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STUDY AND ANALYSIS OF SOME PRACTICAL LIFE USES AND APPLICATIONS OF EXPONENTIAL FUNCTION

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Abstract

We have sorted out some practical uses of exponential function that we are going to study in this report. The report is given more emphasis on real life use of Euler number i.e. e and also on the uses of other exponential function. The sorted out uses of exponential function is based on following topic: real value of money, bit coin value, household saving, production in a factory, and exploitation of forest. Later on, we have also focused on consequences of those problems and find its solution.

Keywords: Euler number e, real value of money, bitcoin, saving, and production and capital used

Introduction

The interesting history of the Euler number e and its corresponding exponential function e^x is deeply entwined with the centuries-long progression of mathematics. Although the ancient Greeks investigated the idea of exponential growth, mathematicians did not start to formalize it until the 17th century. Mathematicians like Johannes Kepler and John Napier worked with logarithms and exponential functions in the 17th century, establishing the foundation for later discoveries.

In the eighteenth century, the Swiss mathematician Leonhard Euler introduced the number e for the first time. After researching compound interest, Euler discovered that when n got closer to infinity, the limit of (1+1/n) ^n approximated a constant number. This constant, known as e, is the foundation of the natural logarithm and is roughly equal to 2.71828. Since it produced the exponential function e^x, Euler's work on e was ground-breaking. There are a lot of uses for this function in different domains like economics, physics, calculus, and engineering.

The amazing link that e^x has with its own derivative is one of its fundamental qualities. To be precise, e^x itself is the derivative of e^x . Due to this characteristic, differential equations—which are essential for explaining natural phenomena—can be solved with great ease using the exponential function.

The exponential function and the Euler number are now fundamental concepts in mathematics and its applications. They are essential ideas in contemporary mathematics, having applications ranging from analysis and calculus to finance and probability theory.

Input-output analysis explores many facets of economic dynamics, claims Sahani (2023). It investigates the connections between producer and consumer excess even in atypical market scenarios when supply is negatively impacted but demand is positively impacted. His research shows that there is still value to be discovered for both producers and consumers in these kinds of situations, highlighting the robustness and complexity of economic systems. This realization has important ramifications for economic policy in addition to being fascinating.

Input-output analysis was used in another study to examine how risk is priced internationally in the global capital market (Sahani et al., 2023). The importance of input-output analysis in separating out the intricate connections between the many global economic sectors and revealing the pathways by which risk is transnationally dispersed is emphasized by Sahani and his associates. These results are relevant to international financial regulation because actions intended to prevent financial contagion can be informed by knowledge of the dynamics of risk transmission.

Additionally, input-output analysis is utilized in the field of non-linear research, revealing the complex relationships among non-linear systems and offering prognostications regarding their conduct (Sahani & Prasad, 2023). The work of Sahani and Prasad demonstrates how flexible input-output analysis is as a tool for comprehending complex systems outside of conventional economic fields.

Now that we are focusing on Africa, input-output analysis is crucial for analysing the issues related to employment and job development that young people in African nations confront (Sahani, 2023). According to Sahani's research, important industries that significantly contribute to young employment include trade, construction, and agriculture. In order to promote economic growth and capitalize on the demographic dividend of Africa's youthful population, it becomes necessary to address the difficulties in various sectors (see [1-6]).

Definitions

Exponential function: it refers to the function of form F(x) = ‘a’ to the power x where a≠1, a>0 and ‘a’ belongs to real number. For example, 2^x, 3^x, 4^x etc. following are the basic form of exponential function. The special form of exponential function is e whose value is equal to 2=7182818284.

$$\boldsymbol{e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}} \qquad :$$

This e is a special type of exponential function which is basically derived from compound interest. However, it can be used for different purpose: This exponential function e helps to find out the growth of anything we can say that compound growth. Similarly, it also helps in finding compound depreciation

Some definitions can be made with help of exponential function are shown below;

i. **Limitless growth function:** A function of the form y=ae^rt in which a and r are considered as constant is known as limitless growth function.

ii. **Limitless decay function:** A function of the form $y=ae^{-rt}$ where a and r are considered as constant is known as limitless decay function.

iii. **Limited growth function:** The function of the form $y=M(1-e^{-rt})$, where M and r are considered constant, is known as limited growth function.

Discussions

Before going up to the problem I'd like to give you the information and concept about real value of money along with its factor like inflation. We are trying to calculate the real value of money over the period of time with the help of exponential function.

What is exactly is the real value of money?

We know that inflation is the rise in general price level or we can also say that decrease in the value of money. Now moving up to our keyword real value of money; real value of money refers to the value of money we get after comparing the current year price level with the base year price level.

Let's summarize the real value of money with the help of a hypothetical example:

Suppose that you could buy ten pens from 50 rupees in year 2012. Now, in the year 2015, you have to spend 70 rupees to buy total of 7 same pens. Of course, here we can observe hike in price of pen from year 2012 to 2015 that what we called inflation. To get the real value of money we have done following operation below:

Let the base year be 2012,

price of ten pens in year 2012=50 rupees,
thus the price of 7 pens in year 2012 = 50/10*7 =35 rupees.

Whereas,
price of 7 pens in year 2015= 70 rupees,
so by comparing the price of 7 pens in year 2015 with base year price gives the real value of money. We can say that 70 rupees is worth 35 rupees as we considered 2012 as a base year.

Based on this very same topic we are going to discuss about the real value of money in context of saving and investment in Nepal.

Problems-1

A person put 500000 rupees in fixed deposit for five year. He deposited the amount in year 2018 then what is the amount of money he'll get after 5 year and also calculate inflation for five years on the sum and find the actual increase in its sum?

We have following details:

Chart for inflation in Nepal:

Year	Inflation rate	Approximate rate
2022	7.65%	8%
2021	4.09%	4%
2020	5.05%	5%
2019	5.57%	6%
2018	4.06%	4%
2017	3.63%	4%

Note*: we've taken approximate value to easier the calculation.

Interest on fixed deposit:

Duration	Interest rate
≥3 up to 6months	6%
≥6 up to 1year	6.5%
≥1 up to 2year	7.2%
≥2 up to 3 year	7.75%
≥3 year	8.10%

Solution:

From the above table,

we have interest (r) = 8.10%=0.081,

principal (P) = 500000 rupees and

time (T) = 5 years,

Firstly,

Amount in 5^{th} year

$= Pe^{rt}$

= 500000e^0.081*5

= 749650 rupees

Secondly,

y=500000^0.081*t

Time (t)	1	2	3	4	5
Amount after t year (y)	542185	587930	637534	691323	749650

From the table, we can get that 500000 rupees at 8.10% rate became 542185 in the first year, similarly 587930 in 2nd year, 637534 in 3rd year, 691323 in 4th year and finally 749650 in 5th year.

But beside this there is a factor called inflation which reduces the value of money p.a. so to get accurate surplus we have to compare it with inflation.

Now,

charging inflation on original sum,

For 1st year, amount = 500000e^0.04*1 = 520405,

For 2nd year, amount = 520405e^0.06*1= 552585,

For 3rd year, amount = 552585e^0.05*1 = 580917,

For 4th year, amount = 580917e^0.04*1 = 604624,

And For 5th year, amount = 604624e^0.08*1 = 654982.

So, in year 2022 the amount 654982 is equal to 500000 in base year 2018.

Remarks:

1. It means that if you can buy x no. of commodity from 500000 in year 2018 then in 2022 same commodity cost you 654982

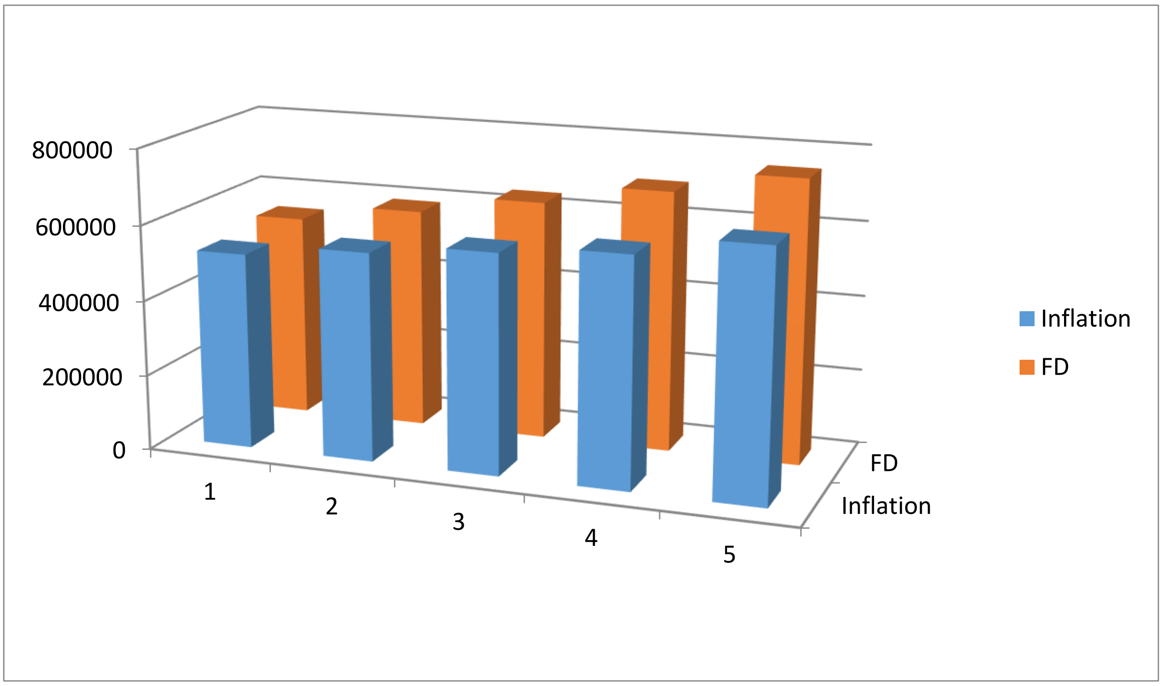
Comparing the fixed deposit amount with inflation:

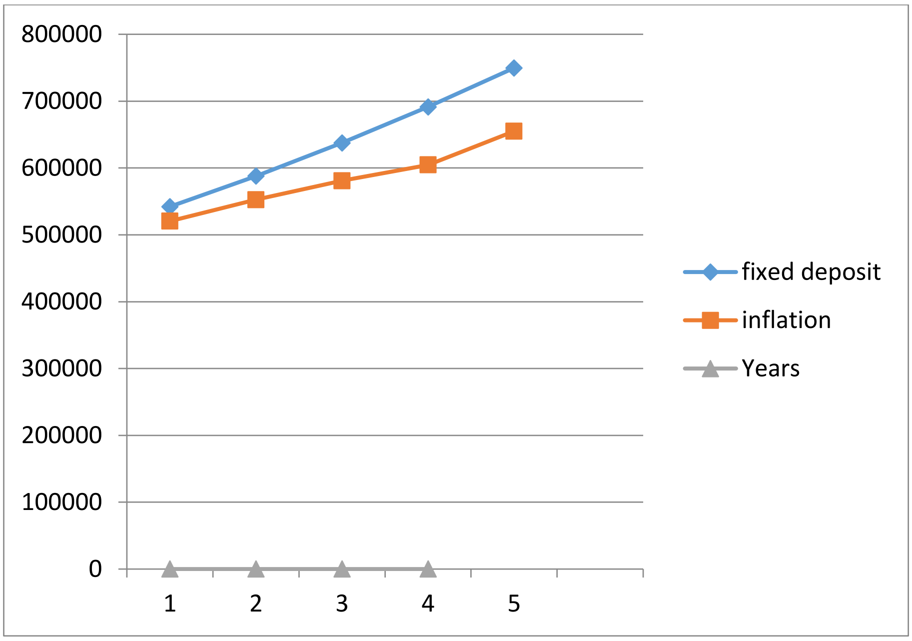
Amount we got after 5 year from FD = 749650 and

Value of 500000 after 5 year due to inflation = 654982.

Since FD provides more amount than the value of 500000 means the consumer will be in surplus. By basic observation we can say that the person will get 749650-500000 = 249650 since he invested only 500000 but that is wrong, due to inflation the amount 500000 became 654982 so that the person's actual surplus is 749650-654982 i.e. 94668. It means he/she will enjoy the surplus of 94668 rupees.

Representation of above data through graph:





Conclusion

1. If you keep your money with yourself that means if you didn't invest it anywhere this results in the automatic decrease in the value of money due to inflation. For

example, suppose you have one lakh rupees and you neither spend it nor invest it anywhere and just kept with yourself and suppose inflation rate is 5% then after a year your money devaluated to 100000-5% of 100000 i.e. 95000 means your 100000 will only worth 95000 after a year.

2. As a conclusion, we should have to aware about the time value of money and need to invest our money in different financial sector or we can buy some valuable commodity whose value appreciates like gold, silver, and land.

Problem-2

A manufacturing company increases its capital by 10% every year which results in 5% increase in its production. If the company invested 500000 in its first year and the production was 1000 units then how many years will it take the company to produce 10000 units in a calendar year and how much capital is required for that?

Solution:

increase in rate of production (r) = 5% p.a. = 0.05,

initial production (P) = 1000 units,

and production required (A) = 10000 units

then time period (T) = ?

With help of exponential function,

we have,

A=Pe^{rt}

or, 10000= $1000e^{0.05t}$

or, 10 =$e^{0.05t}$

Taking log_e on both side

or, $log_e 10 = log_e e^{0.05t}$

or, $log_e 10 = 0.05t * log_e e$

or, t = $\frac{log_e 10}{0.05}$

therefore, t= 46.06 year

It takes approximately 46 years to produce 10000 units if the production increases by the same rate.

Also,

Capital invested in the 46th year (Y) =? , We have initial capital (P) = 500000,

capital increment rate (r) = 10% p.a. = 0.1,

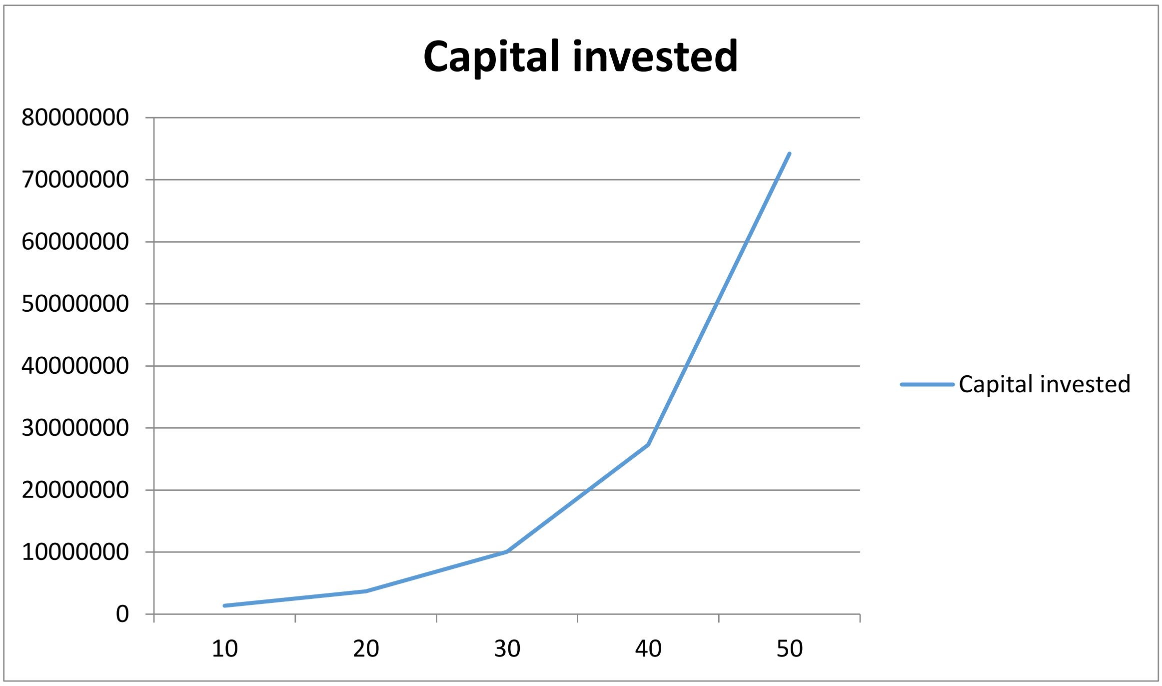
and time (T) = 46 .06 year.

Using exponential function

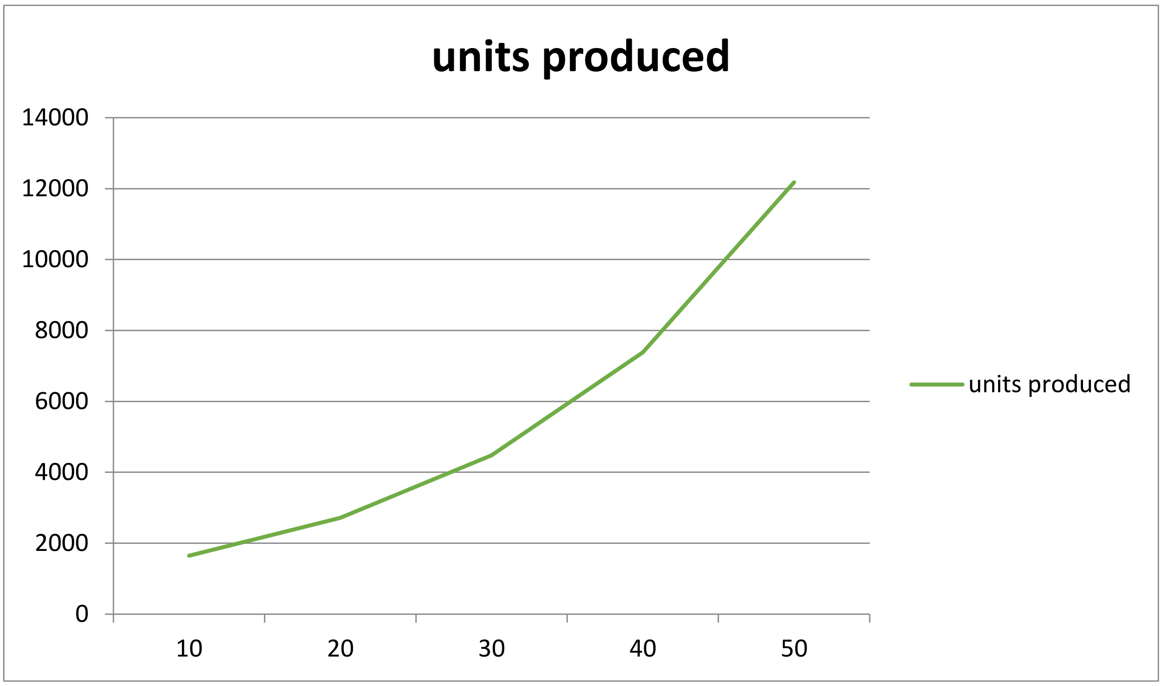
$Y = Pe^{rt} = 500000e^{0.1*46.06}$ = 50041507.92

Therefore the company invest approximately 5cr in 46th year to get 10000 unit production.

Yearly increment in capital observation through following graph:



Production observation through graph



In the above pictures x-axis represents number of years. In the first picture of problem-2 y- axis represents amount of money and in second picture y-axis represents quantity.

Problem-3

A person has bought bit coin of 100 rupees in year 2000. Later on he found that the bit coin value has increased by 75% p.a. from that date to 2020 then what is the value of that bit coin in year 2020?

Solution:

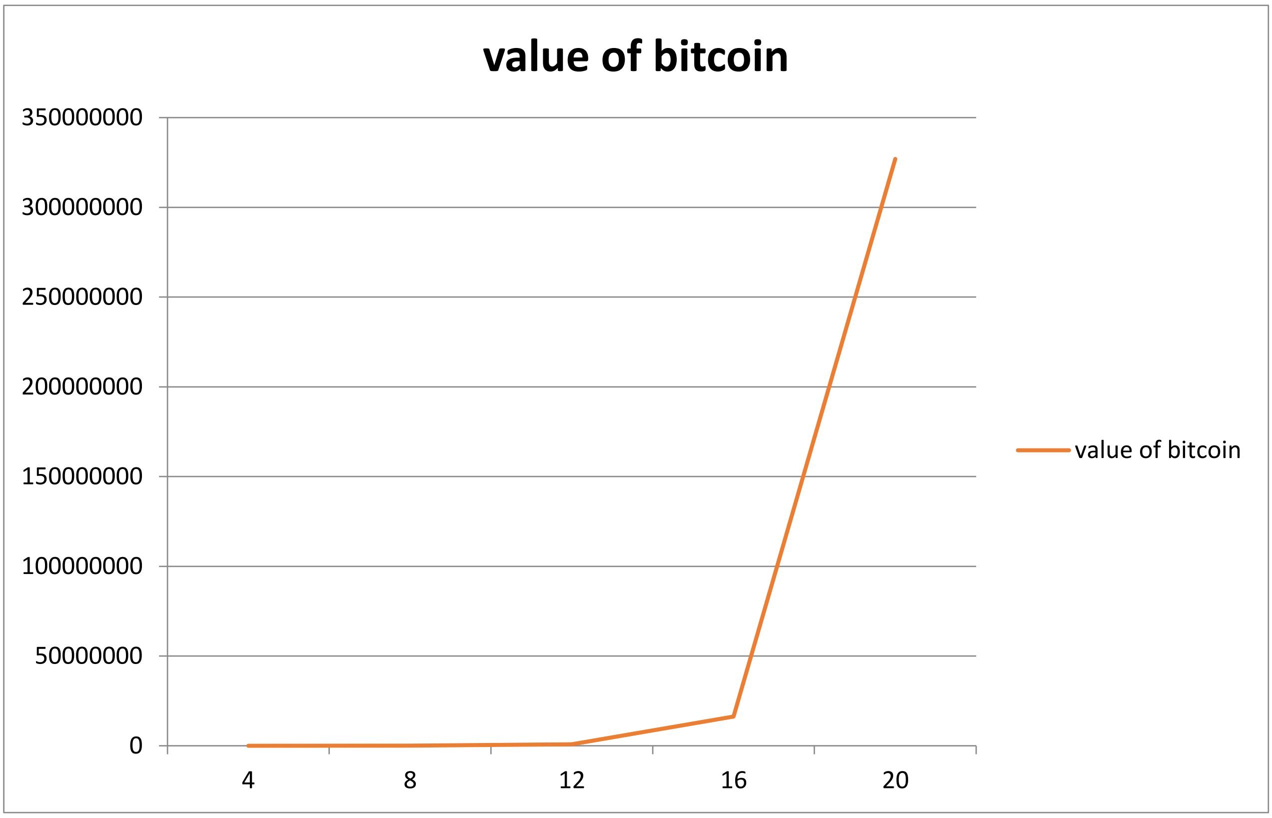
Initial value of bit coin (I) = 100

Rate (r) = 75% = 0.75

Time (t) = 2020-2000 = 20 year

Present value = $Ie^{rt} = 100e^{0.75*20}$ =326901737.2

We are able to find out the increase in the value of bit coin in few steps with help of exponential function.



In the graph we have x-axis as number of years where as y-axis represents value of bit coin over the years. We can see that the line graph is increasing exponentially. From the graph we can see that the initial amount 100 became 2009 in first 4 year after 8 year 1000 became 40351 after 12 year it became 810472 after 16 year 16278765 and at the last after 20 year it became 326 million. Since the rate is very high i.e. 75% P.A. the amount is getting higher and higher with every passing year. Thus exponential is also useful to calculate higher amount of appreciation in the value of bit coin.

Problem-4

Every year deforestation is done and it is increasing by 3% per annum. In 2013, the number of trees cut down is 700000 if the trees were cut down at the same rate then how many trees were cut down up to 2024 find out?

Solution;

Number of trees cut in initial year (P) = 700000

Rate of deforestation (r) = 3% = 0.03

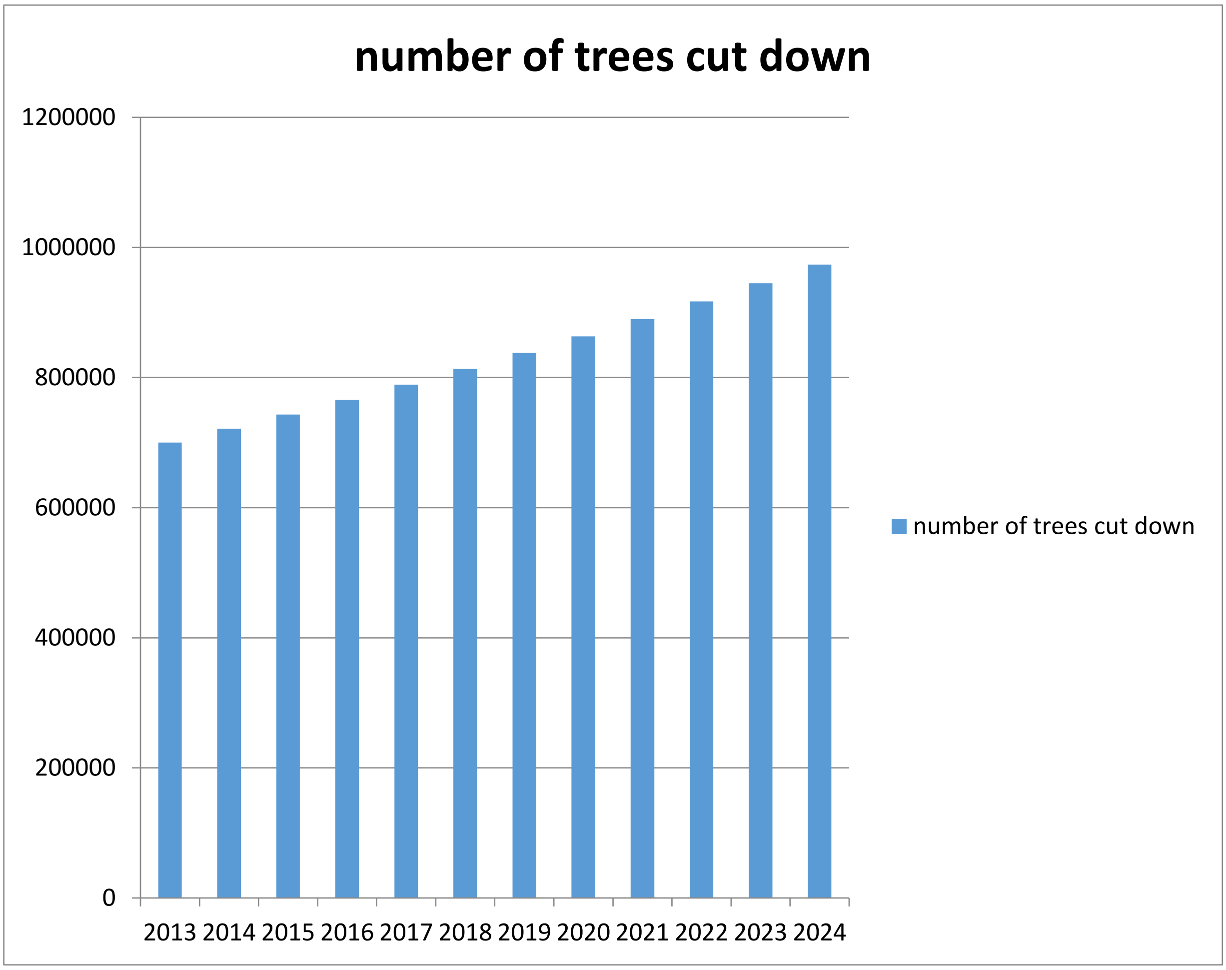
Time (T) = 11 year

Number of trees cut down up to 2024

$= Pe^{rt}$

$=700000e^{0.03*11}$

= 973678



Problem-5

A boy saves 100 rupees in first month, 200 in the next month, 400 in the 3rd month and he saved his money in the same pattern then how much money he would have at the end of 10 months?

Solution:

We can use geometric series to solve this problem, a lot of think why geometric series, Geometric series because it is also a kind of exponential function. We know that the general term of geometric series is ar^{n-1} which itself is an exponential function.

We have,

Saving of first month (a) = 100

Common ratio (r) = $\frac{200}{100} = 2$

No. of month (n) = 10 month

We know that,

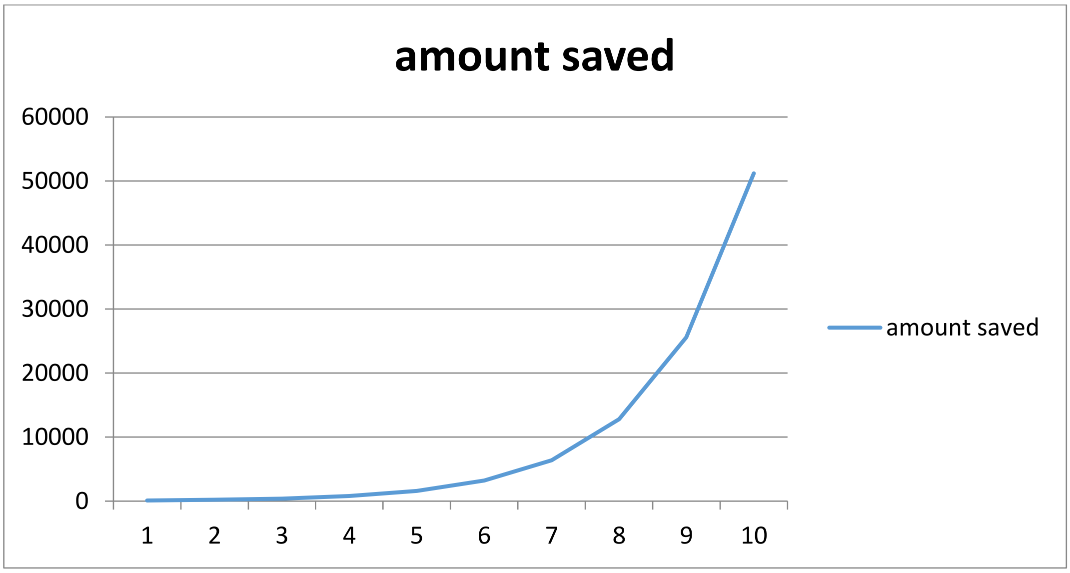
Sum of G.P.

$= \frac{a(r^n-1)}{r-1}$

$=\frac{100(2^{10}-1)}{2-1}$

=102300

It means he will have one lakh twenty three hundred saving in 10 month.



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