples.

Multivariable special functions naturally appear in a variety of mathematical and practical situations. These functions exhibit complicated behaviour and attributes and may be solutions to certain partial differential equations classes. Introducing a new multivariable generalised function by combining two or more functions or extending the generalised function is a study area that is beneficial from an application standpoint. These special functions are essential because of their recurrent and apparent connections, differential equations, series expression, symmetric and convolution performance, and various other significant features. Numerous new extensions of generalised multivariable special functions have been developed as a result of the utility and significant applications of various properties of a multivariable combined special function in challenges involving classical and numerical analysis, number theory, approximation theory, theoretical physics, and other areas of mathematics.

There is a long history of using hypergeometric functions in many areas of mathematical physics, statistics, economics, etc. Due to its applications in several domains, the hypergeometric special function of variables, which has two numerators and one denominator, has grown in importance in recent years [Agarwal et al..(2020), Akhmedova,et al. (2019)]. Using an augmentation of the Pochhammer symbol, various approaches have been taken recently to generalise and enhance hypergeometric function. The reader can review, for instance, the most recent studies of Hidan et al. [Hidan, Muajebah, et al. (2021), and Hidan, M., and M. Abdalla.(2020).], Fuli et al. (2020), Agarwal et al. (2015), Jana et al. [(2019 and (2020)] and Srivastava et al. (2019).

In this study, the general notation has been followed as: $\mathbb{N}, \mathbb{Z}^{-}, and\ \mathbb{C}$ means the set of positive integers i.,e. Natural numbers, negative integers, and the set of complex numbers. Also, the notations $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$ and $\mathbb{Z}_0^{-} = \mathbb{Z}^{-} \cup \{0\}$. The generalised hypergeometric function with m number of numerators and n number of denominators components has the following definition as a series:

$$ {}_mT_n\left(\begin{matrix} c_1, \ldots, c_m \\ d_1, \ldots, d_n \end{matrix}, y\right) = \sum_{p=0}^{\infty} \frac{(c_1)_p \cdots (c_m)_p}{(d_1)_p \cdots (d_n)_p} \frac{y^p}{p!} \tag{1} $$

Where $d_j \in \mathbb{C} \backslash \mathbb{Z}_0^{-}, j = 1, \ldots, n$. The series is convergent for all $y \in \mathbb{C}$ if $m < n$. The series will diverge $\forall\ y \neq 0$ when $m > n + 1$, Unless at least one of the numerator would lie in the $\mathbb{Z}_0^{-}$, for which case the expression is transformed into a polynomial. Furthermore, if $m = n + 1$, the mentioned series will be convergent in the region of unit circle $|y| = 1$ when $\mathbb{R}(\sum d_1 - \sum c_1) > 0$.For $m_1, m_2 \in \mathbb{C}, m_3 \in \mathbb{C} \backslash \mathbb{Z}_0^{-}$, [25] the Gauss hypergeometric special function is represented as:

$$ {}_2T_1\left(\begin{matrix} m_1, m_2 \\ m_3 \end{matrix}, y\right) = \sum_{p=0}^{\infty} \frac{(m_1)_p (m_2)_p}{(m_3)_p} \frac{y^p}{p!} \quad \text{Where } (|y| < 1) \tag{2} $$

The extension of hypergeometric function has been studied in [26] by extending m_j $(1 \leq j \leq p, q)$, which has so many diverse applications in different fields.

The hypergeometric series and its extensions are found in several application-related disciplines of mathematics in the literature. There have been several triple hypergeometric special functions developed and studied. A list of 205 unique triple hypergeometric special functions is shown in Srivastava and Karlsson's study [17, Chapter 3]. Srivastava presented the second order mutivariate hypergeometric special functions H_A, H_B, and H_C in [18,19]. It is well known that H_A is the generalisation of both T_1 and T_2, whereas other two of Srivastava triple hypergeometric functions are generalisations of Appell's special functions T_1 and T_2.

From the above literature there is enough opportunity to study on the extension of hypergeometric special function and their integral transformation. Motivating from this we have studied on the extension and Integral transformation for the general Gauss hypergeometric special function and the Srivastava's triple hypergeometric special function. The following is the paper's outline: Preliminaries definition are discussed in section 2.

Development of the Extended generalised Srivastava's hypergeometric function is discussed in section 3. In section 4, we have discussed the Integral transformation of the extended generalised multivariable hypergeometric functions and section 5 represent their application. overall conclusion of the presented study is presented in section 6.

PRELIMINARy NOTES

Many researchers recently examined higher transcendence hypergeometric type special functions together with extensions, generalisations, and unifications of Euler's Beta function [(1997, and 2004].

The classical Beta function defined by [29]

$$\mathfrak{B}(\alpha,\beta)=\begin{cases}\int_0^1 s^{\alpha-1}(1-t)^{\beta-1}ds, & (\mathcal{R}(\alpha)>0,\mathcal{R}(\beta)>0)\\ \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, & ((\alpha,\beta)\,\epsilon\,\mathcal{C}\ \mathbb{Z}_0^-)\end{cases} \tag{3}$$

The Pochhammer's sign, also known as the shifted factorial since $(1)_n = n!$, can be expressed as:

$$(y)_a \coloneqq \frac{\Gamma(y+a)}{\Gamma(y)}=\begin{cases}1, & (a=0, y\,\epsilon\,\mathbb{C}\backslash\{0\})\\ y(y+1)\ldots(y+a-1), & (y\ \in\mathbb{N},\ \lambda\ \in\ \mathbb{C})\end{cases} \tag{4}$$

The Srivastava triple hypergeometric function $H_A(\cdot)$, $H_B(\cdot)$, and $H_C(\cdot)$ is based on the classical beta function $\mathfrak{B}(\alpha,\beta)$ and the Pochhammer's sign $(y)_a$.

The representation of Srivastava triple hypergeometric function $H_A(\cdot)$ introduced in [19,20]:

$$\begin{aligned}H_A(m_1,m_2,m_3;d_1,d_2;u,v,w) &= \sum_{\alpha,\beta,\delta=0}^{\infty}\frac{(m_1)_{\alpha+\delta}(m_2)_{\alpha+\beta}}{(d_1)_\alpha}\frac{\mathfrak{B}(m_3+\beta+\delta,d_2-m_3)}{\mathfrak{B}(m_3,d_2-m_3)}\frac{u^\alpha}{\alpha!}\frac{v^\beta}{\beta!}\frac{w^\delta}{\delta!},\\ &= \sum_{\alpha,\beta,\delta=0}^{\infty}\frac{(m_1)_{\alpha+\delta}(m_2)_{\alpha+\beta}(m_3)_{\beta+\delta}}{(d_1)_\alpha(d_2)_{\beta+\delta}}\frac{u^\alpha}{\alpha!}\frac{v^\beta}{\beta!}\frac{w^\delta}{\delta!},\end{aligned} \tag{5}$$

Where $m_1, m_2, m_3 \in \mathbb{C}$, also $d_1, and\ d_2 \in \mathbb{C}\backslash\mathbb{Z}_0^-$.

The convergency of the function $H_A(\cdot)$ depends on the $min\{p,q\}\geq 0$; $|u|<L, |v|<M, |w|<N$, where $\mathcal{L}=(1-\mathcal{M})(1-\mathcal{N})$.

We highlight Srivastava's multivariable hypergeometric special function $H_B(\cdot)$ presented by [Srivastava, H. M.(1967)] .In this work. (See also [Srivastava, Hari, and HL0535 Manocha(1984)])

$$H_B(m_1,m_2,m_3;d_1,d_2,d_3;u,v,w) \coloneqq \sum_{\alpha,\beta,\delta=0}^{\infty}\frac{(m_1)_{\alpha+\delta}(m_2)_{\alpha+\beta}(m_3)_{\beta+\delta}}{(d_1)_\alpha(d_2)_\beta(d_3)_\delta}\frac{u^\alpha}{\alpha!}\frac{v^\beta}{\beta!}\frac{w^\delta}{\delta!}$$

$$= \sum_{\alpha,\beta,\delta=0}^{\infty} \frac{(m_1+m_2)_{2\alpha+\beta+\delta}(m_3)_{\beta+\delta}}{(d_1)_\alpha (d_2)_\beta (d_3)_\delta} \times$$

$$\frac{\mathfrak{B}(m_1+\alpha+\delta, m_2+\alpha+\beta)}{\mathfrak{B}(m_1,m_2)} \frac{u^\alpha}{\alpha!} \frac{v^\beta}{\beta!} \frac{w^\delta}{\delta!}, \quad (6)$$

Where $m_1, m_2, m_3 \in \mathbb{C}$ and $d_1, d_2, d_3 \in \mathbb{C}\backslash\mathbb{Z}_0^-$.

The converging criteria is considered as $|u| < L, |v| < M, |w| < N$, where $\mathcal{L} + \mathcal{M} + \mathcal{N} + 2\sqrt{\mathcal{LMN}} = 1$.

The representation of Srivastava multivariable hypergeometric special function $H_C(\cdot)$ introduced in [19,20]

$$H_C(m_1, m_2, m_3; m_4; u, v, w) = \sum_{\alpha,\beta,\delta=0}^{\infty} \frac{(m_1)_{\alpha+\delta}(m_2)_{\alpha+\beta}(m_3)_{\beta+\delta}}{(m_4)_{\alpha+\beta+\delta}} \frac{u^\alpha}{\alpha!} \frac{v^\beta}{\beta!} \frac{w^\delta}{\delta!},$$

$$= \sum_{\alpha,\beta,\delta=0}^{\infty} \frac{(m_2)_{\alpha+\beta}(m_3)_{\beta+\delta}}{(m_4)_\beta} \frac{\mathfrak{B}(m_1+\alpha+\delta, m_4+\beta-m_1)}{\mathfrak{B}(m_1, m_4+\beta-m_1)} \times$$

$$\frac{u^\alpha}{\alpha!} \frac{v^\beta}{\beta!} \frac{w^\delta}{\delta!}, \quad (7)$$

Where $m_1, m_2, m_3 \in \mathbb{C}$ and $m_4 \in \mathbb{C}\backslash\mathbb{Z}_0^-$. The converging criteria is considered as $|u| < L, |v| < M, |w| < N$, where $\mathcal{L} + \mathcal{M} + \mathcal{N} - 2\sqrt{(1-\mathcal{L})(1-\mathcal{M})(1-\mathcal{N})} < 2$.

The Srivastava triple hypergeometric special functions are very useful in analytic continuation, which is discussed in details in [20].An extended version of the beta function $\mathfrak{B}(\alpha, \beta; p)$ is defined by Chaudhry et al. in 1997 [27], which can be expressed as:

$$\mathfrak{B}(\alpha, \beta; p) = \int_0^1 s^{\alpha-1} (1-s)^{\beta-1} exp^{\left[\frac{-p}{s(1-s)}\right]} ds \quad (8)$$

Where $\Re(p) > 0$.

Furthermore extension of the beta function is done by Chaudhry et al in[28] and use this equation to extend the Gauss hypergeometric series ${}_2F_1(.)$. The following is how Choi et al. [24] extended the Beta function such as:

$$\mathfrak{B}(\alpha, \beta; p, q) \equiv \mathfrak{B}_{p,q}(\alpha, \beta)$$

$$= \int_0^1 s^{\alpha-1}(1-s)^{\beta-1} exp\left\{-\frac{p}{s} - \frac{q}{1-s}\right\} d$$

Where $\Re(p), \Re(q) > 0$, If $p = q$ then $\mathfrak{B}_{p,q}(\alpha, \beta) \equiv \mathfrak{B}(\alpha, \beta; p)$.

For the further expansion or extension, the Srivastava triple hypergeometric special functions $H_B^{(r,s)}$, and $H_C^{(r,s)}$can be expressed with additional parameters $r, and\ s$ as:

$$H_B^{(r,s)}(m_1, m_2, m_3; d_1, d_2, d_3; u, v, w)$$

$$= \sum_{\alpha,\beta,\delta=0}^{\infty} \frac{(m_1+m_2)_{2\alpha+\beta+\delta}(m_3)_{\beta+\delta}}{(d_1)_\alpha (d_2)_\beta (d_3)_\delta} \frac{\mathfrak{B}(m_1+r+\alpha+\delta, m_2+s+\alpha+\beta)}{\mathfrak{B}(m_1,m_2)}$$

$$\frac{u^\alpha}{\alpha!}\frac{v^\beta}{\beta!}\frac{w^\delta}{\delta!}, \qquad (10)$$

And,

$$H_C^{(r,s)}(m_1,m_2,m_3;m_4;p,x,z) = \sum_{a,b,c=0}^{\infty} \frac{(m_2)_{a+b}(m_3)_{b+c}}{(m_4)_b}$$

$$\times \frac{\mathfrak{B}(m_1+r+a+c, m_4+s+b-m_1)}{\mathfrak{B}(m_1,m_4+b-m_1)} \frac{p^a}{a!}\frac{x^b}{b!}\frac{z^c}{c!}, \qquad (11)$$

The above equations (11) and (12) will reduce to the general equation (6) and (7) if the value of $r, and\ s$ will be zero.

Convergency of these two equations are same as the general equation.

The Appel hypergeometric multivariable special function $T_1(\cdot)$, can be expressed as:

$$T_1(m_1,m_2,m_3;m_4;u,v) = \sum_{a,b=0}^{\infty} \frac{(m_2)_b (m_3)_a \mathfrak{B}(m_1+a+b, m_4-m_1)}{\mathfrak{B}(m_1,m_4-m_1)} \frac{u^a}{a!}\frac{v^b}{b!} \qquad (12)$$

Details about the Appel hypergeometric special function and integral representation is discussed in [Abreu, et al. (2020)]. The formation of the extended Appel hypergeometric function motivate researchers to think about other special function also.Which is extended based on generalized beta function (8). Also, The extension of hypergeometric multivariable special function is done based on extended beta function by Özarslan and Özergin in [30] and by the equation (9) the extension in multivariable hypergeometric function is done by Parmar and Pogány in [Parmar, R. K., and T. K. Pogány(2017)].

The new extensionof the generalized Pochhammer symbol presented in [15] can be expressed as:

$$(\mathcal{P};p,q)_v = \begin{cases} \frac{\Gamma_q(\mathcal{P}+v;p)}{\Gamma(\mathcal{P})}, & (\Re(p),\Re(q) > 0\ , a = 0, \mathcal{P}, v \in \mathbb{C}) \\ (\mathcal{P};p)_v, & (v = 0, \mathcal{P}, v \in \mathbb{C}\backslash\{0\}) \end{cases} \qquad (13)$$

The integral expression of $(\mathcal{P};p,q)_v$ can be defined as

$$(\mathcal{P};p,q)_v = \sqrt{\frac{2p}{\pi}} \frac{1}{\Gamma(\mathcal{P})} \int_0^\infty x^{\mathcal{P}+v-\frac{3}{2}} exp(-x) \mathcal{J}_{q+\frac{1}{2}}\left(\frac{p}{x}\right) dx \qquad (14)$$

These generalisation of the Pochhammer symbol $(\mathcal{P};p,q)_v$ and the beta function $\mathfrak{B}_{p,q}(\alpha,\beta)$ are very important in different engineering and science field.

By these generalized Pochhammer symbol $(\mathcal{P};p,q)_v$ and the generalized extended beta function $\mathfrak{B}_{p,q}(\alpha,\beta)$, the extension of Generalised Srivastava's Function $H_A(\cdot)$, $H_B(\cdot)$, and $H_C(\cdot)$ have been studied in the next section.

EXTENSION OF THE GENERALISED SRIVASTAVA'S FUNCTION

Several writers have researched integral forms of the Srivastava multivariable hypergeometric special function in response to a few of the above-mentioned expansions of multivariable special functions. We have focused on the Srivastava's special triple hypergeometric special functions which are $H_A(\cdot)$, $H_B(\cdot)$, and $H_C(\cdot)$.

Extension on srivastava triple hypergeometric function $\mathbf{H_A}(\cdot)$

The Srivastava triple hypergeometric function $H_A(\cdot)$ has been studied in several studies [Srivastava, H. M.(1967), Choi, et al.(2012), Choi, et al (2014)]. In this section the extension of generalized Srivastava multivariable hypergeometric special function $H_A(\cdot)$ has been studied.

$\boldsymbol{\mathfrak{B}_{p,q}}$ ***Extension of*** $\boldsymbol{H_A}(\cdot)$**:** The expanded Beta (p,q) function $\mathfrak{B}_{p,q}(x,y)$ specified in equation (9) was used to complete the extension generalized Srivastava triple hypergeometric function $H_A(\cdot)$. The extended function is denoted by $H_{A,p,q}(\cdot)$, and its definition is given by

$$H_{A,p,q}(m_1,m_2,m_3;d_1,d_2;u,v,w) = \sum_{\alpha,\beta,\delta=0}^{\infty} \frac{(m_1)_{\alpha+\delta}(m_2)_{\alpha+\beta}}{(d_1)_\alpha} \frac{\mathfrak{B}_{p,q}(m_3+\beta+\delta,d_2-m_3)}{\mathfrak{B}(m_3,d_2-m_3)} \frac{u^\alpha}{\alpha!}\frac{v^\beta}{\beta!}\frac{w^\delta}{\delta!}, \tag{15}$$

Where $m_1,m_2,m_3 \in \mathbb{C}$ and $d_1,d_2 \in \mathbb{C}\backslash\mathbb{Z}_0^-$.

When $min\{p,q\} \geq 0$; $|u| < L, |v| < K, |w| < G$, where $\mathcal{L} = (1-\mathcal{K})(1-\mathcal{G})$, While $p = 0 = q$ the above extended generalized Srivastava triple hypergeometric function $H_{A,p,q}(\cdot)$ became the classical Srivastava triple hypergeometric function $H_A(\cdot)$ introduced in [Srivastava, H. M.(1967)] shown in the equation (5).

$\boldsymbol{(\mathcal{P};p,q)}$ ***Extension of*** $\boldsymbol{H_A}(\cdot)$**:** The latest extension has been done based on the Pochhammer's symbol as mentioned in equation (4) as:

$$H_{A,(\mathcal{P};p,q)}((\mathcal{P};p,q), m_1,m_2;d_1,d_2;u,v,w)$$
$$= \sum_{\alpha,\beta,\delta=0}^{\infty} \frac{((\mathcal{P};p,q))_{\alpha+\delta}(m_1)_{\alpha+\beta}(m_2)_{\beta+\delta}}{(d_1)_\alpha(d_2)_{\beta+\delta}} \frac{u^\alpha}{\alpha!}\frac{v^\beta}{\beta!}\frac{w^\delta}{\delta!}, \tag{16}$$

Where $\mathcal{P}, m_1, m_2 \in \mathbb{C}$ and $d_1, d_2 \in \mathbb{C}\backslash\mathbb{Z}_0^-$.

Where the convergency depends on $p \geq 0$, $|u| < L, |v| < J, |w| < K$, where $\mathcal{L} + \mathcal{J} + \mathcal{K} = 1 + \mathcal{JK}$.

Extension on Srivastava triple hypergeometric function $\mathbf{H_B}(\cdot)$

In [19] and [20], Srivastava developed the multivariable hypergeometric special function $\mathrm{H_B}(\cdot)$ and their integral forms. Some researchers extended the generalized Srivastava hypergeometric special function $H_B(\cdot)$ [21,22]. The extension of generalized Srivastava multivariable hypergeometric function $H_B(\cdot)$ has been studied in this section.

$\mathfrak{B}_{p,q}$ ***Extension of*** $\boldsymbol{H_B}(\cdot)$***:*** The extension has done by using the Beta (p, q) extended function $\mathfrak{B}_{p,q}(x, y)$ defined in (9) The extended function is denoted by $H_{\boldsymbol{B},p,q}(\cdot)$ which can be defined by

$$H_{B,p,q}(m_1, m_2, m_3; d_1, d_2, d_3; t, x, z)$$

$$= \sum_{\alpha,\beta,\delta=0}^{\infty} \frac{(m_1+m_2)_{2\alpha+\beta+\delta}(m_3)_{\beta+\delta}}{(d_1)_\alpha(d_2)_\beta(d_3)_\delta} \frac{\mathfrak{B}_{p,q}((m_1+\alpha+\delta, m_2+\alpha+\beta)}{\mathfrak{B}(m_1, m_2)} \frac{t^\alpha}{\alpha!} \frac{x^\beta}{\beta!} \frac{z^\delta}{\delta!}, \tag{17}$$

Where $m_1, m_2, m_3 \in \mathbb{C}$ and $d_1, d_2, d_3 \in \mathbb{C}\backslash\mathbb{Z}_0^-$.

The converging region is considered as $|u| < L, |v| < M, |w| < N$, where $\mathcal{L} + \mathcal{M} + \mathcal{N} + 2\sqrt{\mathcal{LMN}} = 1$. Using this formulation, the fundamental classical function in equation (6) when $p = 0 = q$.

$(\boldsymbol{\mathcal{P}}; \boldsymbol{p}, \boldsymbol{q})$ ***Extension of*** $\boldsymbol{H_B}(\cdot)$***:*** The latest extension of Srivastava triple hypergeometric function $H_B(\cdot)$ has been done based on the Pochhammer's symbol as mentioned in equation (4) as:

$$H_{B,(\mathcal{P};p,q)}\left((\mathcal{P}; p, q), m_1, m_2; d_1, d_2, d_3; u, v, w\right)$$

$$= \sum_{a,b,c=0}^{\infty} \frac{((\mathcal{P}; p, q))_{a+c}(m_1)_{a+b}(m_2)_{b+c}}{(d_1)_a(d_2)_b(d_3)_c} \frac{u^a}{a!} \frac{v^b}{b!} \frac{w^c}{c!}, \tag{18}$$

Where $\mathcal{P}, m_1, m_2 \in \mathbb{C}$ also $d_1, d_2, d_3 \in \mathbb{C}\backslash\mathbb{Z}_0^-$.

Where convergency depends on $p \geq 0$, $|u| < L, |v| < M, |w| < N$, where $\mathcal{L} + \mathcal{M} + \mathcal{N} + 2\sqrt{\mathcal{LMN}} = 1$

Extension on Srivastava triple hypergeometric function $\mathbf{H_C}(\cdot)$

Several researchers [Choi, Junesang, Anvar Hasanov, H. M. Srivastava, and Mamasali Turaev(2011)and Choi, Junesang, Arjun K. Rathie, and Rakesh K. Parmar (2014)] have examined the integral interpretations of the functions $H_C(\cdot)$ which were motivated by such modifications of Srivastava triple hypergeometric special functions (as stated above).

$\mathfrak{B}_{p,q}$ ***Extension:*** The extension of Srivastava multivariable hypergeometric special function $H_C(\cdot)$ has been studied in this section, by using the Beta (p,q) extended function $\mathfrak{B}_{p,q}(x,y)$ defined in (9) The extended function is denoted by $H_{C,p,q}(\cdot)$ which can be defined as:

$$H_{C,p,q}(m_1,m_2,m_3;m_4;t,x,z) = \sum_{a,b,c=0}^{\infty} \frac{(m_2)_{a+b}(m_3)_{b+c}}{(m_4)_b} \frac{\mathfrak{B}_{p,q}(m_1+a+c,m_4+b-m_1)}{\mathfrak{B}(m_1,m_4+b-m_1)} \frac{t^a}{a!}\frac{x^b}{b!}\frac{z^c}{c!}, \tag{19}$$

Where $m_1,m_2,m_3 \in \mathbb{C}$ and $m_4 \in \mathbb{C}\backslash\mathbb{Z}_0^-$.

The converging is considered as $|u| < L, |v| < M, |w| < N$, where $\mathcal{L}+\mathcal{M}+\mathcal{N}-2\sqrt{(1-\mathcal{L})(1-\mathcal{M})(1-\mathcal{N})} < 2.$ Using this formulation, the fundamental classical function in (7) when $p = 0 = q$.

$(\boldsymbol{\mathcal{P}};\boldsymbol{p},\boldsymbol{q})$ ***Extension:*** The latest extension of Srivastava triple hypergeometric function $H_C(\cdot)$ has been done based on the Pochhammer's symbol as mentioned in equation (4) can be expressed as:

$$H_{C,(\mathcal{P};p,q)}\left((\mathcal{P};p,q),m_1,m_2;d_1;t,x,z\right) = \sum_{a,b,c=0}^{\infty} \frac{((\mathcal{P};p,q))_{a+c}(m_1)_{a+b}(m_2)_{b+c}}{(d_1)_{a+b+c}} \frac{t^a}{a!}\frac{x^b}{b!}\frac{z^c}{c!}, \tag{20}$$

Where $\mathcal{P},m_1,m_2 \in \mathbb{C}$ and $d_1 \in \mathbb{C}\backslash\mathbb{Z}_0^-$.

The converging criteria is considered as $|u| < P, |v| < Q, |w| < R$, where $\mathcal{P}+\mathcal{Q}+\mathcal{R}-2\sqrt{(1-\mathcal{Q})(1-\mathcal{P})(1-\mathcal{R})} < 2.$

Integral Transformation

In this study the focus is given to the Mellin integral transforms of the special hypergeometric function. The Mellin Integral transformation has been studied for the above mentioned hypergeometric multivariate special functions.

Mellin Integral Transformation

The two-sided Laplace transform's multiplicative counterpart, the Mellin transform, is a type of integral transform used in mathematics. It is strongly linked to the Laplace and Fourier transform, as well as the theory of the related special functions. The theory of

asymptotic expansions, mathematical statistics, and number theory all frequently make use of this integral transform. It is also directly related to the theory of the Dirichlet series.
For the single variable the Mellin integral transformation is defined by

$$\{\mathcal{M}g\}(s) = \Phi(s) = \int_0^\infty r^{s-1} g(r)dr \tag{21}$$

$$\{\mathcal{M}^{-1}\Phi\}(r) = g(r) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} r^{-s}\Phi(s)ds \tag{22}$$

This refers to a line integral performed over the vertical line within a complex plane, according to the notation, and the real portion c must meet a loose lower constraint. The Mellin inverse theorem specifies the circumstances whereby this inversion is valid.
For the several variable functions, the Mellin transformation is defined in the double integration form, or sometimes it is known as the double Mellin transformation. If $g(x,y)$ is locally integrable function with indices $r\ and\ s$ specified in [26, p,193] then the Mellin transformation can be defined as

$$\{\mathcal{M}g(x,y)\}(r,s) = \Phi(r,s)$$
$$= \int_0^\infty \int_0^\infty x^{s-1} y^{s-1} g(x,y)dxdy, \tag{23}$$

which identifies a mathematical function in the terms of analyticity $\mathcal{P} < R(r) < Q\ and\ Q < R(r) < R.$ The inverse of this double Mellin integral transformation can be represented as:

$$\{\mathcal{M}^{-1}\Phi(r,s)\}(x,y) = g(x,y)$$
$$= \frac{1}{(2\pi i)^2}\int_{c-i\infty}^{c+i\infty}\int_{d-i\infty}^{d+i\infty} x^{-r}y^{-s}\Phi(r,s)drds \tag{24}$$

Where $\mathcal{P} < c < Q\ and\ Q < d < R.$

Mellin Transformation for $\mathfrak{B}_{p,q}$ Extension of $H_A(\cdot)$:

The Mellin integral transformation for the extended generalized Srivastava's triple hypergeometric multivariable special function $H_{A,p,q}(\cdot)$ holds for $\Re(p), \Re(q) > 0,$ $\Re(m_3 + r), \Re(d_2 + s - m_3) > 0$ and for all $\Re(r), \Re(s) > 0$ presented by

$$\{\mathcal{M}H_{A,p,q}(m_1, m_2, m_3; d_1, d_2; t, u, x)\}(r,s)$$
$$= \frac{\Gamma(r)\Gamma(s)\mathfrak{B}_{p,q}(m_3+r, d_2+s-m_3)}{\mathfrak{B}_{p,q}(m_3, d_2-m_3)}$$
$$H_A(m_1, m_2, m_3 + r; d_1, d_2 + r + s; t, u, x) \tag{25}$$

The transformation is expressed in the form of generalized Srivastava's triple hypergeometric multivariable special function $H_A(\cdot)$.

The $\mathfrak{B}_{p,q}$ extended generalized Srivastava multivariable hypergeometric special function $H_{B,p,q}(m_1, m_2, m_3; d_1, d_2, d_3; \mathfrak{i}, \mathfrak{h}, \mathfrak{g})$ has the following Mellin transform:

$$\{\mathcal{M}H_{B,p,q}(m_1, m_2, m_3; d_1, d_2, d_3; \mathfrak{i}, \mathfrak{h}, \mathfrak{g})\}(\mu, \rho)$$

$$= \int_0^\infty \int_0^\infty x^{\mu-1} y^{\rho-1} H_{B,p,q}(m_1, m_2, m_3; d_1, d_2, d_3; \mathfrak{i}, \mathfrak{h}, \mathfrak{g}) dxdy,$$

$$= \Gamma(r)\Gamma(s)\, H_B^{(\mu,\rho)}(m_1, m_2, m_3; d_1, d_2, d_3; \mathfrak{i}, \mathfrak{h}, \mathfrak{g}) \tag{26}$$

Where $H_B^{(r,s)}(m_1, m_2, m_3; d_1, d_2, d_3; \mathfrak{i}, \mathfrak{h}, \mathfrak{g})$ is presented in the equation (10) and for all $\Re(\mu), \Re(\rho) > 0$ and $\Re(p), \Re(q) > 0$, $\Re(m_1 + r), \Re(m_2 + s) > 0$ and $d_1, d_2, d_3 \in \mathbb{C}\backslash\mathbb{Z}_0^-$.

The transformation is expressed in the form of Srivastava's multivariable hypergeometric special function $H_B^{(r,s)}$.

The inverse Mellin transformation for $\mathfrak{B}_{\mathrm{p,q}}$ Extension of $\mathrm{H_B}(\cdot)$ can be presented as:

$$\{\mathcal{M}^{-1}\Phi\}(\mu, \rho) = H_{B,p,q}(m_1, m_2, m_3; d_1, d_2, d_3; \mathfrak{i}, \mathfrak{h}, \mathfrak{g})$$

$$= \frac{1}{(2\pi i)^2} \int_{c-i\infty}^{c+i\infty} \int_{d-i\infty}^{d+i\infty} \left(\frac{1}{p}\right)^{\mu} \left(\frac{1}{q}\right)^{\rho} \Gamma(r)\Gamma(s) H_B^{(\mu,\rho)} (m_1, m_2, m_3; d_1, d_2, d_3; \mathfrak{i}, \mathfrak{h}, \mathfrak{g}) d\mu d\rho$$

Where $c > 0 \; and \; d > 0$, $H_B^{(\mu,\rho)}(m_1, m_2, m_3; d_1, d_2, d_3; \mathfrak{i}, \mathfrak{h}, \mathfrak{g})$ is presented in the equation (10) and for all $\Re(\mu), \Re(\rho) > 0$ and $\Re(p), \Re(q) > 0$, $\Re(m_1 + r), \Re(m_2 + s) > 0$ and $d_1, d_2, d_3 \in \mathbb{C}\backslash\mathbb{Z}_0^-$.

Mellin Transformation for $\mathfrak{B}_{\mathrm{p,q}}$ Extension of $\mathbf{H_B}(\cdot)$:

The $\mathfrak{B}_{p,q}$ extended generalized Srivastava multivariable hypergeometric special function $H_{B,p,q}(m_1, m_2, m_3; d_1, d_2, d_3; \mathfrak{i}, \mathfrak{h}, \mathfrak{g})$ has the following Mellin transform:

$$\{\mathcal{M}H_{B,p,q}(m_1, m_2, m_3; d_1, d_2, d_3; \mathfrak{i}, \mathfrak{h}, \mathfrak{g})\}(\mu, \rho)$$

$$= \int_0^\infty \int_0^\infty x^{\mu-1} y^{\rho-1} H_{B,p,q}(m_1, m_2, m_3; d_1, d_2, d_3; \mathfrak{i}, \mathfrak{h}, \mathfrak{g}) dxdy,$$

$$= \Gamma(r)\Gamma(s)\, H_B^{(\mu,\rho)}(m_1, m_2, m_3; d_1, d_2, d_3; \mathfrak{i}, \mathfrak{h}, \mathfrak{g}) \tag{26}$$

Where $H_B^{(r,s)}(m_1, m_2, m_3; d_1, d_2, d_3; \mathfrak{i}, \mathfrak{h}, \mathfrak{g})$ is presented in the equation (10) and for all $\Re(\mu), \Re(\rho) > 0$ and $\Re(p), \Re(q) > 0$, $\Re(m_1 + r), \Re(m_2 + s) > 0$ and $d_1, d_2, d_3 \in \mathbb{C}\backslash\mathbb{Z}_0^-$.

The transformation is expressed in the form of Srivastava's multivariable hypergeometric special function $H_B^{(r,s)}$.

The inverse Mellin transformation for $\mathfrak{B}_{p,q}$ Extension of $H_B(\cdot)$ can be presented as:

$$\{\mathcal{M}^{-1}\Phi\}(\mu,\rho) = H_{B,p,q}(m_1,m_2,m_3;d_1,d_2,d_3;\mathfrak{i},\mathfrak{h},\mathfrak{g})$$

$$= \frac{1}{(2\pi i)^2}\int_{c-i\infty}^{c+i\infty}\int_{d-i\infty}^{d+i\infty} \left(\frac{1}{p}\right)^{\mu}\left(\frac{1}{q}\right)^{\rho}\Gamma(r)\Gamma(s)H_B^{(\mu,\rho)}(m_1,m_2,m_3;d_1,d_2,d_3;\mathfrak{i},\mathfrak{h},\mathfrak{g})\,d\mu d\rho \tag{27}$$

Where $c > 0$ and $d > 0$, $H_B^{(\mu,\rho)}(m_1,m_2,m_3;d_1,d_2,d_3;\mathfrak{i},\mathfrak{h},\mathfrak{g})$ is presented in the equation (10) and for all $\Re(\mu),\Re(\rho) > 0$ and $\Re(p),\Re(q) > 0$, $\Re(m_1+r),\Re(m_2+s) > 0$ and $d_1,d_2,d_3 \in \mathbb{C}\backslash\mathbb{Z}_0^-$.

Mellin Transformation for $\mathfrak{B}_{p,q}$ Extension of $H_C(\cdot)$:

For all $\Re(\mu),\Re(\rho) > 0$ and $\Re(p),\Re(q) > 0$, the Mellin integral transformation for the extended generalized Srivastava's triple hypergeometric multivariate special function $H_{C,p,q}(\cdot)$ is represented by

$$\{\mathcal{M}H_{C,p,q}(m_1,m_2,m_3;m_4;\mathfrak{i},\mathfrak{h},\mathfrak{g})\}(\mu,\rho)$$

$$= \int_0^{\infty}\int_0^{\infty} x^{\mu-1}y^{\rho-1}H_{C,p,q}(m_1,m_2,m_3;m_4;\mathfrak{i},\mathfrak{h},\mathfrak{g})\,dxdy,$$

$$= \Gamma(\mu)\Gamma(\rho)\,H_C^{(\mu,\rho)}(m_1,m_2,m_3;m_4;\mathfrak{i},\mathfrak{h},\mathfrak{g}) \tag{28}$$

Where $\Re(m_1+\mu),\Re(m_2+\rho) > 0$, $m_4 \in \mathbb{C}\backslash\mathbb{Z}_0^-$, and $H_C^{(r,s)}(m_1,m_2,m_3;m_4;\mathfrak{i},\mathfrak{h},\mathfrak{g})$ is defined in the equation (11).

The inverse Mellin transformation for $\mathfrak{B}_{p,q}$ Extension of $H_C(\cdot)$ will follow:

$$\{\mathcal{M}^{-1}\Phi\}(\mu,\rho) = H_{C,p,q}(m_1,m_2,m_3;m_4;\mathfrak{i},\mathfrak{h},\mathfrak{g})$$

$$= \frac{1}{(2\pi i)^2}\int_{c-i\infty}^{c+i\infty}\int_{d-i\infty}^{d+i\infty} \left(\frac{1}{p}\right)^{\mu}\left(\frac{1}{q}\right)^{\rho}\Gamma(r)\Gamma(s)H_C^{(\mu,\rho)}(m_1,m_2,m_3;m_4;\mathfrak{i},\mathfrak{h},\mathfrak{g})\,d\mu d\rho \tag{29}$$

Where $\Re(m_1+\mu),\Re(m_2+\rho) > 0$, $m_4 \in \mathbb{C}\backslash\mathbb{Z}_0^-$, and $H_C^{(r,s)}(m_1,m_2,m_3;m_4;\mathfrak{i},\mathfrak{h},\mathfrak{g})$ is defined in the equation (11)

These Mellin integral transform and Inverse Mellin integral transform will help to derived several properties of the these extended generalized Srivastava's multivariable hypereometric function. There are several other integral transformations like Kumer integral transformation, which can be derived for These extended generalized Srivastava's triple hypergeometric special function.

Importance of the Study and Possible Applications

Some important discoveries from the study of Srivastava triple hypergeometric special functions and their integral transformations have important ramifications for many areas of mathematics and science. The following are the few applications aspects where These extended generalized Srivastava's multivariable hypergeometric function and their Mellin transformation can be useful.

Recursive Function: A recursive function generates a series of phrases by repeating or using its prior term as input. The arithmetic-geometric sequence, which contains words having a common difference between them, is often the basis on which we learn about this function. Languages used for computer programmings, like C, Java, Python, and PHP, heavily utilize this function. The concept of a recursive function, the underlying equation, and the method of constructing the recursive method for the provided sequence will all be covered in this article, along with instances that have been solved. These extended generalized Srivastava's multivariable hypergeometric function and their Mellin transformation are important in recursion methodology.

Fractional Calculus: In fractional calculus, the complete integral or derivatives are taken as a fraction which is very much help in solving real-life problems in Physics, Biological models, Mechanics etc. These extended generalized Srivastava's multivariable hypergeometric function and their Mellin transformation can be useful in the Fractional calculus extensions.

Incomplete Function: Incomplete Functions are also other types of special functions. Incomplete functions are studied to solve critical partial differential equations, it will help to identify the analyticity of a function. Using the use of the incomplete gamma functions $\Gamma(s, x)$, Srivastava et al. [35] recently proposed the incomplete Pochhammer symbols and constructed incomplete hypergeometric functions, whose various intriguing and important aspects and characteristics are studied by these extended generalized Srivastava's multivariable hypergeometric function and their Mellin transformation.

Inversion formulas and uniqueness theorems: The discovery of inversion formulae and uniqueness theorems for different integral transformations has also resulted from research into multivariable special functions. These equations and theorems offer strategies for reversing integral transformations and identifying the one and only answer to a particular puzzle. This discovery has significant ramifications for fields including quantum physics, signal processing, and partial differential equations.

Convergency Analysis: The creation of convergence conditions for various integral transformations is another important result of the research of Multivariable Special Functions. These standards define the circumstances under which integral transforms converge, guaranteeing the accuracy of calculations employing these transforms. The probability theory, mathematical modelling, and numerical analysis are only a few of the disciplines of mathematics and physics where this conclusion has significant ramifications.
Overall, research into Srivastava's triple hypergeometric special functions and associated integral transformations has produced several significant results with significant ramifications for several areas of mathematics and physics. These results offer strong methods for resolving mathematical models and comprehending the behaviour of complicated physical systems, with potential applications in many fields, including quantum physics and signal processing.

CONCLUSION

In this paper we have discussed the extension of generalized Srivastava's triple hypergeometric special function. The earlier works is studied and the motivation of the present work is shown and which is significant in recent research trends. The study on extension of the generalized Srivastava's triple hypergeometric special function $H_A(\cdot)$, $H_B(\cdot)$, and $H_C(\cdot)$ is done based on the recently developed extended beta function $\mathfrak{B}_{p,q}(\alpha,\beta)$ and the generalized Pochhammer's symbol $(\mathcal{P};p,q)_v$ provide a further scope for researchers. In addition, we studied the integral transformation specially Mellin integral transformation which is an important integral transformation for the special multivariable special functions. We studied Mellin integral transformation for the extended generalized Srivastava's multiple variable hypergeometric special function $H_A(\cdot)$, $H_B(\cdot)$, and $H_C(\cdot)$ by using extended (p,q) beta function $\mathfrak{B}_{p,q}(\alpha,\beta)$.

We choose to include several recent papers [8,9,32,33] that address extended generalized Srivastava's multivariable hypergeometric special function and their Mellin transformation and assessment of various characteristics and inequalities in order to inspire more study along the lines indicated in this paper. An extension of the Srivastava's multivariable hypergeometric special function $H_A(\cdot)$, $H_B(\cdot)$, and $H_C(\cdot)$, as described by equation (12-18), may be used to define specific new extensions of several special functions [30,31] and their integral transformations also in uncertainty field like gaussian functions used in [33].

In short, multivariable special functions and associated integral transformations constitute a prominent topic of research in mathematics and physics and continue to significantly influence various domains. Future research in this field will probably provide even more significant discoveries and applications.

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