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Dynamic Economic Analysis from Difference Equation

Nand Kishor Kumar
Tri-Chandra Campus, Kathmandu, Tribhuvan University, Nepal
nandkishorkumar2025@gmail.com

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Abstract

Dynamic economic analysis often involves understanding how economic variables evolve over time. One approach to modeling these dynamics is through the application of difference equations. Difference equations provide a framework for describing the evolution of economic variables in discrete time periods, making them particularly useful for analyzing time series data and economic processes that unfold in steps. In dynamic economic analysis, difference equations are an effective tool for modeling and comprehending how economic variables change with time. These difference equations are used by to study the behavior with economic systems and forecast future trends. Examples of these systems include market dynamics in the cobweb model and capital accumulation in the Solow growth model. Understanding the long-term behavior of complex systems is further aided by stability analysis, which guarantees that the models offer insightful information about economic processes. Stability analysis further aids in understanding the long-term behavior of these systems, ensuring that the models provide meaningful insights into economic processes.

Keywords: Difference Equation, Finite Difference, Iteration Method, Solow Method

Introduction

Introduction some economic variables change due to the change in time. The time considered may be discrete or continuous. In most of the economic studies, focus is given to these economic variables in which we consider the discrete time. For examples; today ' s price of an item depends upon the demand of the same item in the previous day ' s time. The expenditure of a man for a particular month depends upon the same item in the previous day; s time. The expenditure of a man for a particular mother depends upon the previous month's income. The present population of a certain country is estimated according to the population of the same country in a decade ago. These are the examples of dynamic situation but not the static (Allen,1959).

To get the values of an unknown function y(x) of various discrete values of x, uses a difference equation. We derive a function y(x) for any value of x, fulfills the equation. Differential and difference equations are precisely the same; the only distinction is that the former is applicable when the independent variable is continuous, while the latter is only applicable when it takes only discrete values.

The solution of continuous time will be solved by the help of differential equation. But the solution of the discrete time will be solved by the method of finite difference where finite difference is the difference of variety values between two successive intervals of time.

In the field of economics, there are instances where the independent variable is discrete rather than continuous. In other words, x can accept two values, for example, 2 and 4, but not all values between 2 and 4. If y is a function of x, then y can be defined for the x = 2 and x = 4 and not for the whole interval (2, 4). Let's take an example where we are researching a family's finances and we gather data at regular intervals, like once a year. Consequently, we get F as a function of time t, that takes values 0, 1, 2,...., etc., if F = F(t), where F is the savings (Baumol,1974).

In such circumstances, too, we would be interested in examining changes in the dependent variable as a result of changes in the independent variable. Since the independent variable in this situation only takes discrete values rather than continuous ones, the derivative idea is no longer applicable. Finite difference is the study of relations that exist between the values assumed by a function if the independent variable varies by finite leaps. This paper examines dynamic economic issues of time variable of discrete values and also explores the key methods and applications of dynamic economic analysis using difference equations.

Preliminaries

Dynamic Economic Analysis

Dynamic economic analysis studies how economic variables evolve over time and how current economic decisions influence future outcomes. This approach contrasts with static analysis, which examines the economy at a single point in time or under equilibrium conditions. Dynamic analysis is crucial for understanding economic growth, business cycles, and the effects of policy changes (Chiang, 1984).

1. Intertemporal Choice

Definition: The study of how individuals allocate resources (e.g., consumption, savings) over different periods.

Utility Maximization: Individuals seek to maximize their lifetime utility, balancing current and future consumption.

Present Value: The concept that future cash flows are discounted to reflect their present value, influencing consumption and investment decisions.

2. Dynamic Optimization

Determine the optimal path of an economic variable (e.g., capital stock, consumption) over time. Used to find functions that optimize a given integral. Deals with finding control policies (e.g., savings rate) that maximize an objective function over time.

Dynamic Programming: Breaks down a dynamic optimization problem into simpler sub-problems, solved recursively.

3. Economic Growth Models (Dean,2020 & Ouliaris, 2011).

Solow Growth Model: A foundational model examining economic growth for long time driven by capital formulation, and labor.

Endogenous Growth Models: Extend the Solow model by incorporating factors that determine technological progress and innovation within the model.

4. Overlapping Generations Models (OLG)

Structure: Considers multiple generations of agents coexisting at any point in time.

Applications: Analyzes issues like social security, public debt, and intergenerational transfers.

5. Trade Cycle Models

Real Trade Cycle (RTC) Theory: Examines how real shocks (e.g., technology changes) drive economic fluctuations.

New Keynesian Models: Incorporate price and wage stickiness to explain short-term economic fluctuations and the impact of monetary policy.

6. Stochastic Processes and Expectations

Uncertainty: Economic agents make decisions under uncertainty, incorporating expectations about future events.

Stochastic Processes: Used to model random variables that evolve over time, such as stock prices or interest rates.

Rational Expectations: The hypothesis that agents' expectations are consistent with the actual economic model governing the economy.

7. Dynamic General Equilibrium

Definition: A framework where all markets (goods, labor, capital) are in equilibrium over time, and agents optimize their behavior dynamically.

Computable General Equilibrium (CGE) Models: Use numerical methods to solve and simulate dynamic general equilibrium models.

Dynamic economic analysis provides a comprehensive framework for understanding the temporal aspects of economic phenomena, allowing economists to make more accurate predictions and informed policy recommendations.

Finite Difference (Kumar, 2022)

Suppose t and y represent the independent and dependent variables respectively. Suppose y (1) and y (t +1) be the dependent variable over time t and t + 1. Then the difference in time period (t + 1) – (t) is known as the finite difference y(t + 1) – y(t) and is represented by Δ y(t) (Bhusal,2022,& Bajracharya 2078).

$$\therefore \quad \Delta y(t) = y(t+1) - y(t) \text{ is finite difference operator.}$$

For convenient, we write y_1 for y (t) and y_{1+1} for y (t + 1), then

$$\Delta y_t = y_{t+1} - y_1$$

where Δ is said to be the difference operator.

Similarly, the second difference is denoted by $\Delta^2 y_t$ and defined by

$$\begin{aligned}
\Delta^2 y_t &= \Delta(\Delta y_t) = \Delta(y_{t+1} - y_t) \\
&= \Delta y_{t+1} - \Delta y_t \\
&= (y_{t+2} - y_{t+1}) - (y_{t+1} - y_t) \\
&= y_{t+2} - y_{t+1} - y_{t+1} + y_t \\
&= y_{t+2} - 2y_{t+1} + y_t
\end{aligned}$$

Note: y_{1-1} is the value of y over time y – 1 i.e. the value of y, one period before the time t.

Example: Find the first and 2nd difference of the following functions.

$y_t = t^2$, where $t = 2$

Solution:

(a) We know the first difference of y is

$$\Delta y_t = y_{t+1} - y_t$$

When $t = 2$

Then $\Delta y_t = y_3 - y_2 = (3)^2 - (2)^2 = 9 - 4 = 5$

and the second differences is

$$\begin{aligned}
\Delta^2 y_t &= y_{t+2} - 2y_{t+1} + y_t \\
&= y_4 - 2y_3 + y_2 \\
&= (4)^2 - 2 \cdot (3)^2 + (2)^2 \\
&= 16 - 18 + 4 = 2
\end{aligned}$$

Difference equation

Change in the value of some economic variables occurs due to the change in time, which may be discrete or continuous. A difference equation relates the difference between successive values of discrete-time function. It is often used to model the behavior of sequences and discrete dynamical systems. Difference equations can be seen as the discrete analogs of differential equations, which are used for continuous-time systems.

A difference equation involves independent variable, dependent variable and with their successive differences. In the case of differential equation, the pattern of change of a variable y is expressed by derivatives such as $\frac{dy}{dt}$ or $y'(t), \frac{d^2y}{dt^2}$ or $y''(t)$ etc., where t is considered as continuous variable. However, if it is considered as the discrete variable, then the pattern of change of a variable y must be described by time t called "difference" rather than by derivatives or differentials of $y(t)$. If t and y are the independent variable and dependent variable respectively then equation relating y_1 and y_{t+1} is known as the difference equation. For example:

$y_{t+1} = 2y_t$ and $y_{t+1} = 2y_t = 3$

The difference's order is the largest difference that the equation contains. As for example:

$\Delta\ y_t = 4$ is first order difference equation.

Here $\Delta\ y_t = 4$

$\Rightarrow\ y_{t+1}\ \ y_t = 4$

The order is one, because the highest difference

= t + 1 – t = 1

Also, $y_t - y_{t-1} - 0$ is also of order one because highest difference = t - (t -1) = 1 .

It is said to be homogeneous if there is no separate constant term. For example,

$y_{t+1} = ay_t$ is first order homogenous difference equation.

A difference equation of separate constant term is known as non- homogeneous difference equation. For example;

$$y_{t+1} = ay_t = b \tag{1}$$

Main Discussion

Difference Equation in Economics

Suppose the national income y of a specific nation has been rising at a consistent rate g over a ten-year period beginning from base year. This will give us an understanding of how different equations may arise in economics. At every given time t, the rate of growth of y may be expressed as

$$\frac{Y_t - Y_{t-1}}{Y_{t-1}}$$

The ratio of income y_t–1 of the previous period gives the current rate of growth. Rates of growth of income, g, is constant over ten year's intervals. This is the rate of growth of y at a particular point in time.

$\frac{Y_t - Y_{t-1}}{Y_{t-1}}$ = f, t = 1,2,3,-----------10.

Or, y_t = (1+g) y_{t-1} , t = 1,2,-------10. (2)

The values of the variable y at two different times, t and (t – 1), are described by equation (2). It is a difference equation example. The values of the linked variable (y_t and y_{t-1}) are one period behind. Thus, it exemplifies a difference equation of the first order. The maximum number of periods lag determines the order of a difference equation (Beran et.al.2013, & Baillie, 1996).

y_{t-2} - $2y_{t-3}$ = 0 is of order one.

y_t = a(y_{t-1} - y_{t-2}) is of order two. Similarly, the nth order linear constant coefficient differences equation is y = $a_1 y_{t-1}$ +$a_2 y_{t-2}$ +-------+ $a_n y_{t-n}$ + b (3)

Here, a_1, a_2, -----------a_n and b are constants.

It is linear as there are no product terms, a_1, a_2, -----------a_n and b are constants that do not vary with t, and the dependent variable, y, is not raised to any power. If b = 0, this equation will be homogeneous. It is non-homogenous if b ≠ 0. Only the work with difference equations of this particular type of orders one is included in this study (Kalecki, 1935).

Solution of a Difference Equation (Palma, et.al. 2007 & Robinson,2003)

Its solution is a function which satisfies the given equation.

Its solution containing arbitrary constants equal in number to the order of the difference equation is the general solution. But solution got by assigning the particular values is known as particular solution. First order difference equation is solved from following two methods:

1. Iteration method

A first order difference equation describes the pattern of change of y between two consecutive periods only and we may easily use the process of iteration (i.e. a repeated application of the difference equation) to obtain the solution of the equation. There is no problem of finding y_1 from the equation of y_0 is known. Once y_1 is found y_2 can be obtained. Repeating the process, we can find y_3, y_4 etc.

In this method, we use the process of iteration i.e. repeated application of finite difference to get the solution of the difference equation.

Example 1: Solve the difference equation by iteration method

$\Delta y_t = -o.5\, y_t$

$\Rightarrow \; y_{t+1} - y_t = -0.5 y_t$

$\Rightarrow \; y_{t+1} = 0.5 y_t$

$\Rightarrow \; y_t = 0.5 y_{t-1}$ (moving 1 period back)

Put t = 1,2, 3, 4 ,

$$y_1 = 0.05 y_0$$

$$y_2 = 0.5 y_1 = 0.5 \; (0.5\, y_0) = (0.5)^2\, y_0$$

$$y_3 = 0.5 y_2 = (0.5)((0.5)^2 y_0) = (0.5)^2 y_0$$

In the similar manner,

$$y_1 = (0.5)^t y_0$$

$y_t = (0.5)^t y_0$ where y_0 = A

2. General Method

First Order Homogeneous Difference Equation

Suppose first order homogeneous difference equation with constant coefficient is defined as

$$y_{t+1} - a y_1 = 0 \tag{4}$$

Suppose y_t = A B^t is the solution of equation (4) then y_{t+1} = A B^{t+1}

Putting these values in equation (i), then y_{t+1} = A B^{t+1}, putting these values in (i) then

$$A\, B^{t+1} - a.AB^t = 0 = A\, B^t\, (B\text{-}a) = 0$$

A $B^t \neq 0$. The general solution of equation (4) is y_t = A a^t

Now for first order non-homogeneous difference equation, let y_{t+1} = ay_t +b be the equation of that kind.

Iteration method

y_{t+1} = ay_t +b is non-homogeneous equation.

y_t = ay_{t-1} + b (after one period back)

= a (ay_{t-2}+b) +b

a^2y_{t-2} + ab +b

a^2y_{t-2} + (1+ a) b

$a^2(y_{t-3}$ + b) + ab + b

= $a^3y_{t-3} + a^2$b+ ab+b

= a^3y_{t-3} + (1+ a + a^2) b

Proceeding in the same way

$y_1 = a^t y_0 +$ (1 + a + a^2 + ……. + a^{1-1}) b

= A a^t + $\frac{1-a^t}{1-a}$ b; (a ≠ 1) where y_0 = A

If a = 1,

y_1 = A + (1` + 1 + 1 + …… t terms) b.

= A + b t

General method (Teyssiere, et.al. 2007 & Tarasova, 2016)

The general solution of the non-homogeneous first order difference equation is defined by

$$y_{t+1} = ay_t$$

∴ C.F. = Aa^t

P.I. is the particular integral is the solution of the non- homogeneous difference equation

$y_{1+1} = ay_1$ + b (5)

The sum of the complementary function (C.F.) and the particular integral (P. I.) is the general solution as in case of differential equation.

$$y_{t+1} = ay_t$$

∴ CF = Aa^t

P. I. is the particular integral of the non- homogeneous difference equation

For particular integrals, let $y_t = $ k be trial solution of (4) (a$\neq 1$) Then y_{t+1} = k

Now the equation (4) reduces to

K = a k + b

⇒ (1 – a) k = b

∴ k = $\frac{b}{1-a}$

∴ p . 1. = $\frac{b}{1-a}$

∴ the complete solution is

y_1 = CF + pl

= $Aa^t + \frac{b}{1-a}$

If a = 1 , let y_t = kt be the trial solution,

Then y_{t-1} = k(t + 1)

Now from the given non-homogeneous difference equation,

$$y_{t+1} = y_t + b \qquad (\therefore a = 1)$$

⇒ k (t + 1) = kt + b

⇒ k = b

∴ the particular integral (P. l.) = kt = bt

∴ the complete solution is y_1 = A + bt $\quad (\therefore a = 1)$

Thus, general solution of the non- homogeneous linear difference equation $y_{t+1} = ay_t +$ b is

$$= \begin{cases} Aa^t + \frac{b}{1-a} & \text{for a } \neq 1 \\ A + bt & for\ a\ = 1 \end{cases}$$

where A is a constant.

The solution gives the time path of y_r If $= Aa^t$ = 0, then = $y_1 = \frac{b}{1-a}$ is a horizontal line parallel to t – axis known as the equilibrium level of stability.

The time path shown by y_1 depends upon the value of a ranging to (- ∞ < a < ∞) .

First, we find the time path determined by the solution y_t = Aa^t. Then by the rule of translation, we find the time path of y_t = = $Aa^t + \frac{b}{1-a}$.

As it assists in resolving in most common linear difference equation of order one, this approach is known as the general technique. Furthermore, higher order linear difference equations may also be solved using this approach (Tarasov, et.al. 2018).

Example 2. The general and particular solution from iteration method and general method of

$$\Delta\, y_t = 0.2\, y_t \text{ , where } y_0 = 5.$$

Solution: The given difference equation is $\Delta\, y_t$ = 0.2 y_t

$$\Rightarrow y_{t+1} - y_t = 0{,}2\, y_t$$

$$\Rightarrow y_{t+1} - y_t = 1.2\, y_t$$

$$\Rightarrow y_t = 1{,}2\, y_{t-1} \quad \text{(moving one period back)}$$

$$= 1.2\ (1.2\, y_{t-2}) = (1.2)^2\, y_{t-2}$$

$$= (1.2)^2\ (1{,}2\, y_{t-2}) = (1.2)^3\, y_{t-3}$$

After proceeding in the same way

y_t = $(1.2)^t\ y_0$ is the general solution. When t = 0, $y_t = y_0 = 5$

Or, 5 = 1×A , A = 5.

The particular solution is y_t = 5 $(1.2)^t$

General method:

$$y_t = 1{,}2\, y_{t-1} \tag{6}$$

Suppose y_t = A B^t is general solution of difference equation (6).

y_{t-1} = A B^{t-1} is substituted in equation (i).

A B^t = 1.2 AB^{t-1}

A $B^{t-1} \neq 0$, and (B-1.2) = 0 gives B = 1.2

The general solution is

y_t = A B^t = A $(1.2)^t$

When t =0, y_0 = A.1 $\Longrightarrow$ A = 5.

The particular solution is y_t = 5 $(1.2)^t$

Application in Economics

In economics, difference equations are used extensively. The difference equations are particularly useful in dynamic analysis to identifying a time route given the pattern of change of a variable y over time variable t and the time variable t can only take discrete values. We consider a few situations in economics where the issue may be transformed into a difference equation.

Nowadays, motion and change are modeled using difference equations in every branch of research. With the widespread availability of computers, the theory of difference equations has emerged as a crucial tool for economic analysis. If basic ideas (such bifurcations and chaos) and conclusions of modern theory of differential equations are not understood. It is frequently using in cobweb model, Solow Growth Model to analyzing the economic behavior of supply and demand towards the equilibrium level (Chiang,1984).

1. Cobweb Model

The basic Cobweb model assumes that today's demand for any commodity is a function of the present price (P_t), it means Q_d is an unlagged function of price, while today's supply depends upon yesterday's decision about the output, it means $\mathrm{Q_s}$ is a lagged function of price. Hence output is naturally influenced by yesterday's price (P_{t-1}).

$$\therefore\ \mathrm{Q_{st}} = \mathrm{S}(\mathrm{P_{t-1}})$$
$$\mathrm{Q_{dt}} = \mathrm{D}(\mathrm{P_t})$$

The cobweb model is a classic application of difference equations in economics. It describes the dynamics of price and quantity in markets where supply and demand adjust over time. The model assumes that suppliers base their production decisions on past prices.

The model is based on the following three equations

$Q_{dt} = a - \ b\ P_t$ (a, b > 0) (7)

$Q_{st} = c - \ b\, P_{t-1}$ (c, d > 0) (8)

and $Q_{dt} = Q_{st}$ (9)

Using (7) and (8) in (9),

a- b $P_{\ t} = -c + d\, P_{t-1}$

$\Rightarrow$b $P_{\ t} + d\, P_{t-1}$ = a + c

$\Rightarrow \ p_t + \frac{d}{b} p_{t-1} = \frac{a+c}{b}$

(10)

Which in the form $y_1 + ay_{t-1} = b.$

Its solution is the C. F. + P. I,

For C. F.

The corresponding homogeneous difference equation is

$p_t + \frac{d}{b} p_{t-1} = 0$

(11)

C.F. = A $\left(-\frac{d}{b}\right)^t$

For P. l.

Let the trial solution of the P.I. be $p_t = K$ then

$p_{t-1} = \text{K}.$

Now, for equation (11)

$\text{K} + \frac{d}{b}\,\text{k} = \frac{a+c}{b}$

$\Rightarrow \ \left(\frac{b+d}{b}\right) \text{k} = \frac{a+c}{b}$

$\therefore \ \ \text{k} = \frac{a+b}{b+d}$

$\therefore$the general solution is

$p_t \ = \text{A} \left(-\frac{d}{b}\right)^t + \ \frac{a+c}{b+d}$

When t = 0, $p_t = p(\,0),$ then

$$p_0 = \mathrm{A} + \frac{a+c}{b+d}$$

$$\Rightarrow \quad \mathrm{A} = \quad p_0 - \frac{a+c}{b+d}$$

$$\therefore \; p_t = \left(p_0 - \frac{a+c}{b+d}\right) \left(-\frac{d}{b}\right)^t + \frac{a+c}{b+d}$$

If $\bar{P}$ be the equilibrium price for all time t, then

$$p_t = p_{t-1} = \bar{P}$$

Now making (7) and (8) equal,

$$a - b\bar{P} = - c + d\bar{P}$$

$$\Rightarrow (b+d)\bar{P} = a + c$$

$$\therefore \; \bar{P} = \frac{a+c}{b+d}$$

Now, the required solution is

$$p_t = (p_0 - \bar{P}) \left(-\frac{d}{b}\right)^t + \bar{P}$$

2. Solow Growth Model

The Solow Growth Model, also known as the Solow-Swan Model, is a neoclassical model of economic growth that analyzes changes in the economy over time, particularly focusing on the long-run determinants of economic growth. Developed by Robert Solow in 1956, it provides a framework to understand how various factors contribute to economic growth and the steady state of an economy.

The Solow growth model uses difference equations to describe the evolution f capital stock over time. The discrete-time version of the Solow model is given by:

Kt+1=sYt−δKt+Kt

where:

Kt is the capital stock at time t.

S is the savings rate,

Yt is the output at time t

δ is the depreciation rate.

3. Lagged Keynesian Macroeconomic Model

A Lagged Keynesian Macroeconomic Model typically incorporates time lags into the standard Keynesian framework. In macroeconomic theory, the Keynesian model emphasizes the role of aggregate demand in influencing economic output and employment, particularly in the short run. By introducing lags, the model acknowledges that economic agents (such as households, firms, and the government) do not adjust their behavior instantaneously in response to changes in economic conditions. The Lagged Keynesian Macroeconomic Model provides a more realistic representation of the economy by incorporating time lags into the analysis. It allows economists and policymakers to better understand the delayed effects of economic policies and the inertia present in the behavior of economic agents (Tarasova, et.al. 2016).

The income determination model also known as Lagged Keynesian macro-economic model and is assumed to be based on the factors: consumption c_1 and the investment. I_t. That is the national income is equal to the sum of the consumption and investment. Also, it is assumed that consumption is a lagged function of income i.e. a function of income of previous period y_{t-1} or it is simply assumed that I_1 is constant and is denoted by L_0.

Now the model has the following three equations

$$y_t = c_t + I_t \tag{12}$$

$$c_t = by_{t-1} + I_t \tag{13}$$

and $I_t = I_0$ (14)

where b is the marginal propensity to consume and $0 < b < 1.$

Using (13) and (14) in, we have

$$y_1 = by_{t-1} + I_0$$

$$\Rightarrow \quad y_1 = by_{t-1} = I_0$$

Which is a non-homogeneous first order linear difference equation. Hence its solution is equal to the sum of the sum of the C.F. = A $(b)^t$

Particular integral:

Let y_t = k be the trial solution.

Then, y_{t-1} = k. So, from equation (14) k- bk = I_0

$\Rightarrow$ (1- b) k = I_0

$\therefore$ k = $\frac{I_0}{1-b}$

Now the general solution of Keynesian economic model is

$$y_t = \text{A } (b)^t + \frac{I_0}{1-b}$$

When t = 0, $y_t = y_{0,}$, then

$$y_0 = \text{ A } + \frac{I_0}{1-b}$$

$$\therefore \text{ A} = y_0 - \frac{I_0}{1-b}$$

$\therefore$ then general solution is

$$y_t = \left(y_0 - \frac{I_0}{1-b}\right) b^t + \frac{I_0}{1-b}$$

This gives the time path of the general equilibrium income determination model.

Note: If c_t be defined by c_t = a + by_{t-1} where a is the fixed consumption, that the solution lakes the form

$$y_t = \left(y_0 - \frac{I_0+a}{1-b}\right) b^t + \frac{I_0+a}{1-b}$$

Since, b < 1, so $b^t \to \infty$. Hence this model provides y_t always tending to the equilibrium level $\frac{I_)}{1-b}$. This is the main feature of this model which is different from Cobweb model.

Conclusion

Dynamic economic analysis using difference equations provides a robust framework for understanding the temporal evolution of economic variables. From simple linear models to complex nonlinear systems, difference equations help economists capture and analyze the rich dynamics of economic systems. Whether examining stability, predicting cyclical behavior, or exploring the implications of chaos, difference equations are indispensable tools in the economist's toolkit.

As economic systems become increasingly complex, the use of advanced techniques, such as bifurcation analysis and chaos theory, will continue to grow, offering deeper insights into the often-unpredictable world of economic dynamics. From market dynamics in the cobweb model to capital accumulation in the Solow growth model, difference equations help economists analyze the behavior of economic systems and predict future trends.

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