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# A Solution of Airy Differential Equation Via Elzaki Transform

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**Abstract**

The Elzaki transform provides an elegant method to solve the Airy differential equation, leveraging its unique properties to handle the mixed derivative and variable coefficient terms. While the traditional methods remain standard, the Elzaki transform offers a powerful alternative, particularly when coupled with numerical techniques.

**Keywords:** Airy Function; Elzaki Transform; Electromagnetic Theory

## INTRODUCTION

George Biddel Airy discovered a classical equation of mathematical physics which is named with her title name Airy which is Airy differential equation. This Airy equation is applicable in modeling of diffraction of light in rainbow and optics problem.

The Airy differential equation is a special type of Schrödinger's equation. The Airy differential equation formed from Airy function which is very much applicable in microscopy and astronomy producing a point source of light due to diffraction and interference (Airy,1838). Airy function is denoted by Ai is a special function. The functions Ai(x) and Bi(x) are the Airy functions

$$Y= c_1 Ai + c_2 \text{ Bi} \tag{1}$$

The function Ai(x) and the related function Bi(x) are linearly independent solutions to the differential equation, $\frac{d^2y}{dx^2} - xy = 0$ (2)

which is Airy equation or stokes equation. It is mainly second order linear ordinary differential equations e.g. $\ddot{y}$−xy=0 or x"-xy=0

$$x^2y'' + 2xy' - \frac{y}{x} = 0$$

The Airy equation in the complex plane z is w"-z w = 0

This equation can be expressed in integral form as

Y(x) $= c_1 Ai(x) + c_2 Bi(x)$where Airy's function are defined by the improper Riemann integral

$$\text{Ai(x)} = \frac{1}{\pi}\int_0^\infty cos\ cos\left(\frac{t^3}{3} + xt\right)\ \text{dt}$$

$$\text{Bi(x)} = \frac{1}{\pi}\int_0^\infty \left[exp\left(\frac{-t^3}{3} + xt\right) + sin\ sin\left(\frac{t^3}{3} + xt\right)\right] dt$$

**Airy's equation**

The equation of the form y"- t y=0 which is used in physics to model the diffraction of light. The homogenous second order linear differential equation

Y" + p(t) y'+ q(t) y=0 and y"+ xy'+3y=0

(3) where p(t) and q(t ) are function of x and y not constant. Other examples of Airy equations. Laplace's equation is given by $\nabla^2$ R=0 is Laplace operator is also Airy equation. The Airy function $\nabla^2\ \emptyset = \boldsymbol{\rho}\nabla^2$ ∅ $= \boldsymbol{\rho}$ is bi harmonic linear equation.

**Literature Review**

Ansari (2015) has evaluated the fractional Airy transform with the higher order derivatives of the Airy function and the Airy polynomials. This Airy transform in respect of scaling parameter. He has described some of its properties.

Widder (1979) discovered a transform that is called Airy transform which is capable in solving almost Airy equations.

Khan and shah (2015) analytically solved Airy differential equation jointly with natural transform and homotopy perturbation method.

The integral transforms such as Natural transform, Fourier, Mellin, Hankel and Sumudu were extensively used to solve differential equations. Kevser Koklu has solved Airy equation with Natural transform and obtained its function. (Koklu, 2021).

In this article, Airy differential equation is solved with use of Elzaki transform which was recently introduced by Tarig Elzaki (2011). This transformation has been applied in solving ordinary differential equation and partial differential equation.

**Objective**

The objective of this research is to apply the Elzaki transform method to the Airy differential equation.

## METHODS

In this work, Elzaki transform (ET) and its properties is applied to the Airy differential equation for explaining physical phenomenon in terms of an effective analytical form.

**Elzaki Transform**

ELzaki transform defined for functions of exponential order, is in the set A defined by

$$A=\{f(t): \quad \exists M, \ k_1 and k_{2>0\,;|f(t)|<} M e^{|t|} \quad j, \quad t \in -1 \times (0,\infty)\} \tag{4}$$

The Elzaki transform denoted by the operator A $(.)$ defined by the integral equations.

$$T[f(t)] \quad = T(v) = v\int_0^\infty f(t)\, e^{-t/v} \ dt = T(V) \quad t>0 \tag{5}$$

ELzaki transform may be used to solve problems without resorting to the frequency domain. In fact, ELzaki transform which is itself linear preserves linearity (Elzaki, et al. 2012).

**Properties of Elzaki transform**

Theorem1: (Elzaki, et al. 2012). (Convolution) Let F (x, v) and G (x, v) are the Elzaki transform of f (t) and g(t), respectively. Both defined in set A, then

$ET^+ \, [f * g]$ = v F(x, v) G(x, v), where $f * g$ is convolution of two functions defined by

$\int_0^t f(a)g(t-a)\ da = \int_0^t f(t-a)\ g(a)da$ , where ET is short form of Elzaki transform

Theorem2:( Inverse of ET), T(x, x) is the ET of function f(t) in set A, then $[ET]^{-1}$ is defined by $[ET]^{-1}$ [T(x, v]=f(t)= $\int_{Y-IT}^{Y+IT} e^{\frac{t}{v}} T(x,v)\ dx$

where integral is taken along T= Y I n the complex plane T= x + i y. The real number Y is chosen so that T= Y lies on right of all (finite) or count ably infinite singularities.

## RESULTS AND DISCUSSION

The Airy function defined by the integral

W(m) = $\int_0^\infty co\,s\left[\frac{\pi}{2}(w^3 - mw)\right]\ dw$ is the solution of w" $+\frac{\pi^2}{12}$ mw=0

(6)

Suppose the Airy equation $u_{xx} + u_{yy}$ =0

(7 )

u(x,0) = 0, $u_y$(x,0) = $cos\,cos\,x$ , x, y $> 0$

The Elzaki transform of the function f(t) is given

E$[f(t)]$ $= E = v\int_0^\infty f(t)\, e^{-t/v}\ \ dt = T(V) \quad t > 0$

(8)

Let T(v) be the Elzaki transform of u. Then taking Elzaki transform of equation (6)

$\frac{T(x,v)}{v^2}$ -u(x,0)-v$u_y$(x,0) +$\ddot{T}$ (x,0) = 0

$v^2\,\ddot{T}$ (x, v) +T(x,v)= $v^3\ cos\,cos\,x$

(9)

T (x, v) =$\frac{v^3\ coscos\,x}{1-V^2}$

If taking the inverse Elzaki transform equation (9), the solution of equation (7) will be

u(x y)= $cos\,cos\,x\ \ sin\,sin\,hy$ .

Now taking another Airy equation y"-x y=0, and applying ET to both sides of this equation

Gives results ET[y"]-ET[x y] = ET[0] then

$\frac{1}{v^4}T(v,\frac{1}{v}) - \frac{1}{v^3}$ y (0) $-\frac{1}{v}$ y'(0) $- v\frac{d}{dv}$(v T (v,$\frac{1}{v}$)=0

(10)

If y (0) =$c_1$ and y'(0) = $c_2$ in equation (10), then

i.e. $\frac{c_1}{v^6} - \frac{c_2}{v^5}$, on solving this gives

$$T(v) = \frac{1}{v}e^{-\frac{1}{v^6}}\left(c\left(\frac{1}{v}\right) + \frac{\frac{c_1}{v^2}\left(\Gamma\left(\frac{2}{3}\right) - \gamma\left(\frac{2}{3}, -\frac{1}{3v^6}\right)\right)}{3v^2\left(-\frac{1}{v^6}\right)^{\frac{2}{3}}} - \frac{\frac{c_2}{v}\left(\Upsilon\left(\frac{1}{3}\right) - \frac{1}{3v^6}\right) - \frac{2\pi\sqrt{3}}{3\Gamma\frac{2}{3}}}{3v\left(-\frac{1}{v^6}\right)^{\frac{1}{3}}}\right) \tag{11}$$

Where $\Upsilon$ is incomplete gamma function and c(s) = 0, on simplification

$$T(v) = \frac{1}{v3^{\frac{2}{3}}}e^{-\frac{1}{3v^6}}\left(c_1 3^{\frac{1}{3}}\left(\Gamma\left(\frac{2}{3}\right) - \gamma\left(\frac{2}{3}, -\frac{1}{3v^6}\right)\right) + c_2\left(\gamma\left(\frac{1}{3} - \frac{1}{3v^6}\right) - \frac{2\pi\sqrt{3}}{3\Gamma\frac{2}{3}}\right)\right) \tag{12}$$

By applying the inverse ET to T(V) gives

$[ET]^{-1}$T(v)= y(x)

(13)

Y(x) = $c_1 Ai(x) + c_2 Bi(x)$ where AI and Bi are Airy functions at the same time, it is equivalent to the Maclaurin series of the Airy function (Kumar, 2024, Kumar, et al.2023, 2024).

## CONCLUSION

In this work, Elzaki transform based on Natural transform is implemented to solve Airy differential equation and is capable and alternative method in finding an exact solution for the Airy differential equation.

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