Kwaghe International
Journal of Sciences and Technology
ISSN : 0000-0000
Index: Harvard, Boston, Sydney University, Dimensions, Lens, ResearchGet Scilit, Semantic, Google Scholar, Base etc
https://doi.org/10.58578/KIJST.v1i1.3597

Investigation of Positivity, Existence and Uniqueness of a Modified COVID-19 Model

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Article Info:

Submitted:	Revised:	Accepted:	Published:
Jul 1, 2024	Jul 20, 2024	Jul 27, 2024	Jul 31, 2024

Abstract

Corona virus disease is the most dreaded infectious disease all over the whole world. The outburst of the disease made many researchers to step up with research so as to find solution of eradication of the disease. Jummy et al (2021) developed a compartmental differential equation models which they used in studying direct and indirect transmission of COVID-19.We discovered that their model did not consider quarantine, vaccination and partial immunity. We then incorporated quarantine, vaccination and partial immunity into their models to come up with a modified version of Jummy et al model equations. In this research work, we investigated the positivity of the solution of the modified model, the existence and the uniqueness of the solution. The essence of doing these is to be sure that our models can conform to reality in solving the problem of eradication of COVID-19. We discovered that the solution exist, bounded, unique and positive.

Keywords: Invariant region, Existence, Positivity, Uniqueness of the solution

Introduction

Corona virus (popularly called COVID-19) has subjected the whole world into worry considering its rapid spread. It was declared a pandemic by the World Health Organization (WHO) on March 11, 2020(WHO,2020). Corona virus disease is seen as a dreaded disease all over the world. This has prompted many researchers to carry out a series of research works on COVID-19. For instance, Jummy et al (2021);In their work, the studied COVID-19 by considering the impact of direct and indirect transmission of the disease. They observed that indirect transmission has great impact compared to the direct transmission.

The COVID-19 virus spreads to a large extent between people in close contact with each other (within approximately 2 metres). The common incubation period ranges from 1 to 14 days (WHO,2020). In the absence of a definitive treatment modality like a vaccine, physical distancing has been accepted globally as the most efficient strategy for reducing the severity of the disease and gaining control over the disease (Ferguson et al,2020). COVs are widespread among birds and mammals with cemented bats forming the major evolutionary reservoir and ecological drivers of COV diversity (Li et al, 2005). The COV has posed frequent challenges during its course ranging from virus isolation, detection, and prevention to vaccine development (Guan et al,2019). COVID-19 has presented itself as a global pandemic in a short period resulting in a rapid curve shift of infected patients, increasing death rates, a huge global economic burden and widespread mobilization of medical resources across the globe. Being a novel disease (Umakanthan et al,2020). COVID-19 initially has been divided into four types: mild, moderate, severe, and critical cases (National Centre for Disease Control ,2020).

The corona virus is found both in humans and animals and it causes diarrhoea in human beings. COVID-2019 emerged from China in late December. It follows two other corona virus outbreaks, the SARS-CoV and the MERS-CoV. Corona viruses usually circulate among animals but sometimes can jump to humans (Holstein,2020).

The COVID-19 outbreak has generated significant pandemic worry. Pandemic worry refers to the constant, uncontrollable, and unwanted negative thoughts about the future outcomes of disease (Scherr et al, 2017).

Rsearch Methodology

Here, we modified Jummy et al (2021) model and come up with a new compartmental model which is governed by a set of differential equations. The differential equations are formulated by adding two compartments on the models by Jummy et al. (2021). The two compartments are vaccination and Quarantine compartments. We then check our model equations to see how robust our model is by finding the invariant region, positivity of the solution, existence and uniqueness of the solution.

The Equations of the modified model

$$\frac{dS}{dt} = \pi - (1-p_1)(\beta_2 I_A + S\beta_3 I_S)S - (1-p_2)S\beta_1 V + ZV_c + nQ + R\varpi_1 - (\Gamma + \mu_1)S \tag{1}$$

$$\frac{dE}{dt} = (1-p_1)(\beta_{DA} I_A + \beta_{DS} I_S)S + (1-p_2)S\beta_1 V - (\delta + \varpi_2 + \mu_1)E \tag{2}$$

$$\frac{dI_A}{dt} = (1-r)\delta E - (\rho_A + \gamma_A + \omega_1 + \mu_1)I_A \tag{3}$$

$$\frac{dI_S}{dt} = r\delta E - (\rho_S + \mu_S + \gamma_S + \omega_2 + \mu_1)I_S \tag{4}$$

$$\frac{dI_H}{dt} = \alpha_A \gamma_A I_A + \alpha_S \gamma_S I_S + \nu I_W - (\rho_H + \mu_H + \mu_1)I_H \tag{5}$$

$$\frac{dI_W}{dt} = (1-\alpha_A)\gamma_A I_A + (1-\alpha_S)\gamma_S I_S + mQ - (\nu + \rho_W + \mu_W + \mu_1)I_W \tag{6}$$

$$\frac{dR}{dt} = \rho_A I_A + \rho_S I_S + \rho_H I_H + \rho_W I_W - (\varpi_1 + \mu_1)R \tag{7}$$

$$\frac{dV}{dt} = \omega_1 I_A + \omega_2 I_S - (\varphi + \mu_1)V \tag{8}$$

$$\frac{dQ}{dt} = \varpi_2 E - (n + m + \mu_1)Q \tag{9}$$

$$\frac{dV_C}{dt} = \Gamma S - (Z + \mu_1)V_C \tag{10}$$

Existence of Positive Invariant Region

It is important to show that all the state variables of COVID 19 model of system (1) to (10) are non-negative for all $t > 0$ and so the positive invariant region of the model can be obtained on Theorem 1.

Theorem 1:

Solution of the COVID-19 model of equations (1)-(10) is feasible for all time $t \geq 0$ provided that they fall within the invariant region $\Omega. \subset R^{10}$

Proof:

Let $\Omega = \{S, E, I_A, I_S, I_H, I_W, R, V, Q., V_C\} \in R^{10}$ be any solution of the system of equations (10)-(19) such that

$$(S^\circ \geq 0, E^\circ \geq 0, I_A \geq 0, I_S \geq 0, I_H \geq 0, I_W \geq 0,$$
$$R \geq 0, V \geq 0, Q \geq 0, V_c^\circ \geq 0).$$

In the absence of disease, that is

$$(E = I_A, I_S = I_H = I_W = V = 0)$$

the total population becomes

$$N(t) = S(t) + E(t) + I_A(t) + I_S(t) + I_H(t) + I_W + R(t) + V(t) + Q(t) + V_C(t)$$

$$\frac{dN_h}{dt} \le \pi - \mu_1 N_h \tag{11}$$

$$\frac{dN_h}{dt} + \mu_1 N_h \le \pi \tag{12}$$

The integrating factors of the above is $e^{\mu_1 t}$.

Multiplying both sides of the equation (21) by $e^{\mu_1 t}$

$$e^{\mu_1 t}\left(\frac{dN_h}{dt} + \mu_1 N_h\right) \le \pi e^{\mu_1 t} \tag{13}$$

$$\frac{d}{dt}(N_h e^{\mu_1 t}) \le \pi e^{\mu_1 t}$$

$$d(N_h e^{\mu_1 t}) \le \pi e^{\mu_1 t} \tag{14}$$

Integrating both side of (14)

$$\int d(N_h e^{\mu_1 t}) \le \int \pi e^{\mu_1 t} dt$$

$$N_h e^{\mu_1 t} \le \frac{\pi}{\mu_1} e^{\mu_1 t} + C \tag{15}$$

Dividing through equation (15) by $e^{\mu_1 t}$

$$N_h \le \frac{\pi}{\mu_1} + C e^{\mu_1 t} \tag{16}$$

Applying the initial condition $N_h(0) = N_h(0) \ at \ t = 0$

We have;

$$N_h(0) \le \frac{\pi}{\mu_1} + C \tag{17}$$

$$\Rightarrow N_h(0) - \frac{\pi}{\mu_1} \le C \tag{18}$$

Substituting (18) into (16)

$$N_h \leq \frac{\pi}{\mu_1} + (N_h(0) \frac{\pi}{\mu_1})e^{\mu_1 t} \tag{19}$$

By the theorem of differential inequality (Birkhoff and Rota, 1989), we conclude

$$0 \leq N_h \leq \frac{\pi}{\mu_1} \text{ as } t \to \infty. \tag{20}$$

Therefore, Ω is positively invariant

Positivity of the solution

The system in equation (1)-(10) is given below;

$$\frac{dS}{dt} = \pi - (1-P_1)S\beta_{DA}I_A - (1-P_1)S\beta_{DS}I_S - (1-P_2)S\beta_1 V + ZV_C + nQ +$$
$$R\varpi_1 - (\Gamma + \mu_1)S$$
$$\frac{dE}{dt} = (1-p_1)S\beta_{DA}I_A + (1-p_1)S\beta_{DS}I_S + (1-p_2)S\beta_1 V - \delta E - E\varpi_2 - \mu_1 E$$
$$\frac{dI_A}{dt} = (1-r)\delta E - (\rho_A + \gamma_A)I_A - (\omega_1 + \mu_1)I_A$$
$$\frac{dI_S}{dt} = r\delta E - (\rho_S + \mu_S + \gamma_S)I_S - (\omega_2 + \mu_1)I_S$$
$$\frac{dI_H}{dt} = \alpha_A\gamma_A I_A + \alpha_S\gamma_S I_S + \nu I_W - (\rho_H + \mu_H + \mu_1)I_H$$
$$\frac{dI_W}{dt} = (1-\alpha_A)\gamma_A I_A + (1-\alpha_S)\gamma_S I_S - (\nu + \rho_W + \mu_W + \mu_1)I_W + mQ$$
$$\frac{dR}{dt} = \rho_A I_A + \rho_S I_S + \rho_H I_H + \rho_W I_W - (\varpi_1 + \mu_1)R$$
$$\frac{dV}{dt} = \omega_1 I_A + \omega_2 I_S - \mu_1 V$$
$$\frac{dQ}{dt} = E\varpi_2 - (n + m + \mu_1)Q$$
$$\frac{dV_C}{dt} = \Gamma S - (Z + \mu_1)V_C$$

For the model $(1) - (10)$ to be mathematically and epidemiologically well posed, we need to prove that all the state variables are non-negative for all $t \geq 0$.

Theorem 2: Suppose that the initial data are given as

$$\{(S^{\circ} \geq 0, E^{\circ} \geq 0, I_A \geq 0, I_S \geq 0, I_H \geq 0, I_W \geq 0,$$
$$R \geq 0, V \geq 0, Q \geq 0, V_c^{\circ} \geq 0)\} \in \Omega$$

Then, the solution of the set $\{S, E, I_A, I_S, I_H, I_W, R, V, Q, V_C\}$ of the system of the model (1)-(9) is positive for all $t > 0$

Proof

Considering the first equations given as

$$\frac{dS}{dt} = \pi - (1-P_1)S\beta_{DA}I_A - (1-P_1)S\beta_{DS}I_S - (1-P_2)S\beta_1 V + ZV_C + nQ +$$
$$R\varpi_1 - (\Gamma + \mu_1)S,$$

We have that

$$\frac{dS}{dt} \geq -((1-P_1)\beta_{DA}I_A - (1-P_1)\beta_{DS}I_S - (1-P_2)\beta_1 V - (\Gamma + \mu_1))S \tag{21}$$

By the method of separation of variable, we have that

$$\frac{dS}{S} \geq -((1-P_1)\beta_{DA}I_A - (1-P_1)\beta_{DS}I_S - (1-P_2)\beta_1 V - (\Gamma + \mu_1))dt \tag{22}$$

Integrating both sides of the above equation, will give

$$\ln(S) \geq -((1-P_1)\beta_{DA}I_A - (1-P_1)\beta_{DS}I_S - (1-P_2)\beta_1 V - (\Gamma + \mu_1))t + C \tag{23}$$

Where C is the constant of integration.

Taking exponential of both sides, we obtain

$$S(t) \geq e^{-((1-P_1)\beta_{DA}I_A - (1-P_1)\beta_{DS}I_S - (1-P_2)\beta_1 V - (\Gamma + \mu_1))t + C} \tag{24}$$

$$S(t) \geq e^{-((1-P_1)\beta_{DA}I_A - (1-P_1)\beta_{DS}I_S - (1-P_2)\beta_1 V - (\Gamma + \mu_1))t} \times e^{C}$$

$$S(t) \geq e^{-((1-P_1)\beta_{DA}I_A-(1-P_1)\beta_{DS}I_S-(1-P_2)\beta_1 V-(\Gamma+\mu_1))t} \times K \quad (25)$$

Where k is equal to $K = e^{C}$.

At the initial condition, equation (25) becomes

$$S(0) \geq K. \quad (26)$$

Substituting equation (26) into (25), we have

$$S(t) \geq S(0)e^{-((1-P_1)\beta_{DA}I_A-(1-P_1)\beta_{DS}I_S\backslash-(1-P_2)\beta_1 V-(\Gamma+\mu_1))t} \geq 0 \quad (27)$$

From the second equation

$$E = (1-p_1)S\beta_{DA}I_A + (1-p_1)S\beta_{DS}I_S + (1-p_2)S\beta_1 V - \delta E - E\varpi_2 - \mu_1 E \quad (28)$$

$$\frac{dE}{dt} \geq -\left[\delta + \mu_1 + \varpi_2\right]E \quad (29)$$

Applying the method of separation of variables to (29)

$$\frac{dE}{E} \geq -\left[\delta + \mu_1 + \varpi_2\right]dt \quad (30)$$

Integrating (30), we have

$$LnE \geq -\left[\delta + \mu_1 + \varpi_2\right]dt + C \quad (31)$$

Taking exponential of (31)

$$E \geq e^{-\left[\delta+\mu_1+\varpi_2\right]+C}$$

That is

$$E \geq e^{-\left[\delta+\mu_1+\varpi_2\right]t} \bullet e^{C}$$, where $e^{C} = K$

$$E \geq e^{-\left[\delta+\mu_1+\varpi_2\right]t} \bullet K \quad (32)$$

Applying the initial condition, $t=0$ to (32) gives

$$E(0) \geq K \tag{33}$$

$$E = E(0)e^{-\lfloor \delta+\mu_1+\varpi_2 \rfloor}t \geq 0$$

From the third model equation

$$I_A = (1-r)\delta E - (\rho_A + \gamma_A)I_A - (\omega_1 + \mu_1)I_A \tag{34}$$

$$\frac{dI_A}{dt} \geq -(\rho_A + \gamma_A)I_A - (\omega_1 + \mu_1)I_A \tag{35}$$

Applying the method of separation of variables to (44)

$$\frac{dI_A}{I_A} \geq -\lfloor \rho_A + \gamma_A) + \omega_1 + \mu_1 \rfloor dt \tag{36}$$

Integrating (36), we have

$$LnI_A \geq -\lfloor \rho_A + \gamma_A)I_A - (\omega_1 + \mu_1 \rfloor + C \tag{37}$$

Taking exponential of (37)

$$I_A \geq e^{-(\rho_A+\gamma_A)I_A-(\omega_1+\mu_1)} \bullet e^C$$

$$i.e\, I_A \geq e^{-\lfloor \rho_A+\gamma_A+\omega_1+\mu_1 \rfloor}t \bullet K \text{, where } K = e^C$$

$$I_A \geq ke^{-\lfloor \rho_A+\gamma_A)+\omega_1+\mu_1 \rfloor}t \tag{38}$$

Applying the initial condition, $t=0$ gives

$$I_A(0) \geq K \tag{39}$$

Substituting (39) into (38)

$$I_A \geq I_A e^{-\lfloor \rho_A+\gamma_A)+\omega_1+\mu_1 \rfloor}t \geq 0 \tag{40}$$

From the forth model equation

$$I_S = r\delta E - (\rho_S + \mu_S + \gamma_S)I_S - (\omega_1 + \mu_1)I_S \tag{41}$$

$$\frac{dI_S}{dt} \geq -[\rho_S + \mu_S + \gamma_S + \omega_1 + \mu_1]I_S \tag{42}$$

Applying the method of separation of variables to (42)

$$\frac{dI_S}{I_S} \geq -[\rho_S + \mu_S + \gamma_S + \omega 1 + \mu_1]dt \tag{43}$$

Integrating (43) , we have

$$Ln\, I_S \geq -\|\rho_S + \mu_S + \gamma_S + \omega 1 + \mu_1\| + C \tag{44}$$

Taking exponential of (44)

$$I_S \geq e^{-[\rho_S + \mu_S + \gamma_S + \omega 1 + \mu_1] + C}$$

That is

$$I_S \geq e^{-[\rho_S + \mu_S + \gamma_S + \omega 1 + \mu_1]} . e^{C}$$

i.e $I_S \geq e^{-[\rho_S + \mu_S + \gamma_S + \omega 1 + \mu_1]t} \bullet K$

$$I_S \geq k e^{-[\rho_S + \mu_S + \gamma_S + \omega 1 + \mu_1]t} \tag{45}$$

Appling the initial condition, $t = 0$ to (45), gives

$$I_S(0) \geq K \tag{46}$$

Substituting (46) into (45), gives

$$I_S \geq I_S(0) e^{-|\rho_S + \mu_S + \gamma_S + \mu_1|t} \geq 0 \tag{47}$$

From the fifth model equation

$$I_H = \alpha_A \gamma_A I_A + \alpha_S \gamma_S I_S + \nu I_W - (\rho_H + \mu_H + \mu_1) I_H \tag{48}$$

$$\frac{dI_H}{dt} \geq -(\rho_H + \mu_H + \mu_1) I_H \tag{49}$$

Applying the method of separation of variables to (49)

$$\frac{dI_H}{I_H} \geq -\left[\rho_H + \mu_H + \mu_1\right]dt \tag{50}$$

Integrating (50), we have

$$LnI_H \geq -\left[\rho_H + \mu_H + \mu_1\right] + C \tag{51}$$

Taking exponential of (51)

$$I_H \geq e^{-\left[\rho_H + \mu_H + \mu_1\right]} \bullet e^C$$

$$I_H \geq e^{-\left[\rho_H + \mu_H + \mu_1\right]t} \bullet K$$

$$I_H \geq k e^{-\left[\rho_H + \mu_H + \mu_1\right]t} \tag{52}$$

Applying the initial condition, $t = 0$ to (52), gives

$$I_H(0) \geq K \tag{53}$$

Substituting (53) into (52), gives

$$I_H \geq I_H(0) e^{e^{-\left[\rho_H + \mu_H + \mu_1\right]}} t \geq 0 \tag{54}$$

From the sixth model equation

$$I_W = (1-\alpha_A)\gamma_A I_{AI} + (1-\alpha_S)\gamma_S I_S - (\nu + \rho_W + \mu_W) I_W - \mu_1 I_W + mQ$$

$$\frac{I_W}{dt} = (1-\alpha_A)\gamma_A I_{AI} + (1-\alpha_S)\gamma_S I_S + mQ - \left[(\nu + \rho_W + \mu_W) + \mu_1\right] I_W \tag{55}$$

$$\frac{dI_W}{dt} \geq -\left[(\nu + \rho_W + \mu_W) + \mu_1\right] I_W \tag{56}$$

Applying the method of separation of variables to (56)

$$\frac{dI_W}{I_W} \geq -\left[(\nu + \rho_W + \mu_W) + \mu_1\right] dt \tag{57}$$

Integrating (57), we have

$$Ln\, I_W \geq -\left[(\nu + \rho_W + \mu_W) + \mu_1\right] + C \tag{58}$$

Taking exponential of (58)

$$I_W \geq e^{-\left[(\nu+\rho_W+\mu_W)+\mu_1\right]} \bullet e^{C}$$

$$I_W \geq e^{-\left[(\nu+\rho_W+\mu_W)+\mu_1\right]t} \bullet K$$

$$I_W \geq k e^{-\left[(\nu+\rho_W+\mu_W)+\mu_1\right]t} \tag{59}$$

Applying the initial condition $t = 0$ to (59), gives

$$I_W \geq K \tag{60}$$

Substituting (6) into (59), gives

$$I_W \geq I_W(0) e^{-\left[(\nu+\rho_W+\mu_W)+\mu_1\right]t} \geq 0 \tag{61}$$

From the seventh model equation

$$R = \rho_A I_A + \rho_S I_S + \rho_H I_H + \rho_W I_W - (\varpi_1 + \mu_1)R \tag{62}$$

$$\frac{dR}{dt} \geq -\left[\varpi_1 + \mu_1\right]R \tag{63}$$

Applying the method of separation of variables to (63)

$$\frac{dR}{R} \geq -\left[\varpi_1 + \mu_1\right]dt \tag{64}$$

Integrating (64), gives

$$LnR \geq -\left[\varpi_1 + \mu_1\right] + C \tag{65}$$

Taking exponential of (65)

$$R \geq e^{-\left[\varpi_1+\mu_1\right]} + C$$

$$R \geq e^{-\left[\varpi_1+\mu_1\right]t} \bullet e^{C}$$

$$R \geq e^{-[\varpi_1 + \mu_1]t} \bullet K$$

$$R \geq k e^{-|\varpi_1 + \mu_1|t} \quad (66)$$

Applying the initial condition $t = 0$ to (66), gives

$$R \geq K \quad (67)$$

Substituting (67) into (66), gives

$$R \geq R(0) e^{-|\varpi_1 + \mu_1|t} \geq 0 \quad (68)$$

From the eight model equation

$$\frac{dV}{dt} = \omega_1 I_A + \omega_2 I_S - \mu_1 V \quad (69)$$

$$\frac{dV}{dt} \geq -\mu_1 V \quad (70)$$

Applying the method of separation of variables to (70)

$$\frac{dV}{V} \geq -\mu_1 dt \quad (71)$$

Integrating (71), gives

$$LnV \geq |-\mu_1| + C \quad (72)$$

Taking exponential of (72)

$$V \geq e^{-\mu_1 + C}$$

$$V \geq e^{-\mu_1} \cdot e^{C}$$

$$V \geq e^{[-\mu_1]t} \cdot K$$

$$V \geq k e^{[-\mu_1]t} \quad (73)$$

Applying the initial condition $t = 0$ to (73) gives

$$V \geq K \tag{74}$$

Substituting (74) into (73) gives

$$V \geq V(0)e^{\left[-\mu_1\right]t} \geq 0 \tag{75}$$

From the nine model equation

$$Q = \varpi_2 E - (nQ + mQ + \mu_1)Q \tag{76}$$

$$\frac{dQ}{dt} \geq -(n + m + \mu_1)Q \tag{77}$$

Applying the method of separation of variable to (77)

$$\frac{dQ}{Q} \geq -(n + m + \mu_1)dt \tag{78}$$

Integrating (78), we have

$$LnQ \geq -(n + m + \mu_1) + C \tag{79}$$

Taking exponential of (79)

$$Q \geq e^{-(n+m+\mu_1)} + C$$

$$Q \geq e^{-(n+m+\mu_1)} \cdot e^{C}$$

$$Q \geq e^{-(n+m+\mu_1)} \cdot K$$

$$Q \geq k e^{-(n+m+\mu_1)} \tag{80}$$

Applying the initial condition $t = 0$ to (80) gives

$$Q \geq K \tag{81}$$

Substituting (81) into (80)

$$Q \geq Q(0)e^{-(n+m+\mu_1)}t \geq 0 \tag{82}$$

From the ten model equation

$$\frac{dV_C}{dt} = S\Gamma - ZV_C - \mu_1 V_c \tag{83}$$

$$\frac{dV_C}{dt} \geq -(Z - \mu_1)V_c \tag{84}$$

Applying the method of separation of variables to (84)

$$\frac{dV_C}{V_C} \geq -(Z - \mu_1)dt \tag{85}$$

Integrating (85), gives

$$LnV_C \geq -(Z - \mu_1) + C \tag{86}$$

Taking exponential of (86)

$$V_C \geq e^{-(Z-\mu_1)} + C$$

$$V_C \geq e^{-(Z-\mu_1)} \cdot e^C$$

$$V_C \geq k e^{-(Z-\mu_1)} t \tag{87}$$

Applying the initial condition $t = 0$ into (87) gives

$$V_C \geq K \tag{88}$$

Substituting (88) into (87) gives

$$V_C \geq V_C(0) e^{-(Z-\mu_1)} t \geq 0 \tag{89}$$

From the solutions obtained in equations (1)-(9), we see that $(S, E, I_A, I_S, I_H, I_W, R, V, Q, V_C) > 0$, for all t>0.

Thus, we have established that COVID-19 model has positive invariant region and solution.

Existence and uniqueness of the solution of the modified model

In this study, we formulate a theorem on existence and uniqueness of solution for the model system in equation (1)-(10) below.

$$\frac{dS}{dt} = \pi - (1-P_1)S\beta_{DA}I_A - (1-P_1)S\beta_{DS}I_S - (1-P_2)S\beta_1 V + ZV_C + nQ +$$
$$R\varpi_1 - (\Gamma + \mu_1)S$$
$$\frac{dE}{dt} = (1-p_1)S\beta_{DA}I_A + (1-p_1)S\beta_{DS}I_S + (1-p_2)S\beta_1 V - \delta E - E\varpi_2 - \mu_1 E$$
$$\frac{dI_A}{dt} = (1-r)\delta E - (\rho_A + \gamma_A)I_A - (\omega_1 + \mu_1)I_A$$
$$\frac{dI_S}{dt} = r\delta E - (\rho_S + \mu_S + \gamma_S)I_S - (\omega_2 + \mu_1)I_S$$
$$\frac{dI_H}{dt} = \alpha_A\gamma_A I_A + \alpha_S\gamma_S I_S + \nu I_W - (\rho_H + \mu_H + \mu_1)I_H$$
$$\frac{dI_W}{dt} = (1-\alpha_A)\gamma_A I_A + (1-\alpha_S)\gamma_S I_S - (\nu + \rho_W + \mu_W + \mu_1)I_W + mQ$$
$$\frac{dR}{dt} = \rho_A I_A + \rho_S I_S + \rho_H I_H + \rho_W I_W - (\varpi_1 + \mu_1)R$$
$$\frac{dV}{dt} = \omega_1 I_A + \omega_2 I_S - \mu_1 V$$
$$\frac{dQ}{dt} = E\varpi_2 - (n + m + \mu_1)Q$$
$$\frac{dV_C}{dt} = \Gamma S - (Z + \mu_1)V_C$$

Theorem 3: Considering the system of equation (1)-(10);

The model system$(1)-(10)$, defined as

$\dfrac{df_i}{dx_j}$ $for\ i = j$ Should be write in compact form as

$$x_i' = f_i(t,x), x(t_0) = x_0, \ for\ set\ of\ function\ f_i,$$
$$for\ all\ f_i = f_1, f_2 \ldots\ldots f_{10}\ where\ i = 1,2,3,\ldots\ldots n \tag{90}$$

where,

$x = (S, E, I_A, I_S, I_H, I_W, R, V, Q, V_C)$.

And

$f(t,x) = (f_1(t,x), f_2(t,x), \ldots, f_{10}(t,x))^T$

$f_1 = \frac{dS}{dt}, f_2 = \frac{dE}{dt}, \ldots, f_{10} = \frac{dV_C}{dt}$.

If D' denotes the region $|t - t_0| \le a,\ \ |x - x_0| \le b,\ \ x = (x_1, x_2, \ldots x_n),\ \ x_0 = (x_{10}, x_{20}, \ldots x_{n0})$. And suppose that $f(t, x_1)$ satisfies the Lipshitz Condition $|f(t,x_1) - f(t,x_2)| \le k|x_1 - x_2|$ whenever the pair $f(t, x_1)$ and $f(t, x_2)$ belong to D' where k is positive constant, then there exists a constant $\delta > 0$ such that there exists a unique continuous vector solution X(t) of the system x' = f(t, x), as defined above, in the interval $|t - t_0| \le \delta$. It is important to note that the condition is satisfied by

the requirement that $\frac{\partial f_i}{\partial x_j},\ i, j = 1,2,\ldots$ be continuous and bonded in D'.

The existence and uniqueness of solution

Considering our model defined in equation (1)-(10), we are interested in the region $1 \le \alpha \le R.$ and we look for a bounded solution of the form

$0 \le R \le \infty$

Theorem 4

Let D denotes the region $0 \le \alpha \le R,$ then each of the equations of the system (1)-(10) have a unique solution which is bounded in the region D (Egbetade and Ibrahim,2012).

Proof:

We are saddled with the responsibility to show that $\frac{\partial f_i}{\partial x_j},\ i,j=1,2,3,4,5,6,7,8,9,10$ are continuous and bounded in D in order to be able to prove the existence and uniqueness of solution for the system of equations obtainable in our model.

From equations (1)

$$f_i(S,E,I_A,I_S,I_H,I_W,R,V,Q,V_c)$$

$$\frac{\partial f_1}{\partial S}=(S,E,I_A,I_S,I_H,I_W,R,V,D,Q,V_c)=\pi-(1-P_1)S\beta_{DA}l_A$$
$$-(1-P_1)S\beta_{DS}l_S-(1-P_2)S\beta_1 V+ZV_C+nQ+R\varpi_1-(\Gamma+\mu_1)S$$
$$=-(1-P_1)\beta_{DA}l_A-(1-P_1)\beta_{DS}l_S-(1-P_2)\beta_1 V-(\Gamma+\mu_1)$$
$$\left|\frac{\partial f_1}{\partial S}\right|=\left|-(1-P_1)\beta_{DA}l_A-(1-P_1)\beta_{DS}l_S-(1-P_2)\beta_1 V-(\Gamma+\mu_1)\right|<\infty \tag{91}$$

Also, from (2)

$$f_2(S,E,I_A,I_S,I_H,I_W,R,V,Q,V_c)$$
$$f_2=(S,E,I_A,I_S,I_H,I_W,R,V,Q,V_c)$$
$$\frac{\partial f_2}{\partial E}=(1-p_1)S\beta_{DA}I_A+(1-p_1)S\beta_{DS}I_S+(1-p_2)S\beta_1 V-\delta E-E\varpi_2-\mu_1 E$$
$$-\delta-\varpi_2-\mu_1$$
$$\left|\frac{\partial f_2}{\partial E}\right|=\left|-\delta-\varpi_2-\mu_1\right|<\infty \tag{92}$$

From equation (3)

$$f_3(S, E, I_A, I_S, I_H, I_W, R, V, Q, V_c)$$

$$\frac{\partial f_3}{\partial I_A} = (1-r)\delta E - (\rho_A + \gamma_A)I_A - (\omega_1 + \mu_1)I_A$$

$$= -(\rho_A + \gamma_A + \omega_1 + \mu_1)$$

$$\left|\frac{\partial f_3}{\partial I_A}\right| = \left|-(\rho_A + \gamma_A + \omega_1 + \mu_1)\right| < \infty \tag{93}$$

From equation (4)

$$f_4(S, E, I_A, I_S, I_H, I_W, R, V, Q, V_c)$$

$$\frac{\partial f_4}{\partial I_S} = r\delta E - (\rho_S + \mu_S + \gamma_S)I_S - (\omega_2 + \mu_1)I_S$$

$$-(\rho_S + \mu_S + \gamma_S + \omega_2 + \mu_1)$$

$$\left|\frac{\partial f_4}{\partial I_S}\right| = \left|-(\rho_S + \mu_S + \gamma_S + \omega_2 + \mu_1) < \infty\right| \tag{94}$$

From equation (5)

$$f_5(S, E, I_A, I_S, I_H, I_W, R, V, Q, V_c)$$

$$\frac{\partial f_5}{\partial I_H} = \alpha_A \gamma_A I_A + \alpha_S \gamma_S I_S + \nu I_W - (\rho_H + \mu_H + \mu_1)I_H$$

$$= -(\rho_H + \mu_H + \mu_1)$$

$$\left|\frac{\partial f_5}{\partial I_H}\right| = \left|-(\rho_H + \mu_H + \mu_.)\right| < \infty \tag{95}$$

From equation (6)

$$f_6(S,E,I_A,I_S,I_H,I_W,R,V,Q,V_c)$$
$$\frac{\partial f_6}{\partial I_W}=(1-\alpha_A)\gamma_A I_A+(1-\alpha_S)\gamma_S I_S-(\nu+\rho_W+\mu_W+\mu_1)I_W+mQ$$
$$=-(\nu+\rho_W+\mu_W+\mu_1)$$
$$\left|\frac{\partial f_6}{\partial I_W}\right|=\left|-(\nu+\rho_W+\mu_W+\mu_1)\right|<\infty \tag{96}$$

From equation (7)

$$f_7(S,E,I_A,I_S,I_H,I_W,R,V,Q,V_c)$$
$$\frac{\partial f_7}{\partial R}=\rho_A I_A+\rho_S I_S+\rho_H I_H+\rho_W I_W-(\varpi_1+\mu_1)R$$
$$=-(\varpi_1+\mu_1)$$
$$\left|\frac{\partial f_7}{\partial R}\right|=\left|-(\varpi_1+\mu_1)\right|<\infty \tag{97}$$

From equation (8)

$$f_8(S,E,I_A,I_S,I_H,I_W,R,V,Q,V_c)$$
$$\frac{\partial f_8}{\partial V}=\omega_1 I_A+\omega_2 I_S-\mu_1 V$$
$$=-\mu_1$$
$$\left|\frac{\partial f_8}{\partial V}\right|=\left|-\mu_1\right|<\infty \tag{98}$$

From equation (9)

$$f_9(S, E, I_A, I_S, I_H, I_W, R, V, Q, V_c)$$

$$\frac{\partial f_9}{\partial Q} = E\varpi_2 - (n + m + \mu_1)Q$$

$$= -(n + m + \mu_1)$$

$$\left|\frac{\partial f_9}{\partial Q}\right| = \left|-(n + m + \mu_1)\right| < \infty \tag{99}$$

From equation (10)

$$f_{10}(S, E, I_A, I_S, I_H, I_W, R, V, Q, V_c)$$

$$\frac{\partial f_{10}}{\partial V_c} = \Gamma S - (Z + \mu_1)V_C$$

$$= -(Z + \mu_1)$$

$$\left|\frac{\partial f_{11}}{\partial V_c}\right| = \left|-(Z + \mu_1)\right| < \infty \tag{100}$$

Hence, since clearly the Partial Derivatives for the set of function $f_i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ with respect to $(S, E, I_A, I_S, I_H, I_W, R, V, Q, V_c)$, created from equation (1)-(10) are continuous and they exist for the system of the model equations proposed; and they are also bounded, then we can remark according to Theorems 3 and 4 that, there exists a unique solution for the system of equations of our model in (1)-(10) in the region D as defined.

From the above test, it is proven that the model equation has a unique solution and that the model exists.

Results

Here, we present the findings on the basic properties of the model equations (1)-(10) as follows;

Basic properties of the model

(1) The close-set Ω of the model is shown to be positively invariant (Theorem 1).

(ii) The state variables of the model (1)-(10) are proved to be non-negative for all $t \geq 0$ (Theorem 2).

(iii) The existence and uniqueness of solution of the model is established and shown to stay in the region Ω for all $t \geq 0$ (Theorem 3 and Theorem 4).

Conclusion

COVID-19 started with massive death all over the world which made people think that the world has come to an end. From this study, we investigated the positivity, existence and uniqueness of the modified COVID-19 model. We present the existence, uniqueness and positivity of the model solution. It is proved that the solution exists and that the solution is unique. This shows that the modified model is mathematically and epidemiologically wee posed, meaning that it is a good model for studying COVID-19.

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