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On the Application of Exponentiated Burr V Distribution and Its Extension

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Abstract

In this research, the application of Exponentiated Burr-V (EBV) distribution is presented and compared with other competing distributions which include Exponentiated Pareto distribution, Exponentiated Lomax distribution, Exponentiated Gumbel distribution, and Exponentiated Generalized Inverse Exponential distribution. The maximum likelihood estimation method was used in estimating the EBV and the four competing distributions. The loglikelihood (LL) and Akaike Information Criteria (AIC) was applied for determining the best fitted distribution and the distribution with the largest LL and smallest AIC is considered the best fitted distribution. The data used is on bladder cancer which is a widely used data from Lee and Wang (2003), Lemonte and Cordeiro (2011), Luz, (2012), and Kazeem *et al.,* (2014). The results obtained showed that the EBV distribution has the largest LL value of 2950.726 and the smallest AIC value of –5895.452. The LL and AIC values imply that the EBV distribution is a very competitive distribution in fitting the bladder cancer data and it is an appropriate distribution for fitting asymmetric or negatively skewed and high kurtosis datasets.

Keywords: Burr V, Maximum likelihood, Exponentiated, AIC, Loglikelihood

INTRODUCTION

The idea of exponentiated distribution was introduced by Gupta *et al.* (1998). Person & Jesper (2010) proposed the Exponentiated Gumbel (EG) distribution. They compared Gumbel and the Exponentiated Gumbel on the estimation of returns levels of significant wave height. Due to the nature of the data collected shows evidence of extreme values. From the empirical results, they concluded that exponentiated Gumbel gives a closet fit than Gumbel and recommends the uses of EG for estimation of returns levels of significant wave height. Hasnain *et al.*, (2015) proposed Exponentiated Moment Exponential (EME) distribution. They provided various proved of statistical and mathematical properties while the maximum likelihood estimator (MLE) were used and fitted to an artificial and real dataset. Raja & Mir (2011) conduct an empirical study of eight distribution such as the Exponentiated Weibull (EW), Exponentiated Exponential (EE), Exponentiated Lognormal (EL), Lognormal, Weibull, Gamma, Exponentiated Gumbel (EG) and Gumbel using two-lifetime data. Exponentiated distributions were subjected to one of the two datasets while lognormal, Gumbel, Gamma, Weibull was estimated using the other dataset. From the results, EL and EE were better.

Dara & Ahmad (2012) studied various moment distributions and developed some basic properties such as Moments, Skewness, Kurtosis, hazard function, Moment Generating Function (MGF) of Exponentiated Moment Exponential and Exponentiated Weighted Exponential distribution. Dikko & Agboola (2017) worked on exponentiated skewed student t distribution. They proposed the distribution and also derived some statistical properties such as the CDF, Hazard function, Survival function, quantile function, rth Moment, and Characteristic function. Andrade *et al.* (2015) proposed a four parameters model within this class of the exponentiated generalized Gumbel distribution. They obtained explicit expressions for the ordinary moments, generating and quantile functions, mean deviations, Bonferroni and Lorenz curves and Rényi entropy. The density function of the order statistic was derived and the method of maximum likelihood is used to estimate model parameters. Cordeiro *et al.,* (2013) proposed the exponentiated generalized class distribution. They propose a new method of adding two parameters to a continuous distribution that extends the idea first introduced by Lehmann (1953) and studied by Nadarajah and Kotz (2006). They derive some mathematical properties including the ordinary moments, generating function, mean deviations and order statistics.

Also, Mathew *et al.* (2024) introduced a one parameter entropy transformed exponential distribution where some properties of the distributions were estimated and found to be a stable and competing distribution in fitting left and right skewed data sets. Also, David *et al.* (2024) introduced the new sine inverted exponential distribution and a reliability analysis with new sine inverse Rayleigh distribution was performed by David *et al.* (2023). In this research, the application of the EBV distribution which was theoretically introduced by Idi & Agboola (2020) is carried out by fitting it to bladder cancer data.

METHODS

Probability distribution of Burr V (BV) distribution

$$g(x) = \alpha\beta e^{-\tan x}\sec^2 x\left(1+\beta e^{-\tan x}\right)^{-\alpha-1} \tag{1}$$

Probability distribution of Exponentiated Burr V (EBV) distribution (Idi & Agboola, 2020)

$$f(x) = \alpha\beta\gamma e^{-\tan x}\sec^2 x\left(1+\beta e^{-\tan x}\right)^{-\alpha(\gamma-1)}\left(1+\beta e^{-\tan x}\right)^{-\alpha-1} \tag{2}$$

Cumulative distribution of EBV

A random variable X is said to have an exponentiated distribution function if its distribution function (pdf) is given as

$$F(x) = \left(1+\beta e^{-\tan x}\right)^{-\alpha\gamma} \tag{3}$$

The plotted probability density function (PDF) and cumulative density function (CDF) for different parameter values are presented in Figure 1 and Figure 2.

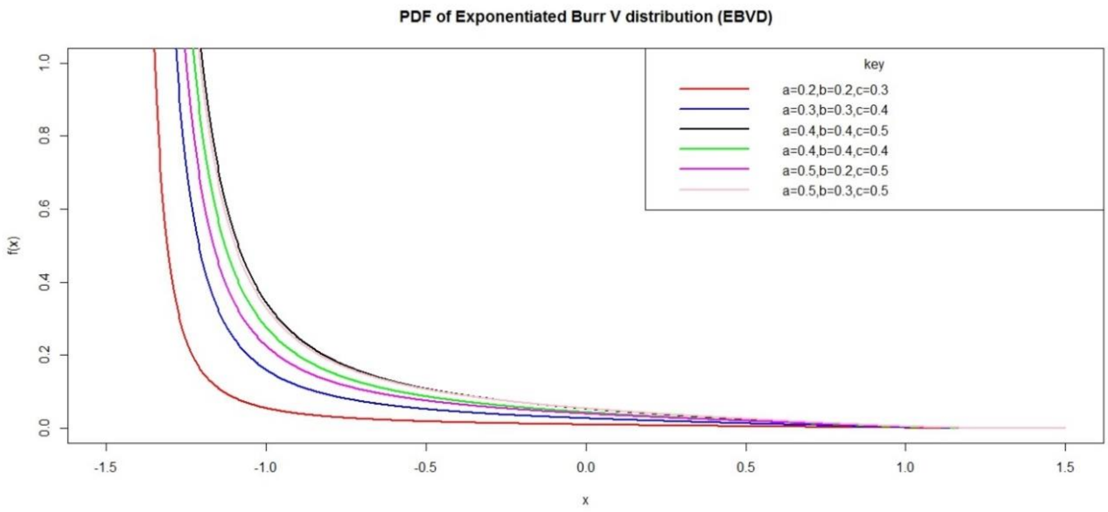

Figure 1. The PDF Plot of EBV Distribution

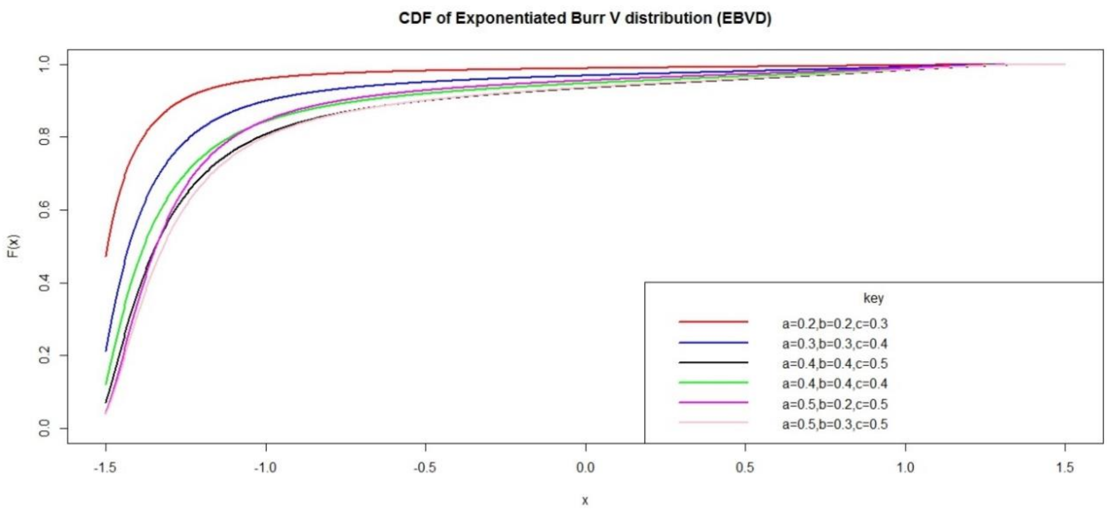

Figure 2. The CDF Plot of EBV Distribution

Where, a and β are the location parameters and γ is the shape parameter of the EBV distribution in equation (1) and (2).

The estimation of the survival function and hazard functions can be found in Idi & Agboola (2020). Also, other properties like the quantile function, moments, moment generating function, characteristics function, order statistics, and asymptoticity of the EBV were estimated.

Table 1: The Competing Distributions PDF

Probability distribution of Exponentiated Pareto (EP) distribution $f(x)=\alpha\beta^{\gamma}\gamma\left[1-\left(\frac{\beta}{x}\right)^{\gamma}\right]^{\alpha-1}x^{-\gamma-1}$
Exponentiated Lomax (EL) distribution $f(x)=\alpha\beta\lambda\left[1-(1+\lambda x)^{-\beta}\right]^{\alpha-1}(1+\lambda x)^{-(\beta+1)}$
The probability distribution of Exponentiated Gumbel (EG) distribution $f(x)=\alpha\beta x^{\alpha-1}Exp\left(-\beta x^{-\alpha}\right)$
Probability distribution of Exponentiated Generalized Inverse Exponential (EGIE) distribution $f(x)=\alpha\beta\gamma x^{-2}Exp\left(-\frac{\gamma}{x}\right)\left[1-Exp\left(-\frac{\gamma}{x}\right)\right]^{\alpha-1}*\left[1-\left(1-Exp\left(-\frac{\gamma}{x}\right)\right)^{\alpha}\right]^{\beta-1}$

MLE of the Parameters of the distribution and its extensions

Let $X_1, X_2,...X_n$ be a random sample from a population X with probability density function $f(x:\theta)$ where θ the parameters with unknown. The likelihood function $L(\theta)$ is defined as the product of the joint density of the random variables $X_1, X_2,...X_n$ given as

$$L(\theta)=\prod_{i=1}^{n}f(x;\theta) \tag{4}$$

The value of θ that maximizes the likelihood function $L(\theta)$ is called the maximum likelihood estimators of θ which is denoted by $\hat{\theta}$.

$$L(x:\alpha,\beta,\gamma)=\prod_{i=1}^{n}g(x:\alpha,\beta,\gamma) \tag{5}$$

Model Selection

The Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) is used for model selection and is defined as:

$$\text{AIC} = -2\log \Lambda(L) + 2\Lambda \tag{6}$$

where, L is the likelihood of the model and $\mathit{\Lambda}$ is the number of parameters in the model

$$\text{BIC} = -2 \log \mathit{\Lambda}(L) + \mathit{\Lambda} \log(n) \tag{7}$$

where, n is the sample size.

Application of real-life data

A widely used data from Lee & Wang (2003), Lemonte & Cordeiro (2011), and Kazeem *et al.,* (2014) given below is used in estimating the parameters of the EBV distribution and other competing distributions. The data is on remission times (in months) of a random sample of 128 patients suffering from bladder cancer.

0.08, 2.09, 3.48, 4.87, 6.94 , 8.66, 13.11, 23.63, 0.20, 2.23, 3.52, 4.98, 6.97, 9.02, 13.29, 0.40, 2.26, 3.57, 5.06, 7.09, 9.22, 13.80, 25.74, 0.50, 2.46 , 3.64, 5.09, 7.26, 9.47, 14.24, 25.82, 0.51, 2.54, 3.70, 5.17, 7.28, 9.74, 14.76, 26.31, 0.81, 2.62, 3.82, 5.32, 7.32, 10.06, 14.77, 32.15, 2.64, 3.88, 5.32, 7.39, 10.34, 14.83, 34.26, 0.90, 2.69, 4.18, 5.34, 7.59, 10.66, 15.96, 36.66, 1.05, 2.69, 4.23, 5.41, 7.62, 10.75, 16.62, 43.01, 1.19, 2.75, 4.26, 5.41, 7.63, 17.12, 46.12, 1.26, 2.83, 4.33, 5.49, 7.66, 11.25, 17.14, 79.05, 1.35, 2.87, 5.62, 7.87, 11.64, 17.36, 1.40, 3.02, 4.34, 5.71, 7.93, 11.79, 18.10, 1.46, 4.40, 5.85, 8.26, 11.98, 19.13, 1.76, 3.25, 4.50, 6.25, 8.37, 12.02, 2.02, 3.31, 4.51, 6.54, 8.53, 12.03, 20.28, 2.02, 3.36, 6.76, 12.07, 21.73, 2.07, 3.36, 6.93, 8.65, 12.63, 22.69

RESULTS AND DISCUSSION

Descriptive statistics

Table 2 shows the summary statistic of the bladder cancer patients data. From the results of the statistics, it can be observed that the data is negatively skewed with a value of -0.1315 and high kurtosis with the value of 3.2230. Also, the variance and standard deviation are far from the mean and median.

Table 2. Summary Statistics of Bladder Cancer Patients

Statistic	Value
Mean	2.760
Median	2.835
Variance	0.7946
Standard deviation	0.8915
Minimum	0.390
Maximum	4.900
1st Qua.	2.178
3rd Qua.	3.277
Skewness	-0.1315
Kurtosis	3.2230

Parameter Estimates and Goodness of Fit Test

Table 3 presents the MLE parameter estimates and goodness of fit results. It can be observed that the EBV performed better than the other competing distribution in fitting the bladder cancer patient data set because it has the largest LL value, smallest AIC and BIC values compared to the other competing distributions.

Table 3. Parameter Estimation of EBV Distribution and Its Extensions

Distribution	Estimates	LL	AIC	BIC	Rank
BV	$\alpha = 0.00917$ $\beta = 5.689$ $\gamma = 1.000$	1215.394	-2424.79	-7279.72	4
EBV	$\alpha = 9.848$ $\beta = 4.378$ $\gamma = 4.614$	2950.726	-5895.45	-17691.7	1
EGD	$\alpha = 2.728$ $\beta = 1.155$	2448.2	-4896.4	-9784.37	2
EPD	$\alpha = 27.8242$ $\beta = 2.9342$	61.89078	-119.782	-239.134	5
EGIED	$\alpha = 1.096$ $\beta = 1.929$ $\gamma = 2.712$	2119.03	-4232.06	-12701.5	3
ELD	$\alpha = 6.430$ $\beta = 1.597$ $\gamma = 9.437$	-95.5828	197.1656	586.1399	6

CONCLUSION

In this study, the application of EBV is presented and compared with five other competing distributions. The parameter estimates are obtained through the method of MLE and the best fitted distribution was determined using *LL* and information criteria (AIC and BIC). The model corresponding to the smallest AIC and BIC is regarded as the best fitted distriution. In this case, the EBV distribution has the largest LL value of 2950.726, smallest AIC value of -5895.452, and the smallest BIC value of -1.7691.7. The new distribution is highly competitive and performed better for fitting the bladder cancer patients data and based on this it can be concluded that the EBV distribution is an appropriate distribution for fitting asymmetric or negatively skewed and high kurtosis datasets.

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